On kinetic contact forces on the balls of radial ball bearings

Katica (Stevanović) Hedrih

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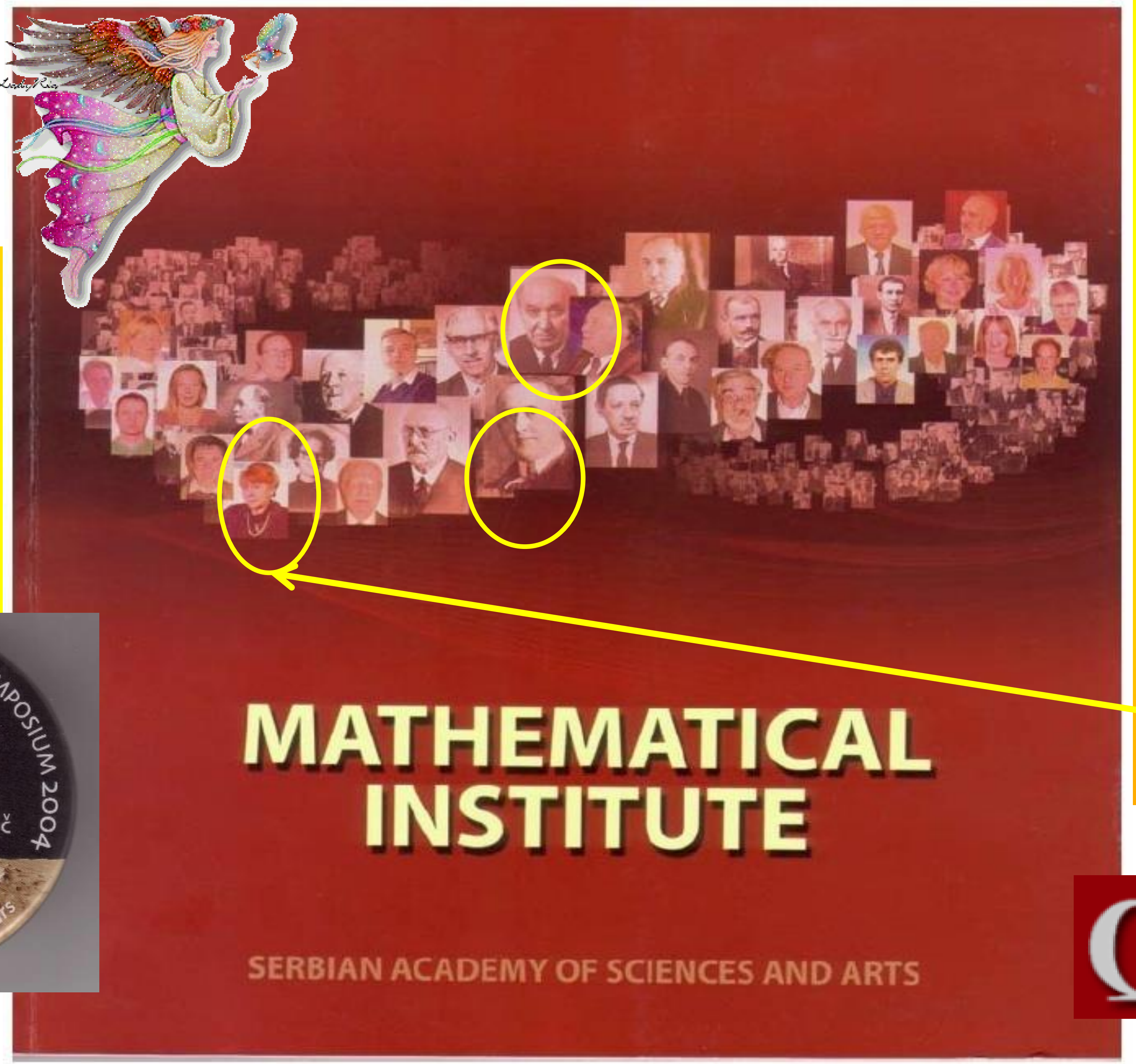

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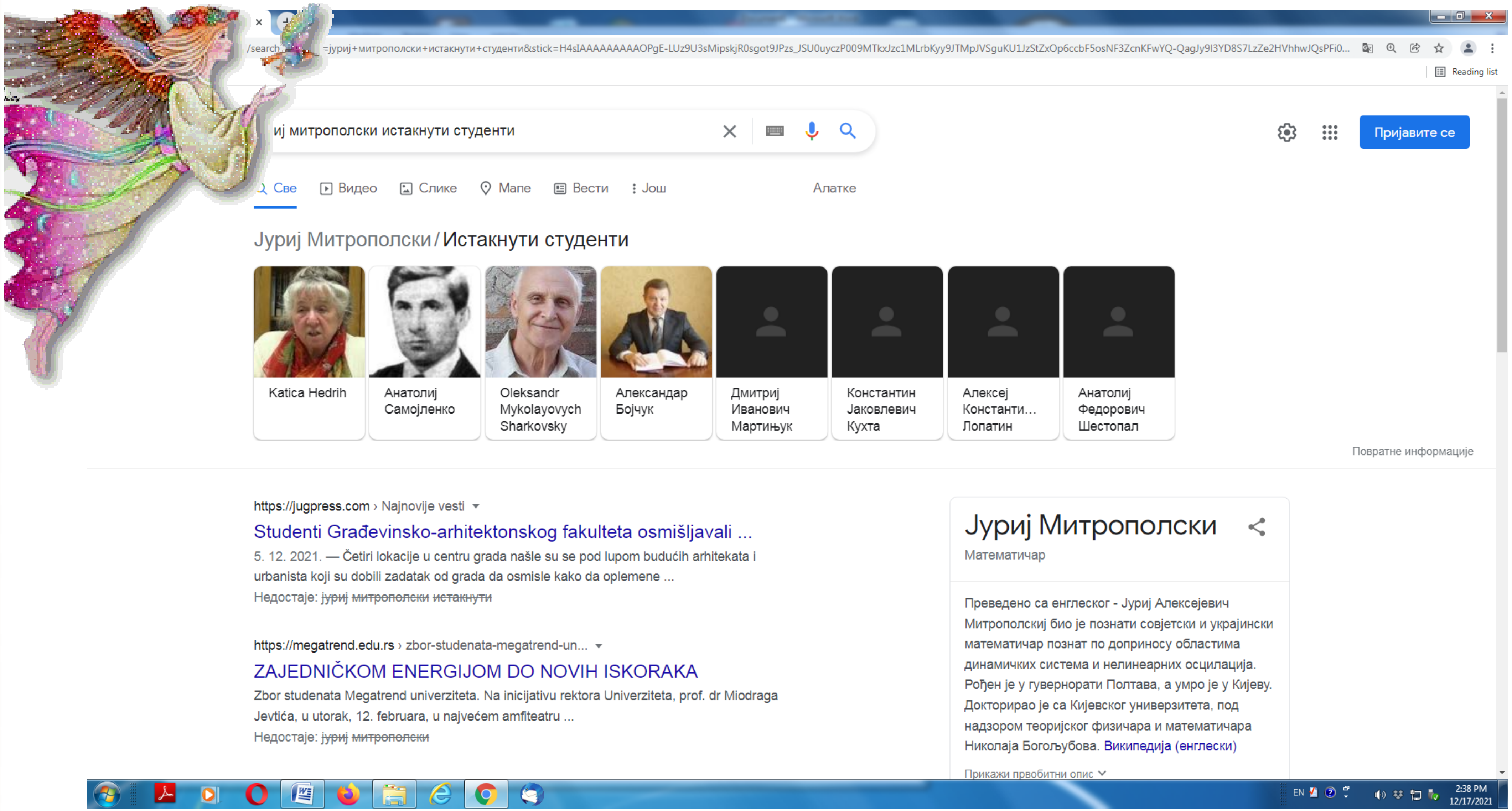


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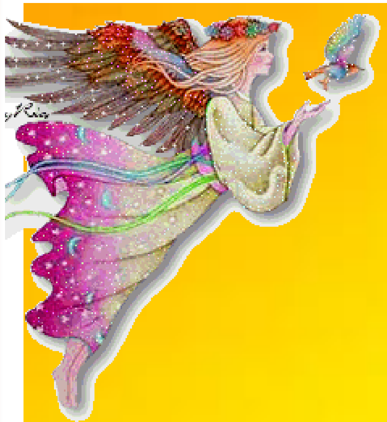

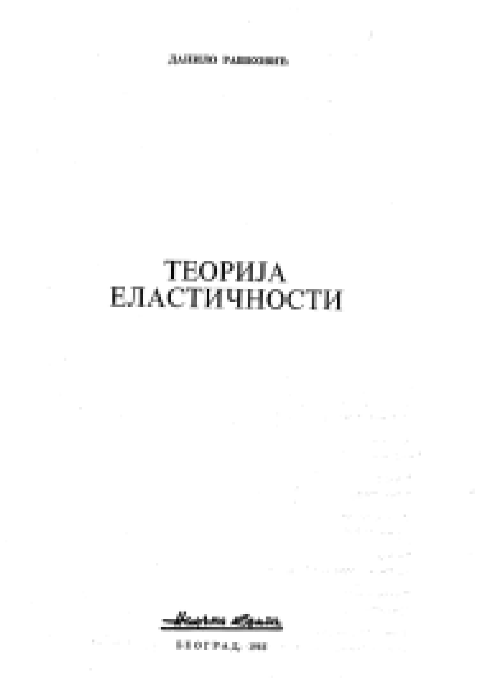
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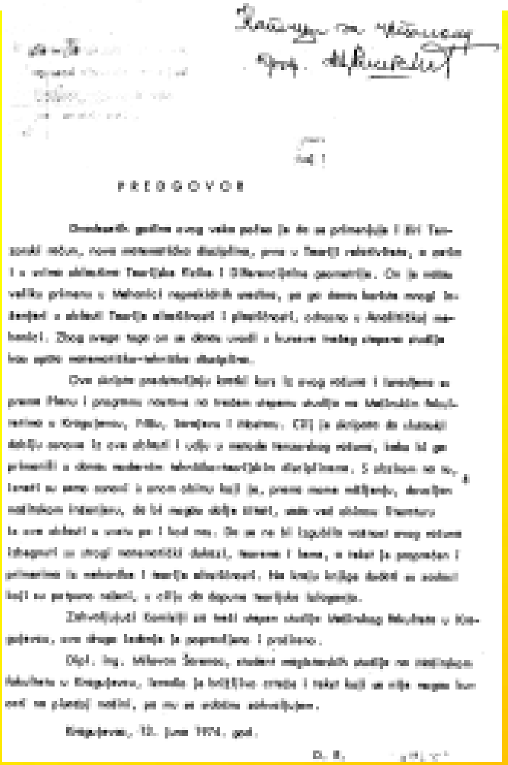
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Vladimir Mefodyevich
Matrosov

(08.05.1932-17.04.2011)

I COLLOQUIUM ANS BELGRADE, MAY 2005
Sreda, 11. maj 2005:

16:00-16:40, **Vladimir Matrosov** (predsednik ANN-Moskva, i akademik RAN),
O Evro-azijskoj strategiji stabilnog razvoja u 21. veku - nelinearna naucno - obrazovna istrazivanja.

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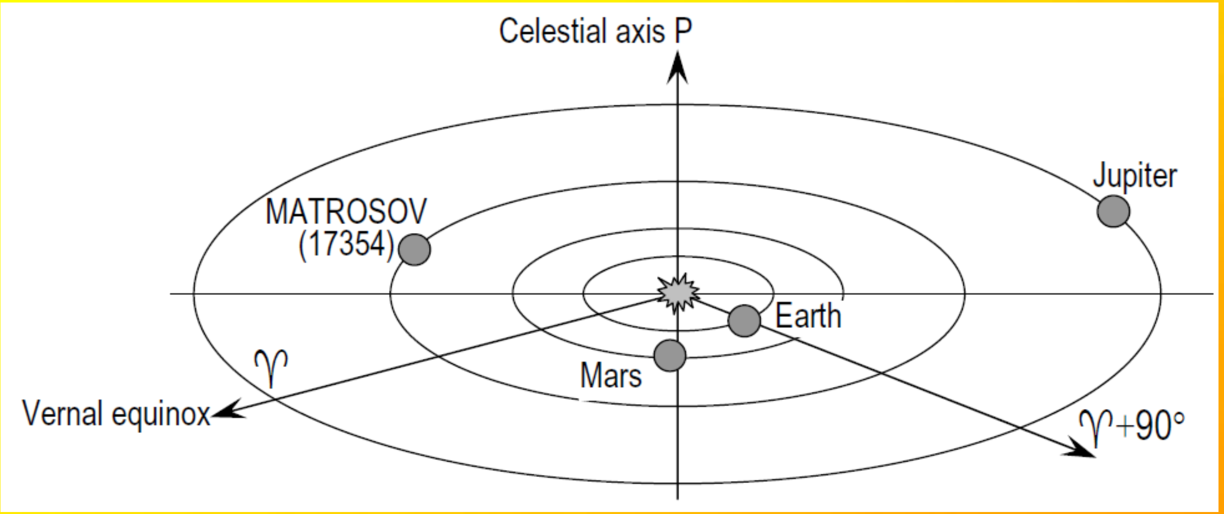

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dedicated to the 275th Anniversary of the Russian Academy of Sciences Saint-Petersburg, June 22-24, 1999.

