Schwarzschild-de Sitter metric type solutions of a Nonlocal de Sitter Gravity Model

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Nonlocal Modification of GR

Nonlocal square root gravity model

$$S = \frac{1}{16\pi G} \int_M \sqrt{R - 2\Lambda}\, F(\Box) \sqrt{R - 2\Lambda} \sqrt{-g}\, d^4x,$$

where $F(\Box) = 1 + \mathcal{F}(\Box) = 1 + \sum_{n=1}^{\infty} f_n \Box^n + \sum_{n=1}^{\infty} f_{-n} \Box^{-n}$.

- Construction

$$\begin{aligned} R - 2\Lambda = \sqrt{R - 2\Lambda}\sqrt{R - 2\Lambda} &\to \sqrt{R - 2\Lambda}\, F(\Box) \sqrt{R - 2\Lambda} \\ &= R - 2\Lambda + \sqrt{R - 2\Lambda}\, \mathcal{F}(\Box) \sqrt{R - 2\Lambda} \end{aligned}$$

Equations of motion

Equation of motion are

$$-\frac{1}{2}g_{\mu\nu}\sqrt{R-2\Lambda}\mathcal{F}(\Box)\sqrt{R-2\Lambda}+R_{\mu\nu}W-K_{\mu\nu}W+\frac{1}{2}\Omega_{\mu\nu}=-(G_{\mu\nu}+\Lambda g_{\mu\nu})$$

$$\Omega_{\mu\nu}=\sum_{n=1}^{\infty}f_n\sum_{l=0}^{n-1}S_{\mu\nu}(\Box^l\sqrt{R-2\Lambda},\Box^{n-1-l}\sqrt{R-2\Lambda})$$
$$-\sum_{n=1}^{\infty}f_{-n}\sum_{l=0}^{n-1}S_{\mu\nu}(\Box^{-(l+1)}\sqrt{R-2\Lambda},\Box^{-(n-l)}\sqrt{R-2\Lambda}),$$

$$K_{\mu\nu}=\nabla_\mu\nabla_\nu-g_{\mu\nu}\Box,$$

$$S_{\mu\nu}(A,B)=g_{\mu\nu}\nabla^\alpha A\nabla_\alpha B-2\nabla_\mu A\nabla_\nu B+g_{\mu\nu}A\Box B,$$

$$W=\frac{1}{\sqrt{R-2\Lambda}}\mathcal{F}(\Box)\sqrt{R-2\Lambda}.$$

Eigenvalue problem

If we assume $\Box\sqrt{R-2\Lambda} = p\sqrt{R-2\Lambda}$, EOM are simplified to

$$W = \mathcal{F}(p)$$

$$\Omega_{\mu\nu} = \mathcal{F}'(p) S_{\mu\nu}(\sqrt{R-2\Lambda}, \sqrt{R-2\Lambda}),$$

$$(G_{\mu\nu} + \Lambda g_{\mu\nu})\,(1+\mathcal{F}(q)) + \frac{1}{2}\mathcal{F}'(q) S_{\mu\nu}(\sqrt{R-2\Lambda}, \sqrt{R-2\Lambda}) = 0.$$

It is evident that EOM are satisfied if $\mathcal{F}(p) = -1$ and $\mathcal{F}'(p) = 0$.

Schwarzschild-de Sitter-type metric

We want to investigate our model outside the spherically symmetric massive body. Since this model is a nonlocal generalization of general relativity with the cosmological constant Λ, it is natural to consider a generalization of the Schwarzschild-de Sitter metric starting from the standard Schwarzschild expression

$$\mathrm{d}s^2 = -A(r)\mathrm{d}t^2 + \frac{1}{A(r)}\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\varphi^2.$$

The corresponding scalar curvature R of above metric is

$$R = \frac{2 - 2A(r) - 4rA'(r) - r^2A''(r)}{r^2} = \frac{1}{r^2}\frac{\partial^2}{\partial r^2}\left[r^2(1 - A(r))\right].$$

As we concluded earlir, to find a solution of EoM it is sufficient to solve an eigenvalue problem $\Box\sqrt{R-2\Lambda} = q\sqrt{R-2\Lambda}$. Note that here d'Alembertian $\Box$ acts in the following way:

$$\Box u(r) = A(r)\, u''(r) + (A'(r) + \frac{2}{r}\, A(r))\, u'(r) = \frac{1}{r^2}\frac{\partial}{\partial r}\big[r^2 A(r)\frac{\partial u}{\partial r}\big],$$

where $u(r)$ is any differentiable scalar function.