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May 12-14, 2010,
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Participants of ICDVC-2010-
The Third International Conference on Dynamics, Vibration and Control,
12-14 May 2010, Hangzhou, China, Chinese Society of Theoretical and Applied Mechanics
http://saa.zju.edu.cn/icdvc2010

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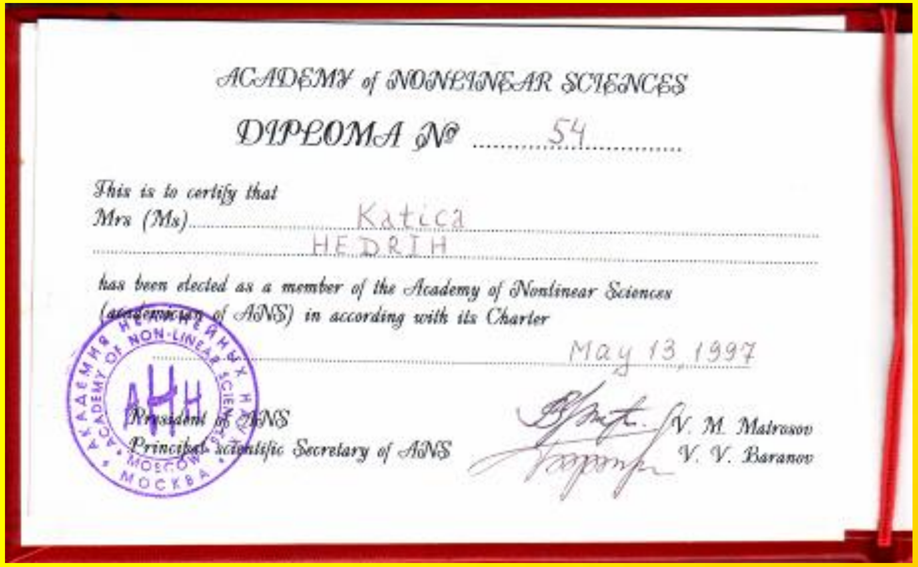

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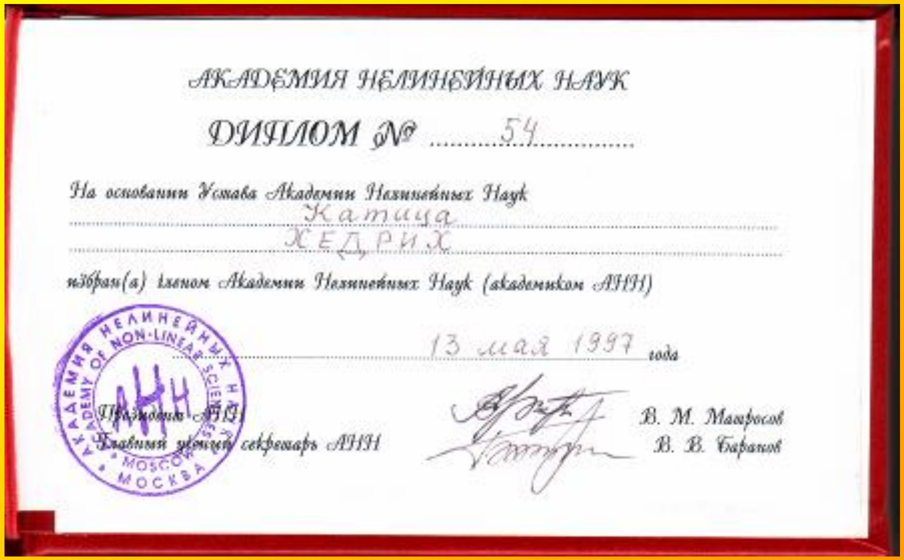

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Катица ХЕДРИХ

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13 мая 1997 года

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On kinetic contact forces on the balls of radial ball bearings

Katica (Stevanović) Hedrih

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ABSTRACT

In the paper, the kinetic forces on the balls of the radial ball bearings of the multi-stage gear reducers, i.e. the multiplier of the revolutions of the main shaft, were determined. The kinetic contact forces of balls and circular guides, stationary and moving radial ball bearing due to the occurrence of:

a* centrifugal forces of unbalanced gears fixed on the shafts

and

b* when eccentricity of the center of mass of a pair of balls with one diameter occurs in a radial ball bearing, due to the difference in their mass density, at equal radii. **of mass of the balanced part of the gear.**

The number of revolutions of the balls in rolling, without sliding, and the change of contact points in which kinetic contact forces occur, for one revolution of each of the shafts and reduced to the main shaft, were determined. It is determined for the cases of radial ball bearings with four pairs of balls and with six pairs of balls in radial ball bearings.

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The centrifugal force, which occurs due to the eccentricity of the corresponding material point, is equal to the product of the mass of the material point and its normal deflection due to the angular speed of rotation vratial,

$$\mathbf{F}_{c,m_{ik}} = -m_{ik} a_{N,m_{ik}} = -m_{ik} R_{ik} \omega_i^2 = -m_{ik} R_{ik} \dot{\vartheta}_i^2$$

It acts in the radial direction and rotates together with the shaft, with the angular velocity of the shaft.

Due to the appearance of these centrifugal forces, kinetic pressures appear on the radial balls of the shaft bearing. Those kinetic pressures on the radial ball bearings lead to the appearance of contact forces between the balls rolling on the stationary circular groove, on which they roll, and in the dynamical contact points of the movable circular groove, which rotates at the angular velocity of the shaft to which it is rigidly connected, assuming that the shaft rotates at a constant angular velocity.

Another source of centrifugal forces is the eccentricity of the center of mass of one or more pairs of balls in a radial ball bearing. The centripetal force of a pair of balls on one diameter is equal to the product of the sunn masses of the two balls and the normal acceleration of its center of mass rotating at the angular velocity of the shaft, assuming that the shaft rotates at a constant angular speed.

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We have assumed that all the balls of the radial ball bearing are all with the еьуал spherical contour surfaces of equal radii. But we also introduced the assumption that in one or more pairs of balls, there are balls with different mass densities. That difference in the mass densities of the balls in a pair on one diameter is the cause of the eccentricity of the center of mass of one pair on one diameter. Now we can write that the centrifugal force that occurs due to the eccentricity of the center mass of the ball pair on one diameter:

Keywords:

Nonlinear dynamics,
roller bearing balls,
phase trajectory portraits.
shaft rotation angular velocity,
statopnary circle path of rolling,
rotatimg circle path of vontact rolling.

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https://doi.org/10.1007/978-3-030-96964-6_15

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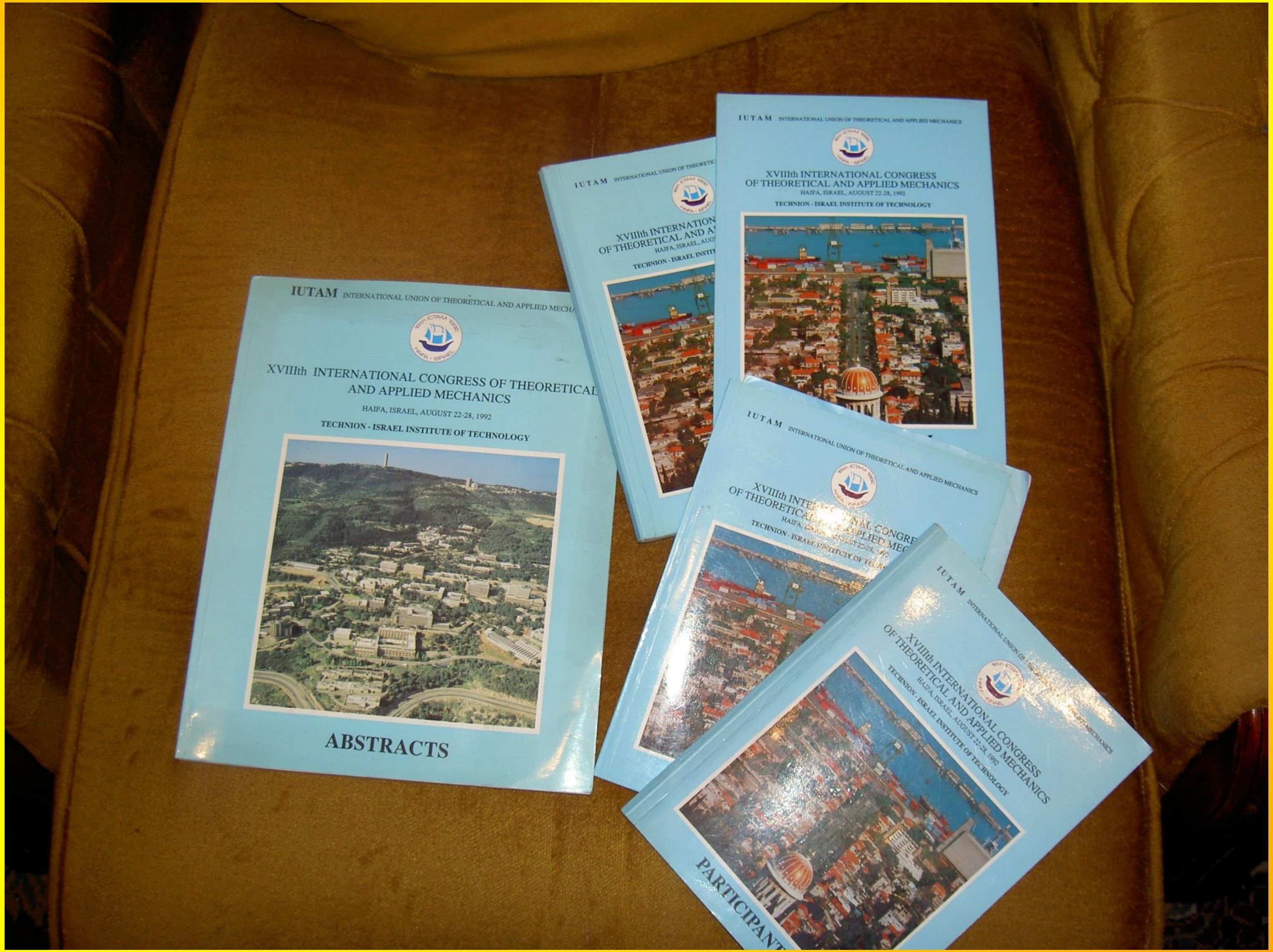

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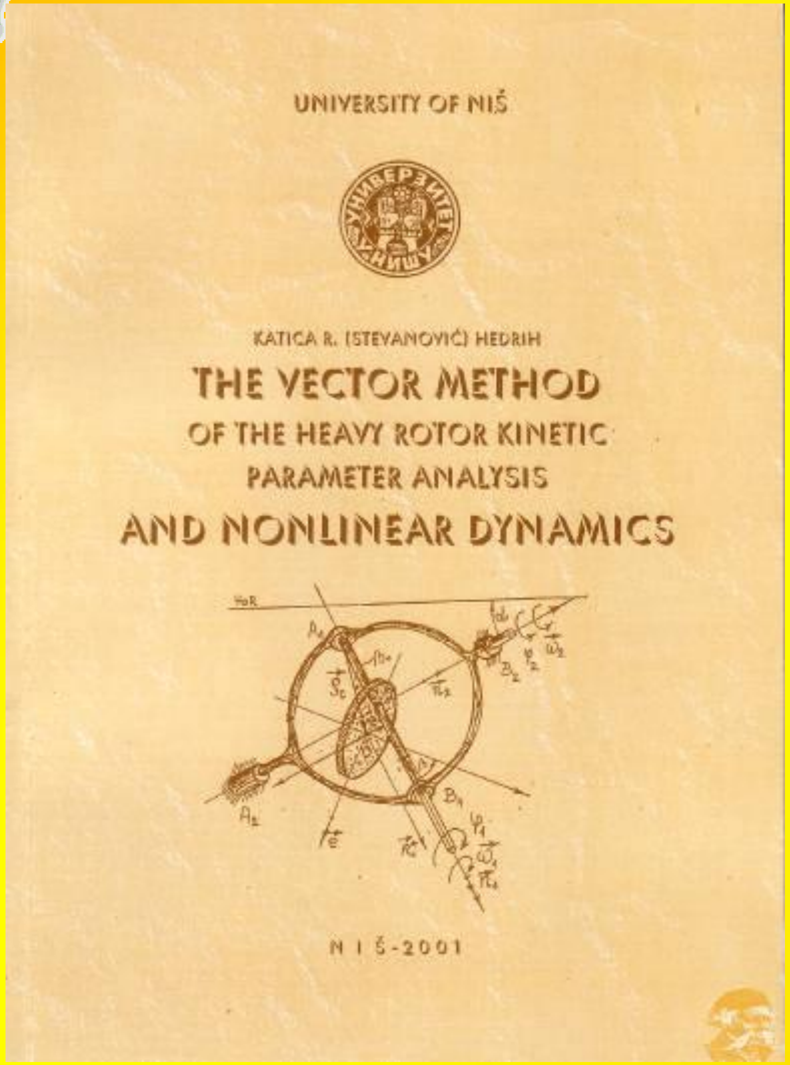

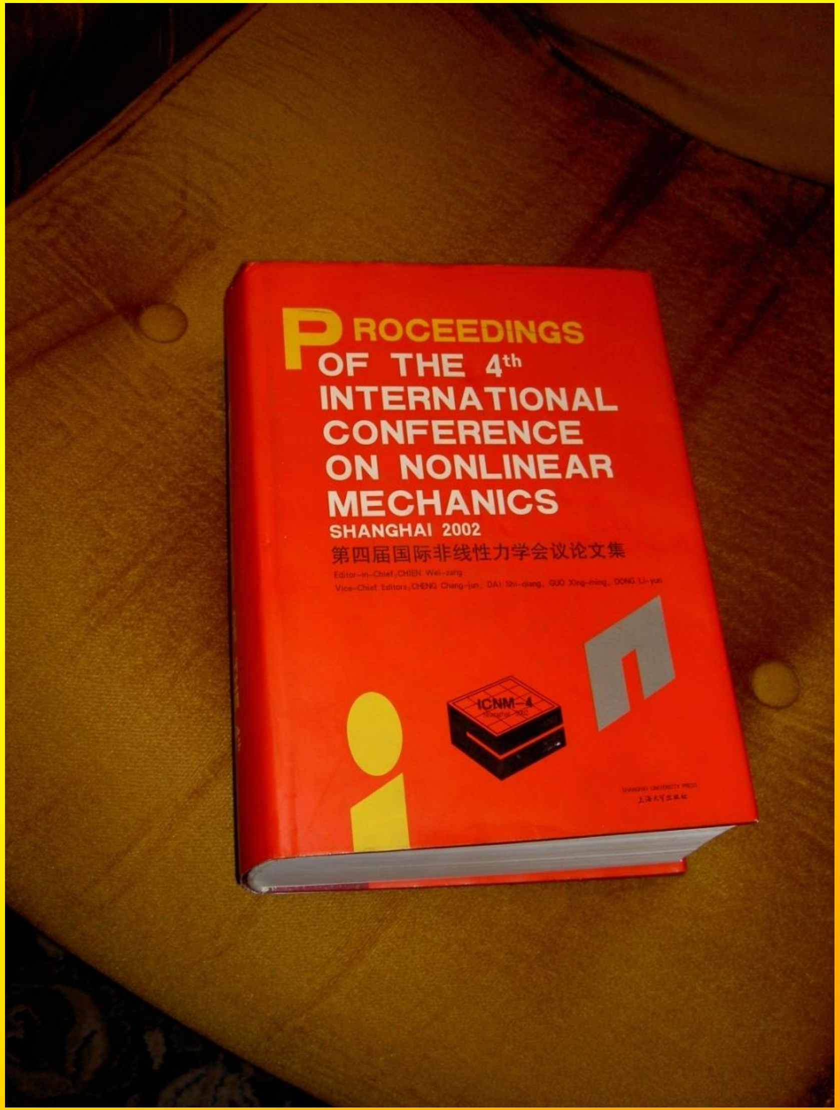

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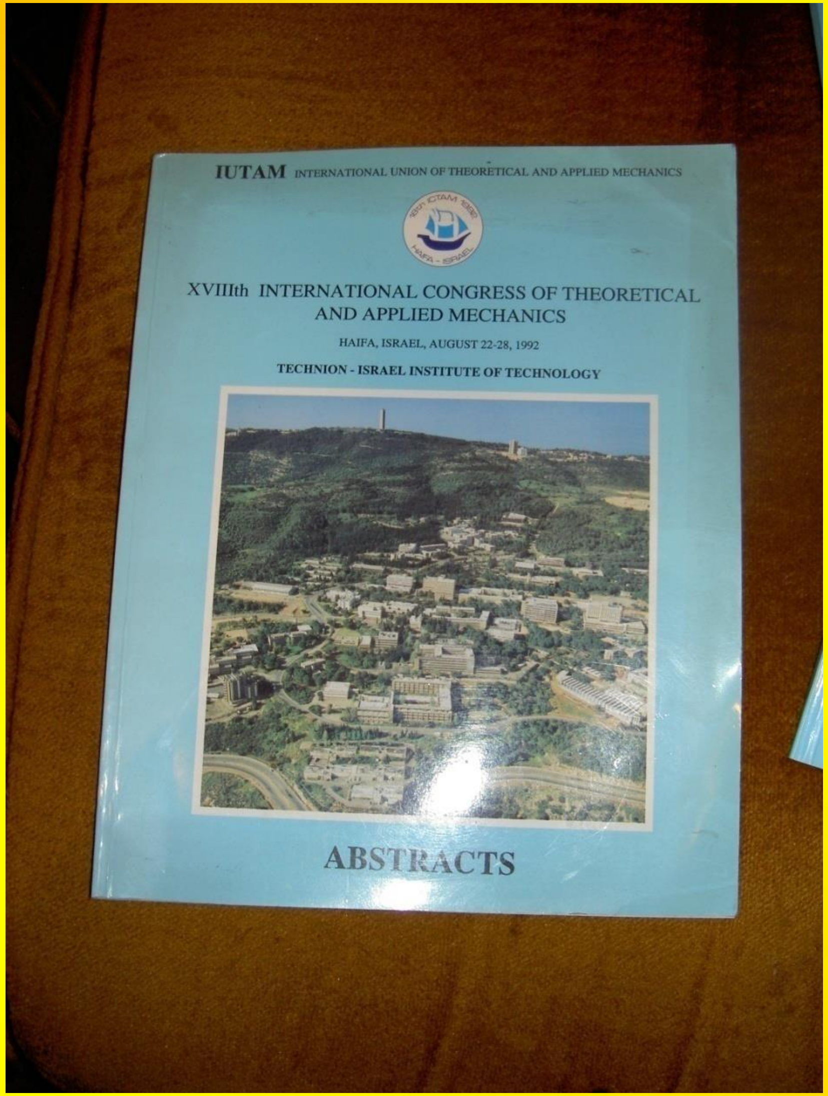
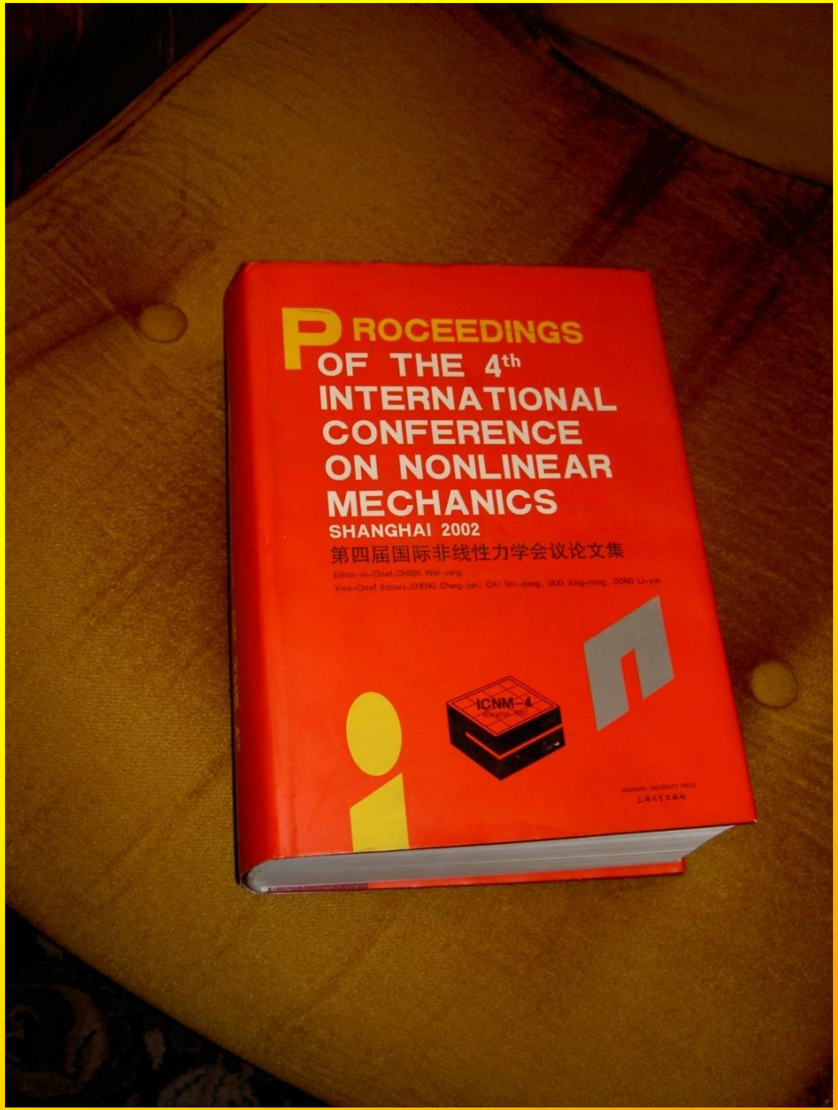
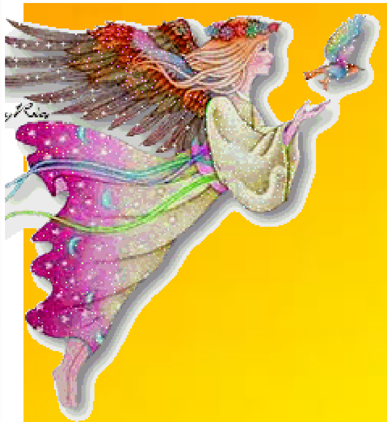


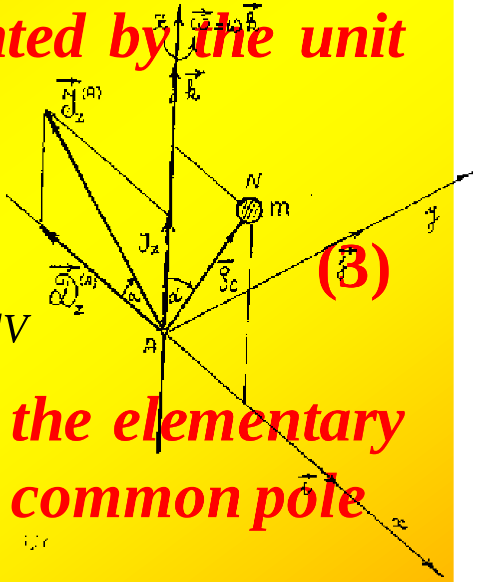




3* Vector $\vec{\mathfrak{J}}_{\vec{n}}^{(O)}$ ***of the body mass inertia moment at the point*** O ***for the axis oriented by the unit vector*** $\vec{n}$ ***:***

$$\vec{\mathfrak{J}}_{\vec{n}}^{(O)} \overset{def}{=} \iiint_V [\vec{\rho},[\vec{n},\vec{\rho}]]dm \qquad (3)$$

$dm = \sigma dV$



where $\vec{\rho}$ ***is the position vector of the elementary body mass*** dm ***with respect to the common pole*** O .

The spherical and the devivational parts of the inertia moment vector and of the inertia tensor are analyzed.

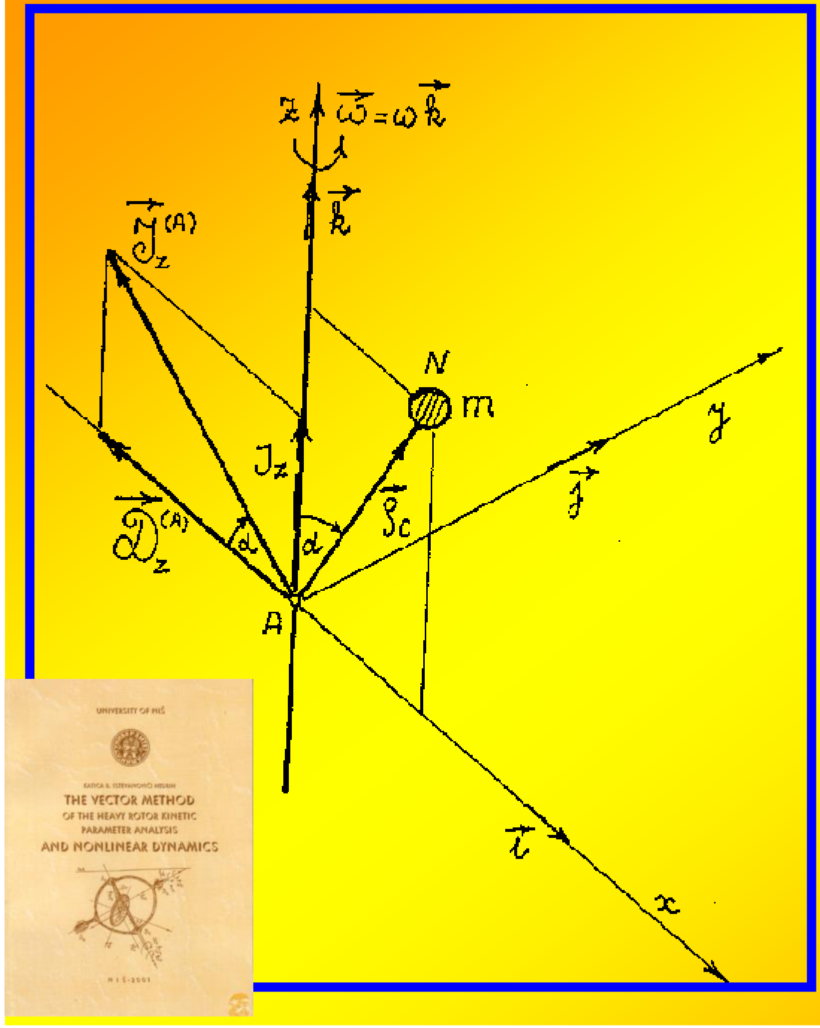


Figure 1. b*

The graphical presentation of the vector of mass particle's mass inertia moment for the reference point and an oriented axis and of the corresponding deviational plane.