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where $u(r)$ is any differentiable scalar function. Let us now consider function $A(r)$ in the form

$$A(r) = 1 - \frac{\mu}{r} - \frac{\nu}{r^2} - \frac{\Lambda}{6} r^2 - f(r),$$

where μ and ν are some parameters to be discussed later. Then one can show that for our choice of $A(r)$ holds

$$R(r) = \frac{1}{r^2}\frac{\partial^2}{\partial r^2}\Big[r^2\big(1 - A(r)\big)\Big] = 2\Lambda + \frac{1}{r^2}\frac{\partial^2}{\partial r^2}\Big[r^2 f(r)\Big].$$

Denoting $u(r)=\sqrt{R-2\Lambda}$ and using previous results we obtain

$$\Box\sqrt{R-2\Lambda}=\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2A(r)\frac{\partial}{\partial r}\sqrt{R-2\Lambda}\right]=q\sqrt{R-2\Lambda}.$$

To get an approximative solution we take $A(r)\approx 1$, what is applicable when

$$\left|\frac{\mu}{r}\right|\ll 1,\quad \left|\frac{\nu}{r^2}\right|\ll 1,\quad |\Lambda r^2|\ll 1,\quad |f(r)|\ll 1,$$

Under these conditions $\Box$ operator becomes a Laplacian $\triangle$

$$\triangle\sqrt{R-2\Lambda} = \frac{1}{r^2}\frac{\partial}{\partial r}\Big[r^2\frac{\partial}{\partial r}\sqrt{R-2\Lambda}\Big] = q\,\sqrt{R-2\Lambda}.$$

One can easily find eigenfunctions of $\triangle$ operator

$$u(r) = \frac{C_1}{r}\ e^{\sqrt{q}\ r} + \frac{C_2}{r}\ e^{-\sqrt{q}\ r}.$$

The next step is looking for general solution of

$$R(r) - 2\Lambda = \frac{1}{r^2}\frac{\partial^2}{\partial r^2}\left[r^2 f(r)\right] = u^2(r),$$

which is

$$f(r) = -\frac{\Lambda r^2}{6} - \frac{C_1^2}{4qr^2}\ e^{2\sqrt{q}\ r} - \frac{C_2^2}{4qr^2}\ e^{-2\sqrt{q}\ r} - C_1 C_2 + \frac{C_3}{r} + \frac{C_4}{r^2}.$$

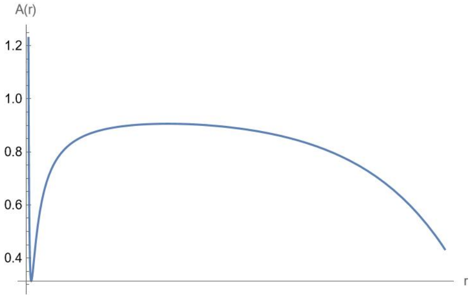
Then the value of $A(r)$ and $R(r)$ become

$$A(r) = 1 - \frac{\mu}{r} - \frac{\nu}{r^2} - \frac{\Lambda r^2}{6} - \frac{C_1^2}{4qr^2}\ e^{2\sqrt{q}\ r} - \frac{C_2^2}{4qr^2}\ e^{-2\sqrt{q}\ r} - C_1 C_2,$$

$$R(r) = 2\Lambda + \left(\frac{C_1}{r}\ e^{\sqrt{q}\ r} + \frac{C_2}{r}\ e^{-\sqrt{q}\ r}\right)^2.$$

$$A(r) = 1 - \frac{\mu}{r} - \frac{\nu}{r^2} - \frac{\Lambda r^2}{6} - \frac{C_1^2}{4qr^2}\ e^{2\sqrt{q}\ r} - \frac{C_2^2}{4qr^2}\ e^{-2\sqrt{q}\ r} - C_1 C_2$$

where q, C_1 and C_2 are free parameters and their values should be determined by observations.



Some relevant references

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THANK YOU FOR YOUR ATTENTION!