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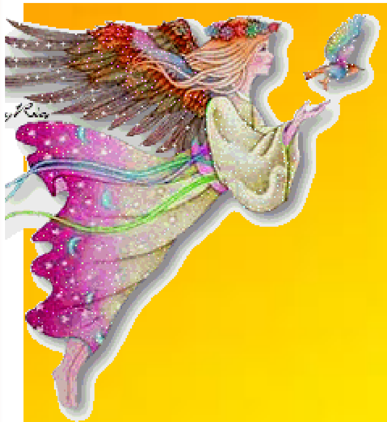

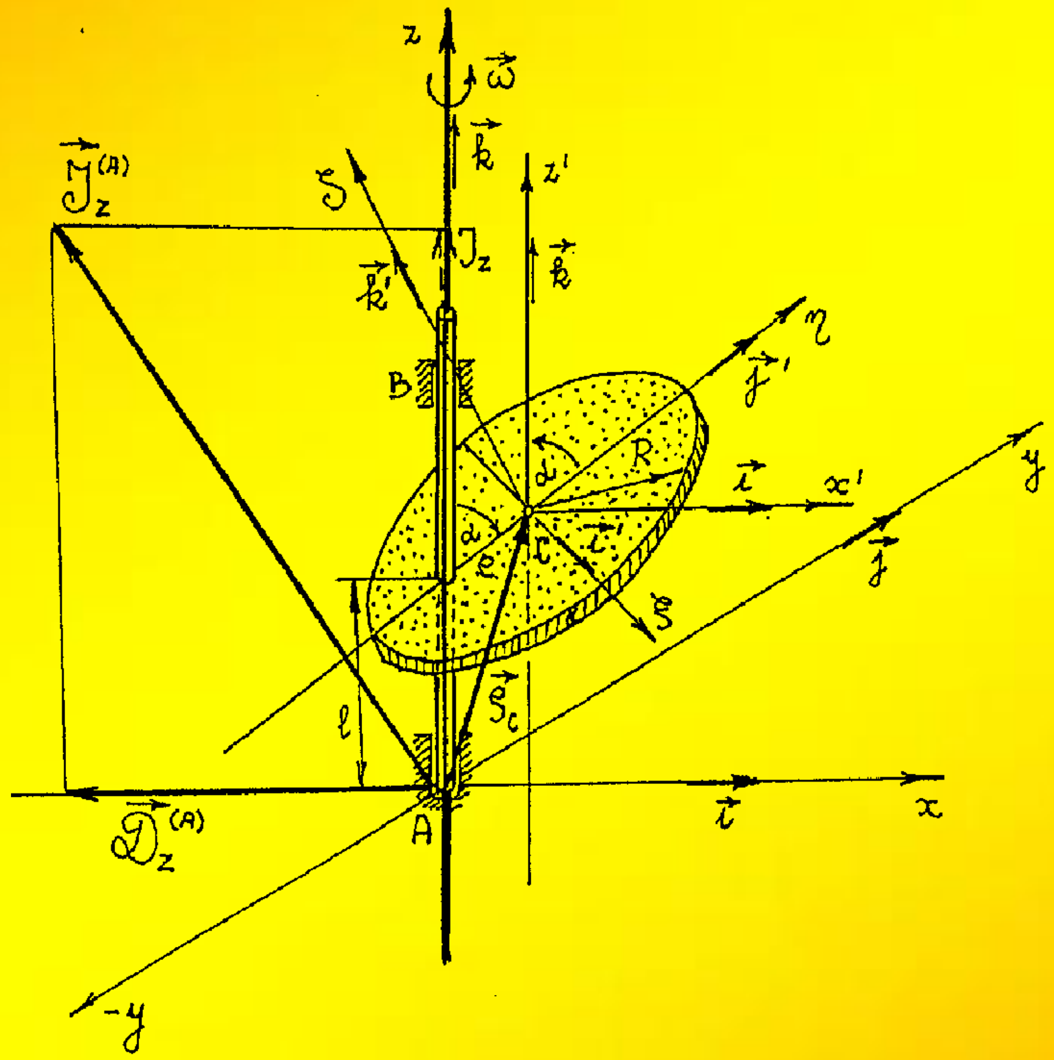

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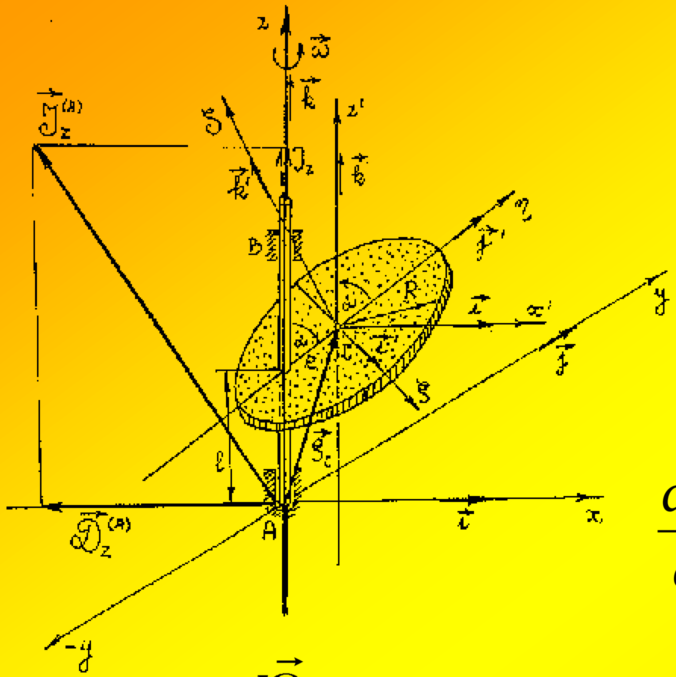

$$\vec{\mathfrak{J}}_{\vec{n}}^{(O)} \overset{def}{=} \iiint_V [\vec{\rho},[\vec{n},\vec{\rho}]]dm$$

$$\vec{\mathfrak{S}}_O^{(\vec{n})} = \iiint_V [\vec{n},\vec{r}]dm$$

$$\mathfrak{R} = \sqrt{\dot{\omega}^2 + \omega^4}$$

$$\frac{d\vec{\mathfrak{K}}}{dt} = \vec{\mathfrak{R}}_1 \left|\vec{\mathfrak{S}}_{\vec{n}}^{(A)}\right| = \sum_{k=1}^{k=N} \vec{F}_k + \vec{F}_A + \vec{F}_B$$

$$\frac{d\vec{\mathfrak{L}}_A}{dt} = \dot{\vec{\omega}} J_{\vec{n}}^{(A)} + \dot{\omega}\vec{\mathfrak{D}}_{\vec{n}}^{(A)} + \omega\left[\vec{\omega},\vec{\mathfrak{D}}_{\vec{n}}^{(A)}\right] =$$

$$= \dot{\vec{\omega}} J_{\vec{n}}^{(A)} + \left|\vec{\mathfrak{D}}_{\vec{n}}^{(A)}\right| \vec{\mathfrak{R}}_2 = \sum_{k=1}^{k=N} \left[\vec{\rho}_k, \vec{F}_k\right] + \left[\vec{\rho}_B, \vec{F}_B\right]$$

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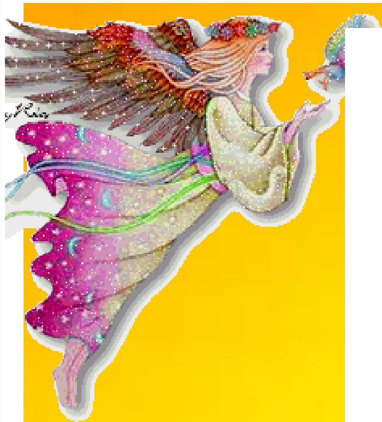

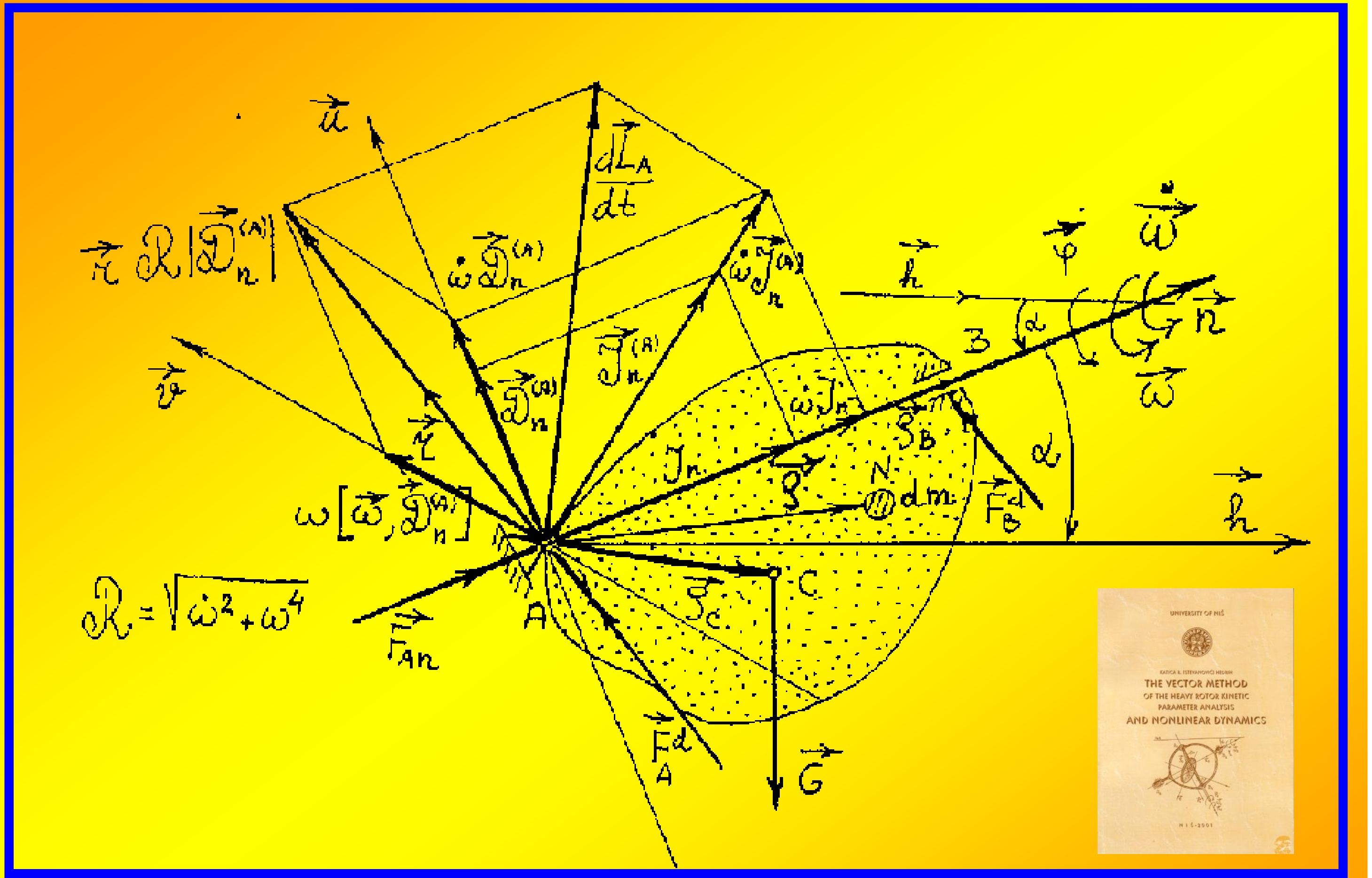

Figure 12. ***The graphical presentation of the kinetic vectors of rotors with inclined rotation axis.***

Following the expressions (67) and (70), as well as the expression (68) and (71), we can write the following two vector equations:

$$\frac{d\vec{\mathfrak{K}}}{dt} = \mathfrak{R}\left|\vec{\mathfrak{S}}^{(A)}_{\vec{n}}\right|\vec{\mathrm{r}}_1 = \sum_{k=1}^{k=N}\vec{\mathrm{F}}_{\mathrm{k}} + \vec{\mathrm{F}}_A + \vec{\mathrm{F}}_B + \vec{\mathrm{G}} \qquad (125)$$

$$\frac{d\vec{\mathfrak{L}}_A}{dt} = \dot{\vec{\omega}}\,\mathrm{J}^{(A)}_{\vec{n}} + \left|\vec{\mathfrak{D}}^{(A)}_{\vec{n}}\right|\vec{\mathfrak{R}} = \sum_{k=1}^{k=N}\left[\vec{\rho}_k, \vec{\mathrm{F}}_{\mathrm{k}}\right] + \left[\vec{\rho}_C, \vec{\mathrm{G}}\right] + \left[\vec{\rho}_B, \vec{\mathrm{F}}_B\right] \qquad (126)$$

These two vectorial equations are kinetic equations of dynamic equilibrium of the body rotating around the stationary axis under the action of the active force system $\vec{\mathrm{F}}_{\mathrm{k}}$.

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2* the equations for determining the bearings' kinetic pressures, that is, pressures upon the bearings, $\vec{F}_A$ and $\vec{F}_B$, that is, their components in the axis direction $\vec{n}$ and normal to the rotation axis:

$$\vec{F}_{A\vec{n}} = \left(\vec{F}_A,\vec{n}\right)\vec{n} = -\vec{n}\sum_{k=1}^{k=N}\left(\vec{F}_k,\vec{n}\right) - \vec{n}\left(\vec{G},\vec{n}\right) \qquad (128)$$

$$\vec{F}_{A\vec{T}} = -\vec{F}_B + \vec{\mathfrak{R}}_1\left|\vec{\mathfrak{S}}_{\vec{n}}^{(A)}\right| - \left[\vec{n},\left[\vec{G},\vec{n}\right]\right] - \sum_{k=1}^{k=N}\left[\vec{n},\left[\vec{F}_k,\vec{n}\right]\right] \qquad (129)$$

$$\vec{F}_B = \frac{1}{\rho_B}\vec{\mathfrak{R}}\left|\vec{\mathfrak{D}}_{\vec{n}}^{(A)}\right| - \frac{1}{\rho_B}\left[\vec{n},\left[\left[\vec{\rho}_C,\vec{G}\right],\vec{n}\right]\right] - \frac{1}{\rho_B}\sum_{k=1}^{k=N}\left[\vec{n},\left[\left[\vec{\rho}_k,\vec{F}_k\right],\vec{n}\right]\right] \qquad (130)$$

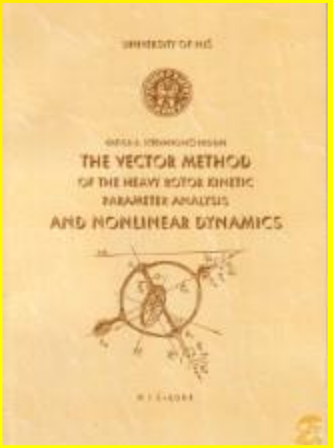
where is: $\vec{\mathfrak{R}} = \mathfrak{R}\,\vec{r}$, $\mathfrak{R} = \sqrt{\dot{\omega}^2 + \omega^4}$. **(131)**

The rotator $\vec{\mathfrak{R}} = \mathfrak{R}\,\vec{r}$ is rotating and increasing by the angular velocity and by the angular acceleration.

If we now multiply scalarly and vectorialy these equations (125) and (126) with the unit vector $\vec{n}$, and, having in mind that the $\vec{\rho}_B = \rho_B \vec{n}$, we obtain:

1* the rotation equation around the axes oriented by the unit vector $\vec{n}$ in the form:

$$\left(\vec{\mathfrak{J}}_{\vec{n}}^{(A)}, \dot{\vec{\omega}}\right) = \left(\left[\vec{\rho}_C, \vec{G}\right], \vec{n}\right) + \sum_{k=1}^{k=N} \left(\left[\vec{\rho}_k, \vec{F}_k\right], \vec{n}\right) \qquad (127)$$



THEOREM 2.

Vector expression of angular momentum derivatives of the rigid N bodies, multi coupled rotations, around no intersecting axis in all cases, placed bodies on the each axis, between other terms, contain sum of products by intensity of rigid N bodies mass deviation moment vectors

$$\left|\mathfrak{D}_{\mathbf{i}\vec{n}_j}^{(o_K)}\right| = \left|\left[\vec{n}_j, \left[\iiint_{V_i}\left[\vec{n}_j, \vec{\rho}_i\right] dm_i, \vec{n}_j\right]\right]\right|, \quad i = 1,2,3..N, \quad j = 1,2,3..K$$

for the axes oriented by unit vectors of component coupled rotation axes through pole on the rigid N bodies self-rotation axis and vector rotators defined by:

$$\vec{\mathfrak{R}}_{\mathbf{i}_j} = \dot{\omega}_j \frac{\vec{\mathfrak{D}}_{\mathbf{i}\vec{n}_j}^{(o_K)}}{\left|\mathfrak{D}_{\mathbf{i}\vec{n}_j}^{(o_K)}\right|} + \omega_j^2\left[\vec{n}_j, \frac{\vec{\mathfrak{D}}_{\mathbf{i}\vec{n}_j}^{(o_K)}}{\left|\mathfrak{D}_{\mathbf{i}\vec{n}_j}^{(o_K)}\right|}\right] \quad i = 1,2,3..N, \quad j = 1,2,3..K$$

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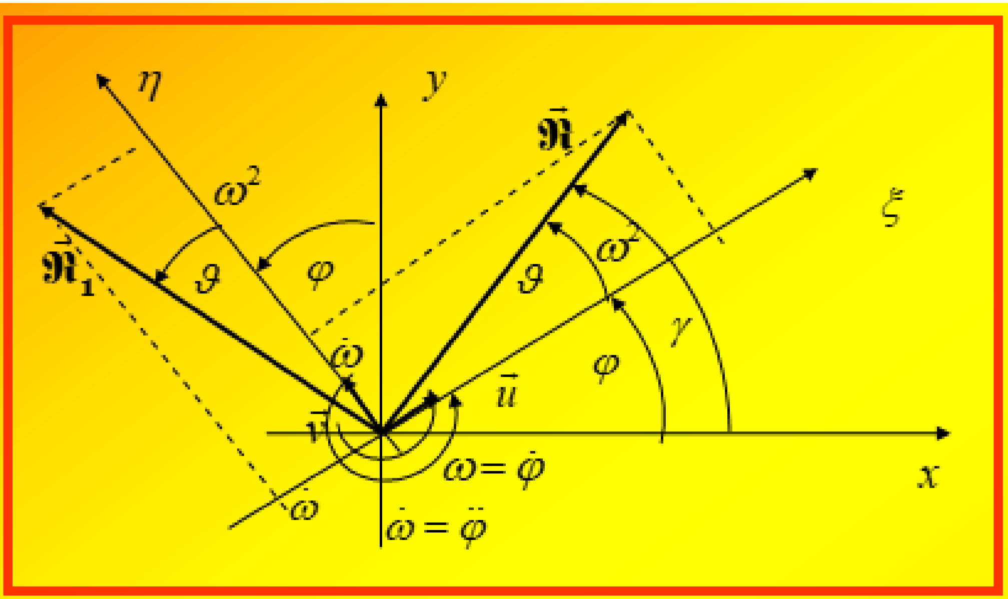

$$\omega = \dot{\varphi}$$

$$\dot{\omega} = \ddot{\varphi}$$

$$\ddot{\omega} = \dddot{\varphi}$$

$$tg\vartheta = \frac{\dot{\omega}}{\omega^2}$$

$$\omega_{\mathfrak{R}\mathfrak{R}} = \dot{\varphi} + \dot{\vartheta} = \omega + \omega_r$$

$$\vec{u} = \vec{i}\cos\varphi + \vec{j}\sin\varphi$$

$$\vec{v} = -\vec{i}\sin\varphi + \vec{j}\cos\varphi$$

$$\dot{\vec{u}} = \dot{\varphi}\vec{v} \qquad \dot{\vec{v}} = -\dot{\varphi}\vec{u}$$

$$\dot{\vec{u}} = -\vec{i}\,\dot{\varphi}\sin\varphi + \vec{j}\,\dot{\varphi}\cos\varphi = \dot{\varphi}\vec{v}$$

$$\dot{\vec{v}} = -\vec{i}\,\dot{\varphi}\cos n\varphi - \vec{j}\,\dot{\varphi}\sin\varphi = -\dot{\varphi}\vec{u}$$

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$$\vartheta = arctg\frac{\dot{\omega}}{\omega^2}$$

$$\gamma = \varphi + \vartheta$$

$$\gamma = \varphi + arctg\frac{\dot{\omega}}{\omega^2}$$

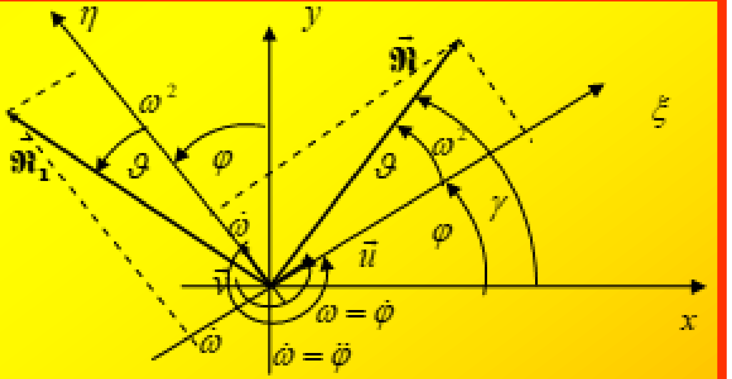
$$\dot{\gamma} = \dot{\varphi} + \dot{\vartheta} = \omega + \dot{\vartheta}$$

$$\dot{\gamma} = \dot{\varphi} + \frac{d}{dt}\left(arctg\frac{\dot{\omega}}{\omega^2}\right)$$

$$\dot{\vartheta} = \omega\chi(\omega,\dot{\omega},\ddot{\omega})$$

$$\dot{\vartheta} = \omega\frac{\ddot{\omega}\omega - 2\dot{\omega}^2}{\omega^4 + \dot{\omega}^2}$$

$$\chi(\omega,\dot{\omega},\ddot{\omega}) = \frac{\ddot{\omega}\omega - 2\dot{\omega}^2}{\omega^4 + \dot{\omega}^2}$$



$$\ddot{\vartheta} = \frac{\left(\ddot{\omega}\omega^2 - 2\dot{\omega}^3 - 2\omega\dot{\omega}\ddot{\omega}\right)\left(\omega^4 + \dot{\omega}^2\right) - \left(4\omega^3\dot{\omega} + 2\dot{\omega}\ddot{\omega}\right)\left(\ddot{\omega}\omega^2 - 2\omega\dot{\omega}^2\right)}{\left(\omega^4 + \dot{\omega}^2\right)^2}$$

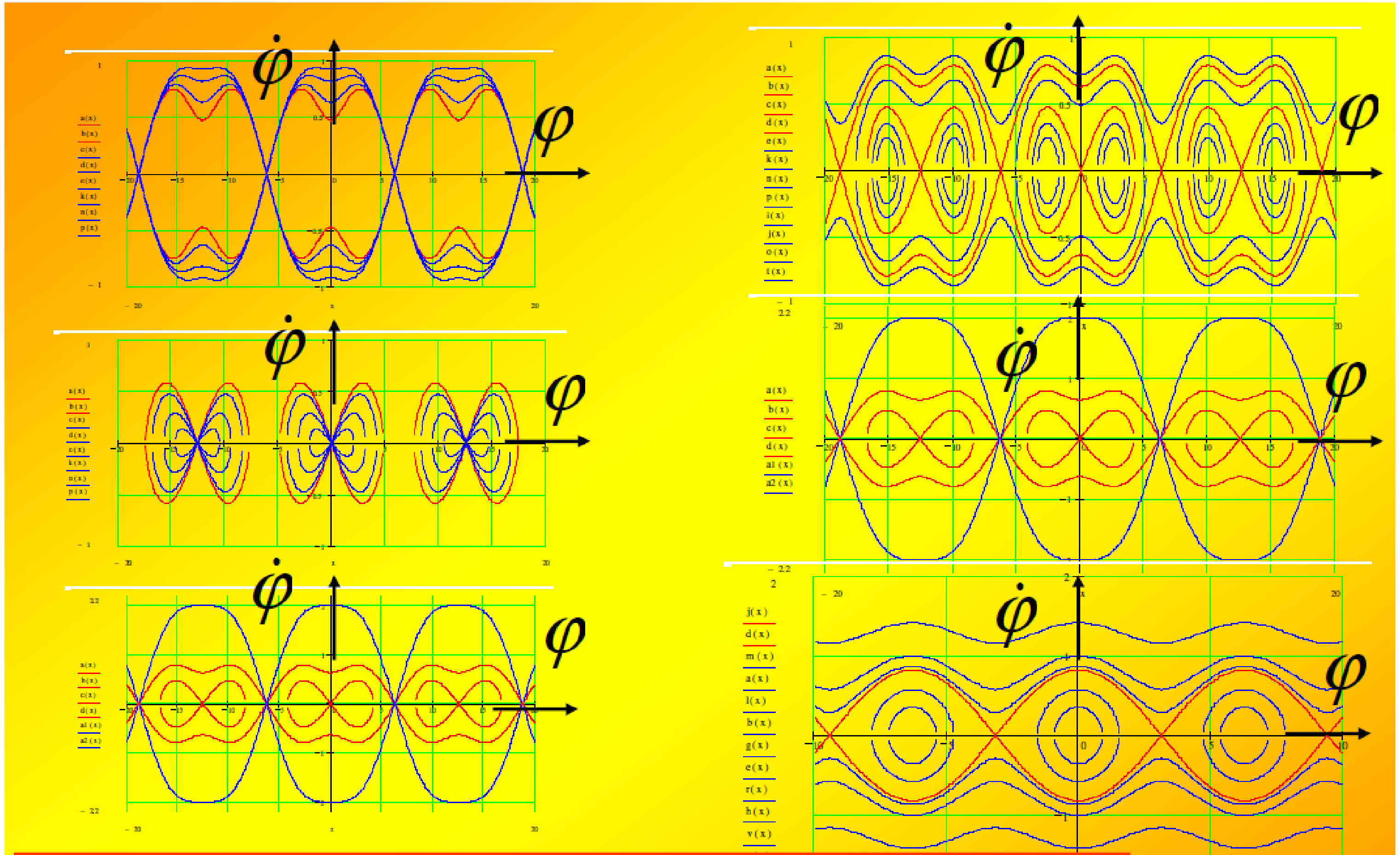
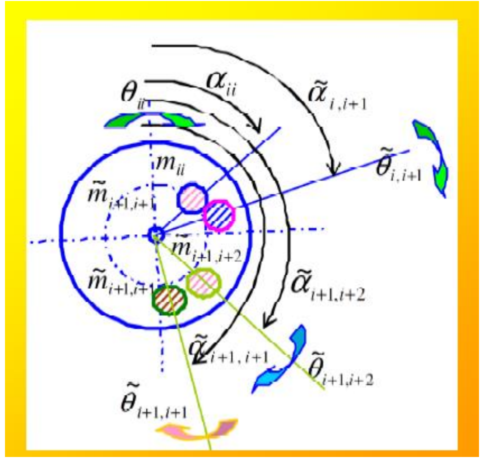


Figure 4 . Characteristic graphs of potential energy and transformation of homoklinic trajectories of nonlinear dynamic of reductor for different system parameters.



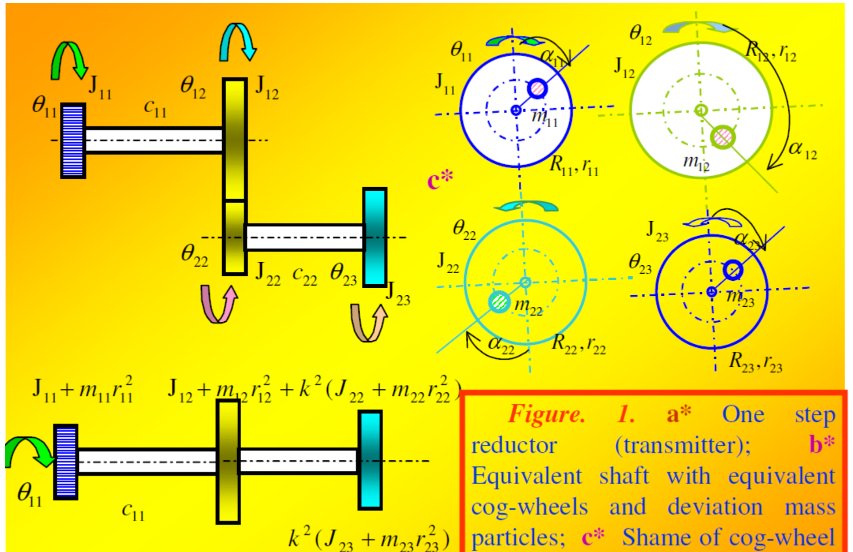


Figure. 1. **a*** One step reductor (transmitter); **b*** Equivalent shaft with equivalent cog-wheels and deviation mass particles; **c*** Shame of cog-wheel with debalance of masses on cog-wheel with deviation properties.

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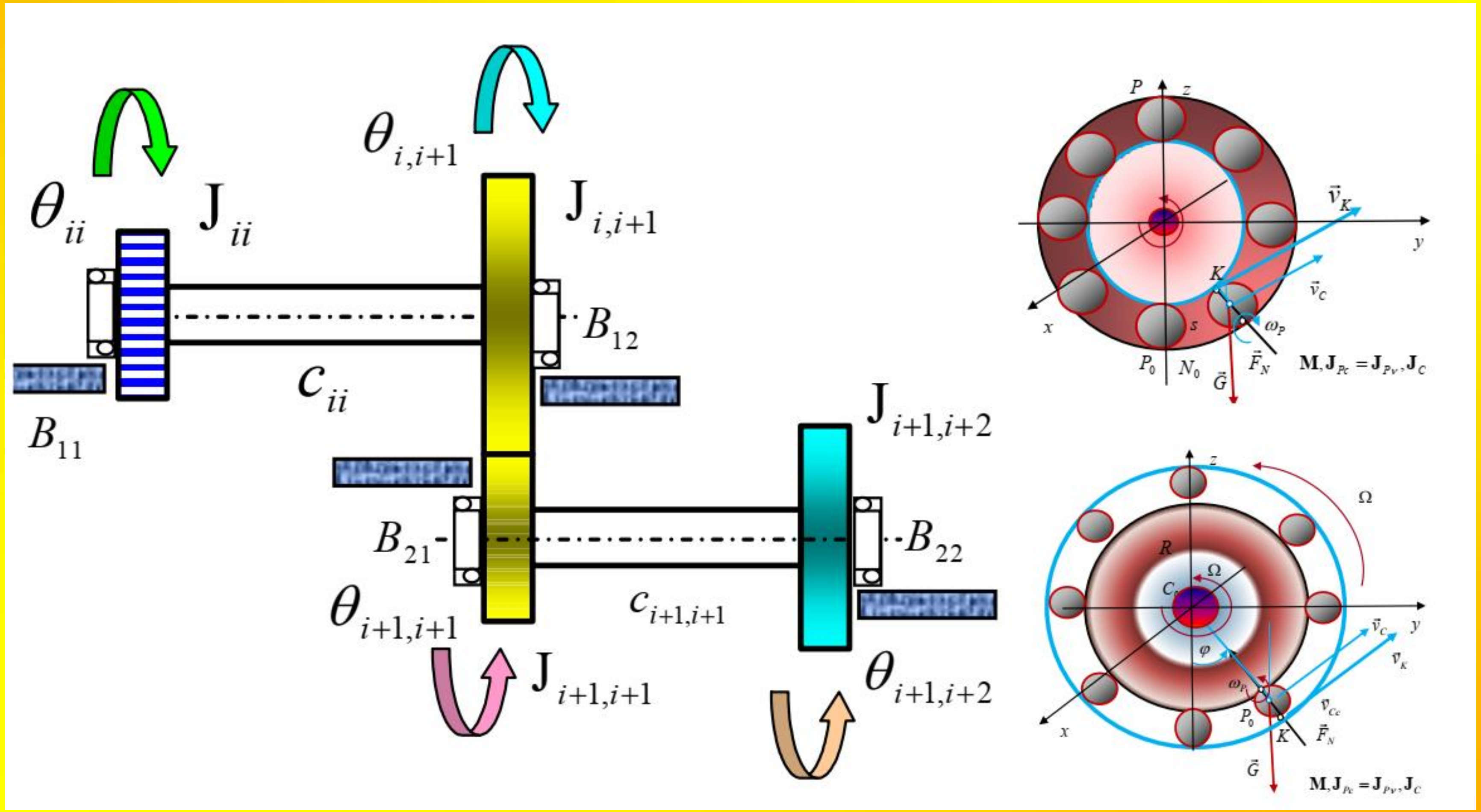

Figure 4. Configuration of radial ball bearings on the shafts of a two-stage gear transmission with unbalanced gears (with debalances in the form of eccentrically placed material points)

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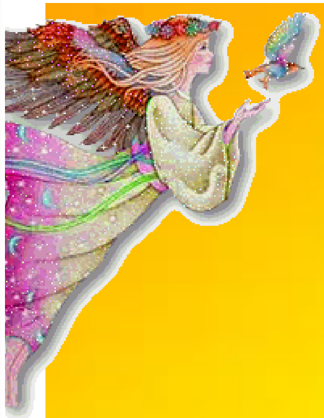

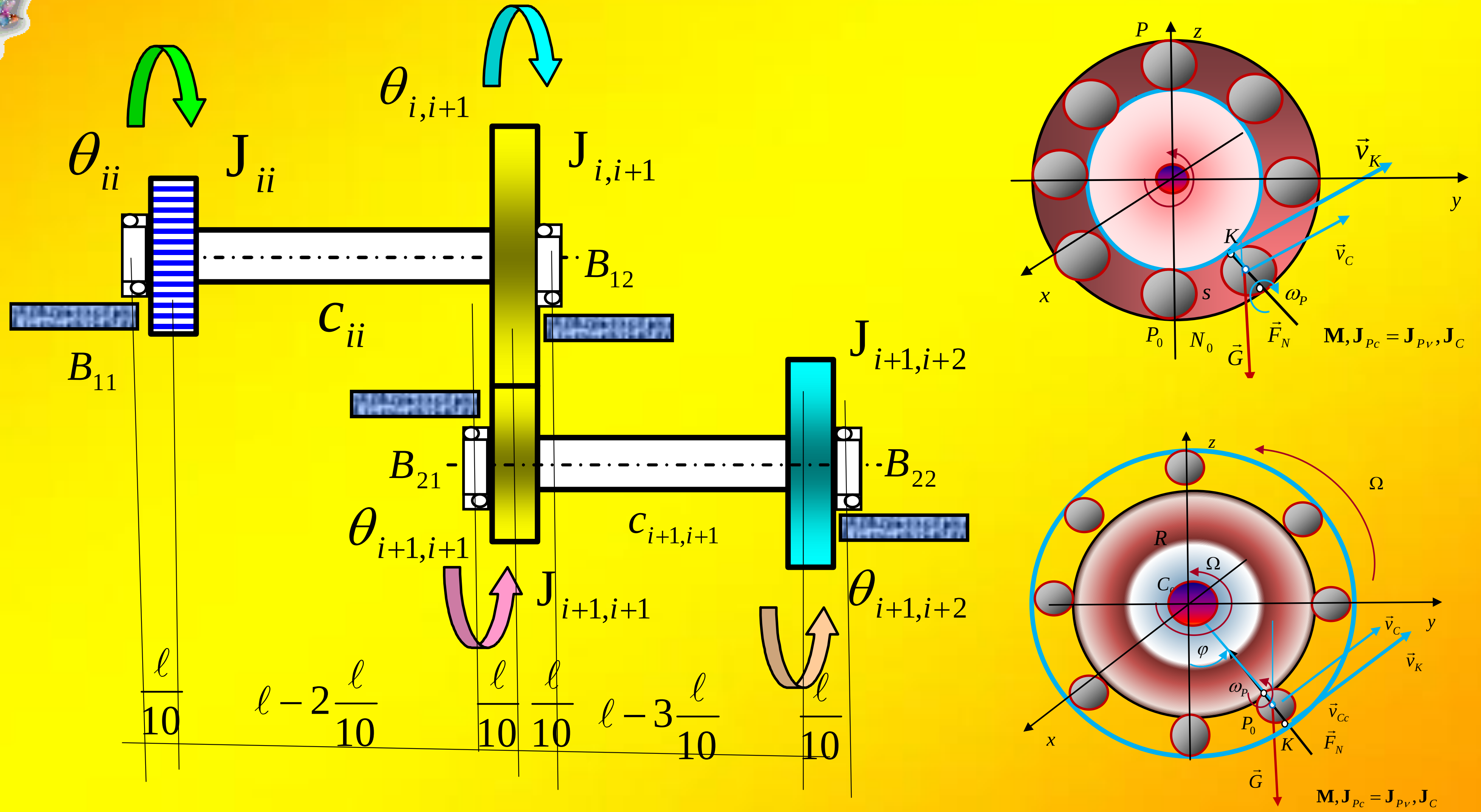

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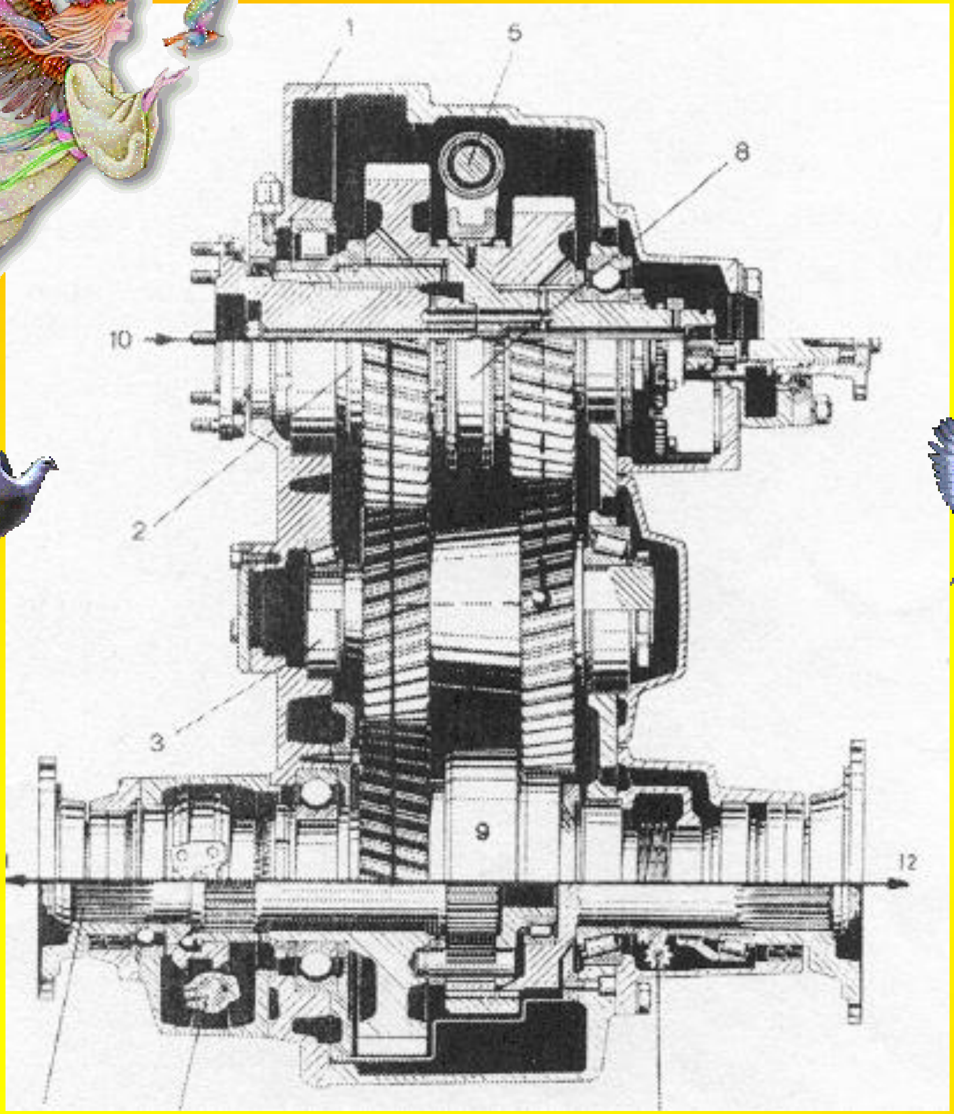

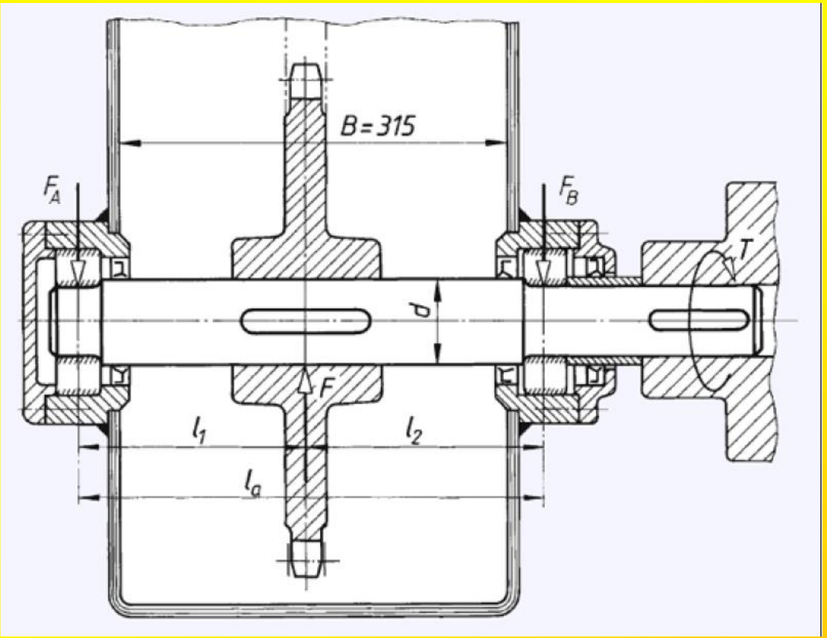

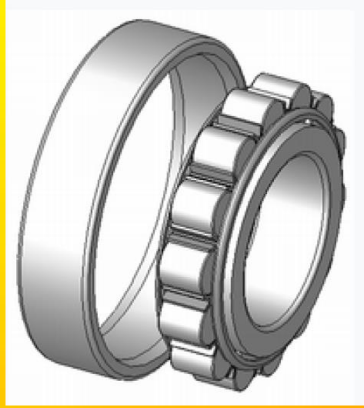

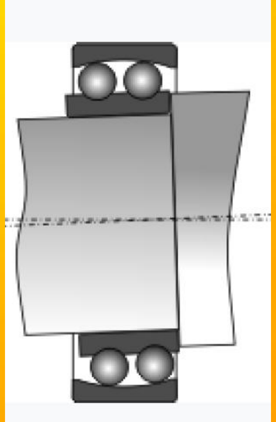

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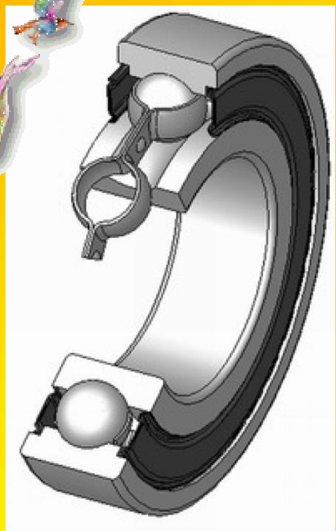

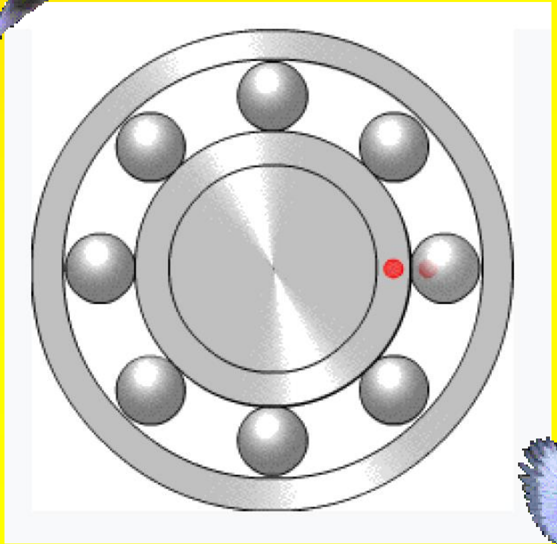

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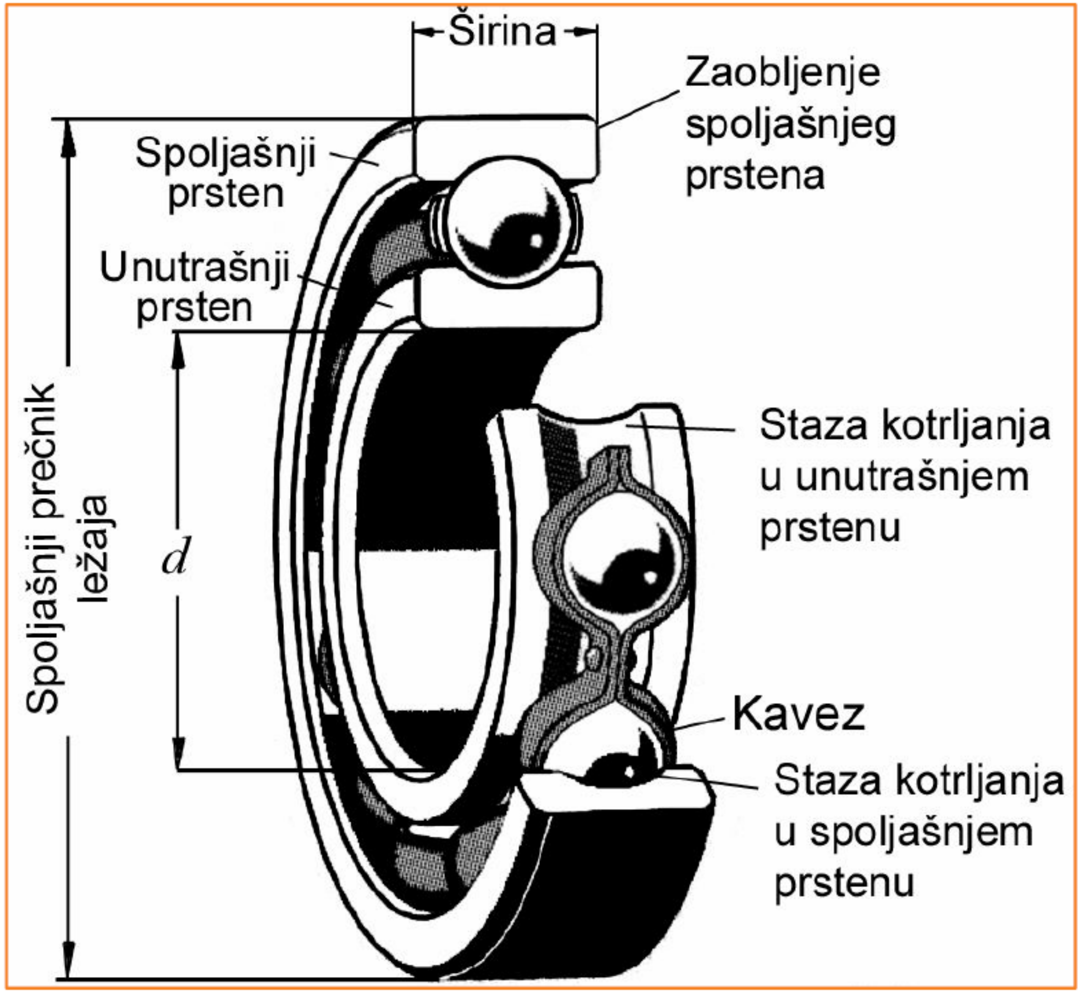

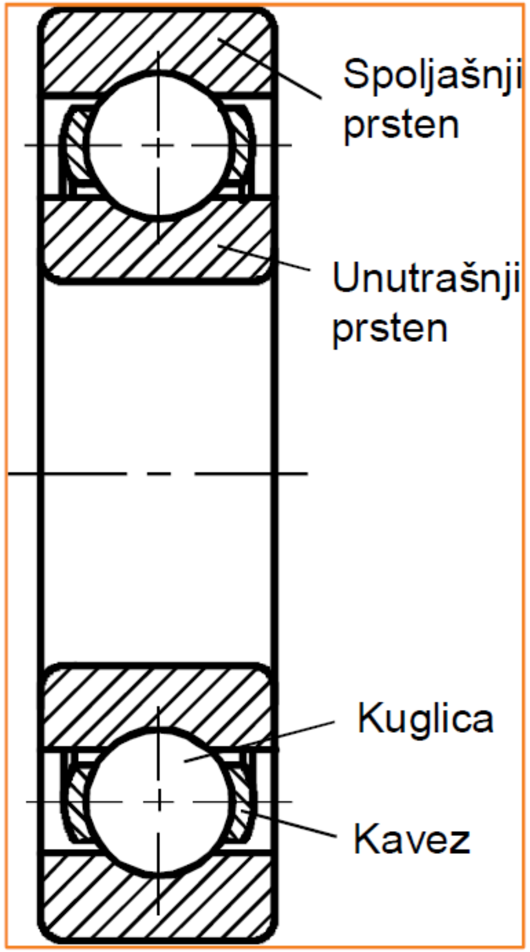

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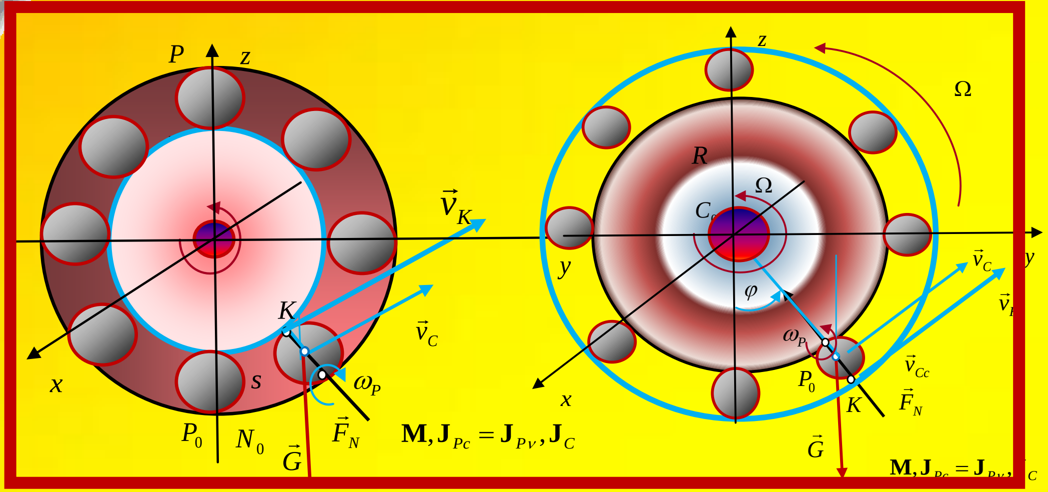

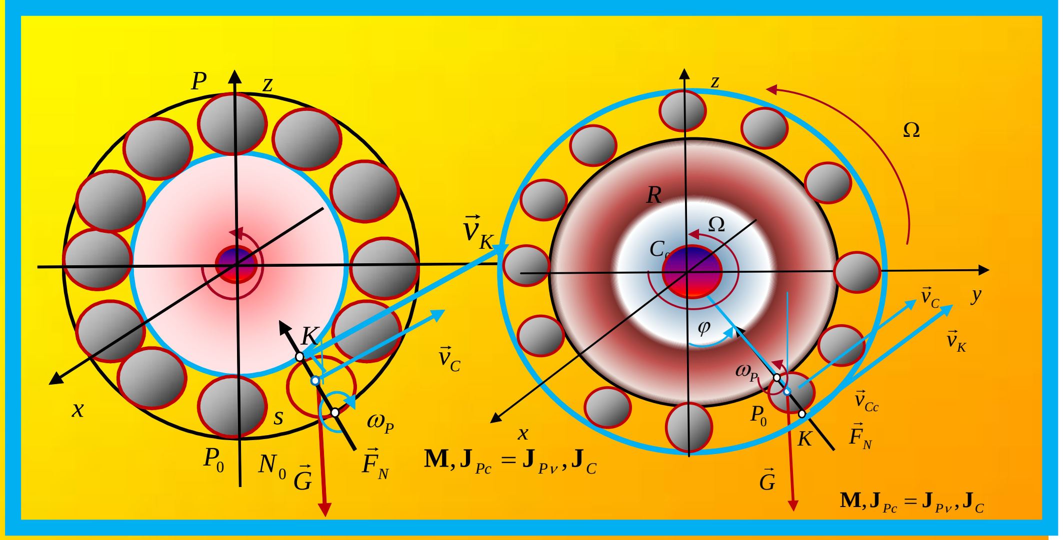

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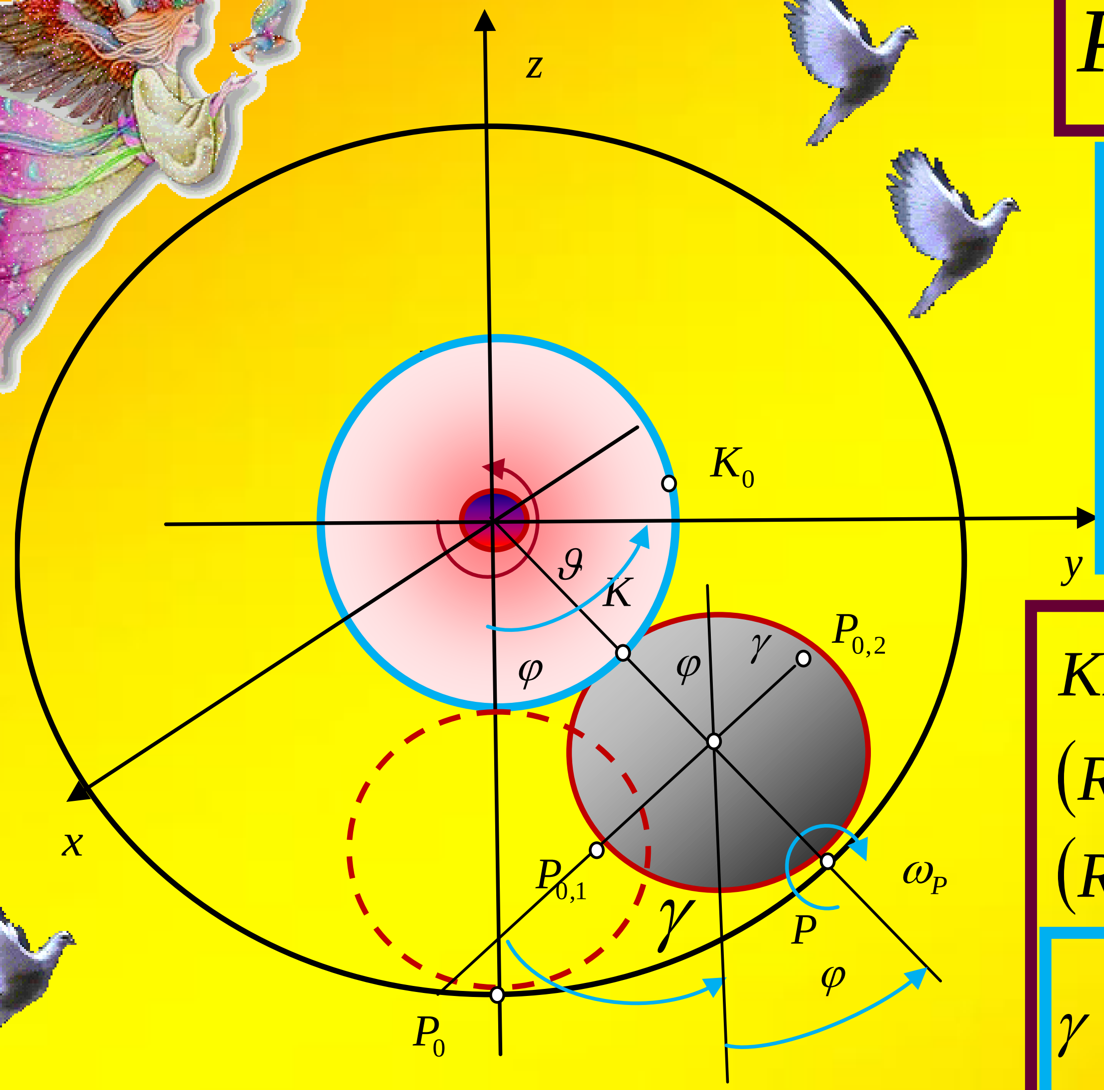

$$P_0P = PP_{0,1} = KK_0 = KP_{0,2}$$

$$P_0P = PP_{0,1}$$

$$R\varphi = r(\varphi + \gamma)$$

$$\gamma = \frac{(R-r)\varphi}{r} \Rightarrow \omega_P = \dot{\gamma} = \frac{(R-r)\dot{\varphi}}{r}$$

$$KK_0 = KP_{0,2}$$

$$(R-2r)\vartheta - (R-2r)\varphi = r(\varphi + \gamma)$$

$$(R-2r)\vartheta = (R-r)\varphi + r\gamma = 2r\gamma$$

$$\gamma = \frac{(R-2r)\vartheta}{2r} \Rightarrow \omega_P = \dot{\gamma} = \frac{(R-2r)\dot{\vartheta}}{2r}$$

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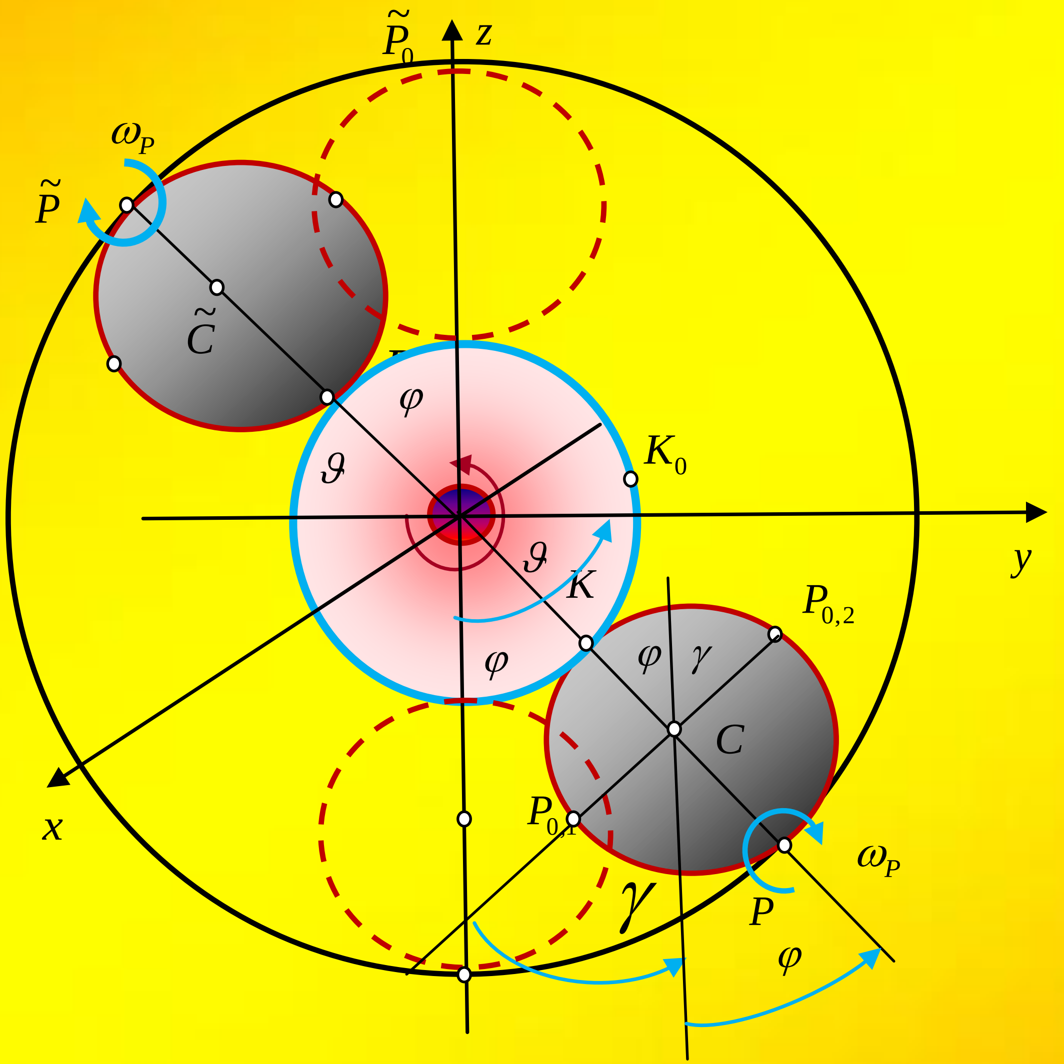

$$v_C = (R-r)\dot{\varphi} = r\omega_P$$

$$\omega_P = \frac{(R-r)\dot{\varphi}}{r}$$

We now use the kinematic connection of the contact of the ball and the movable circular groove, at the point K of contact, by determining its velocity v_K, as a point K, which belongs to the movable circular groove, which rotates at at angular angle velocity $\dot{\vartheta} = \Omega$, but also belongs to the ball contact pount K in rolling at a current angular velocity $\omega_P = \frac{(R-r)\dot{\varphi}}{r}$ of ball rolling in the form:

$$\omega_P = \frac{(R-2r)\dot{\vartheta}}{2r}$$

$$v_K = (R-2r)\dot{\vartheta} = 2r\omega_P$$

$$\omega_P = \frac{(R-2r)\dot{\vartheta}}{2r}$$

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We have now obtained that the angular velocity $\dot{\vartheta}$ of rotation of a fixed circular groove in the function of an independent generalized coordinate $\dot{\varphi}$ is in the following form:

$$\dot{\vartheta} = 2\frac{(R-r)\dot{\varphi}}{(R-2r)}$$

$$\omega_P = \frac{(R-r)\dot{\varphi}}{r}$$

$$\omega_P = \frac{(R-2r)\dot{\vartheta}}{2r}$$

We see that this angular velocity $\dot{\vartheta}$ is greater than the angular velocity $\dot{\varphi}$ according to the independent generalized coordinate φ.

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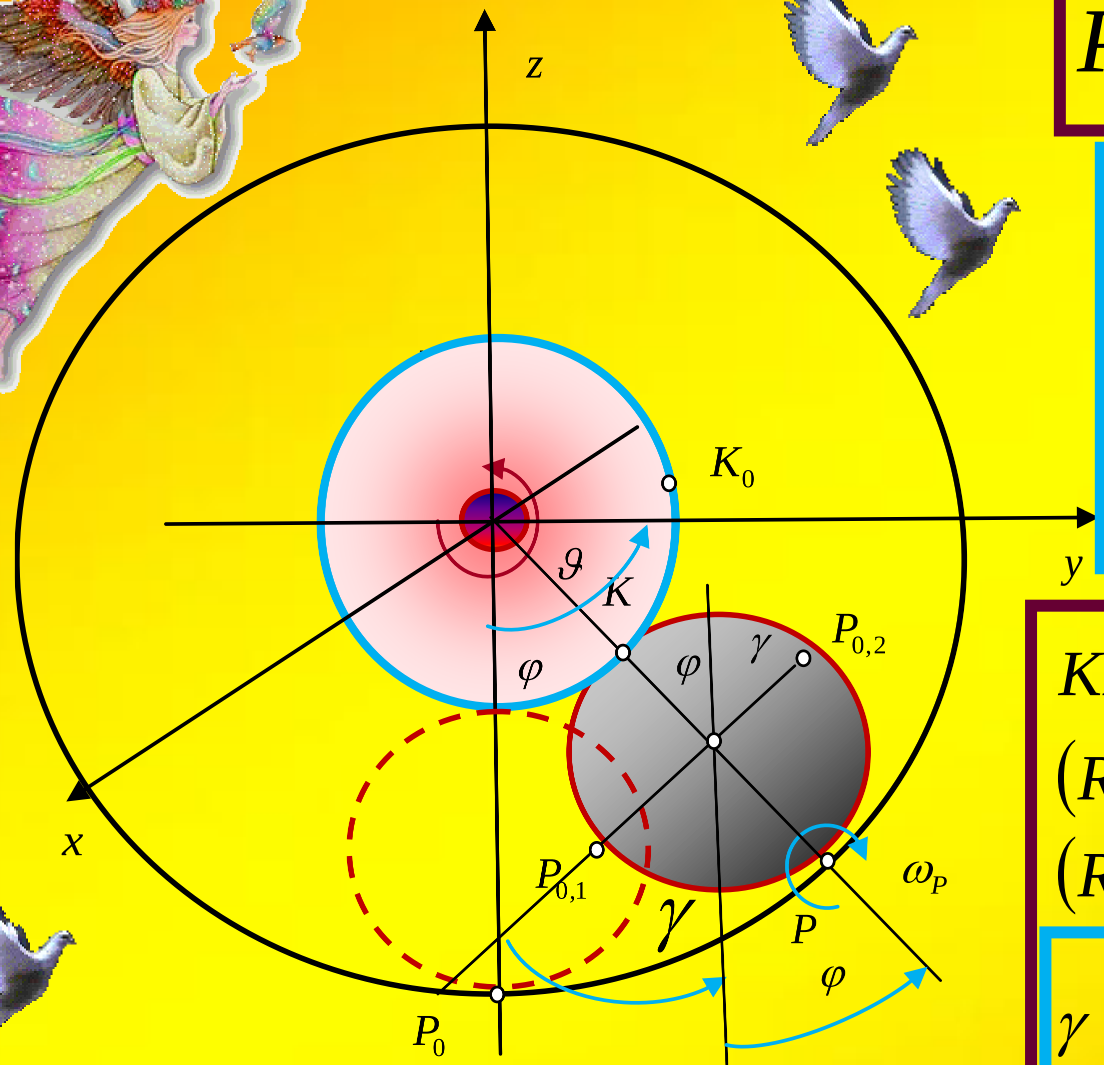

$$P_0P = PP_{0,1} = KK_0 = KP_{0,2}$$

$$P_0P = PP_{0,1}$$

$$R\varphi = r(\varphi + \gamma)$$

$$\gamma = \frac{(R-r)\varphi}{r} \quad \Rightarrow \quad \omega_P = \dot{\gamma} = \frac{(R-r)\dot{\varphi}}{r}$$

$$KK_0 = KP_{0,2}$$

$$(R-2r)\vartheta - (R-2r)\varphi = r(\varphi + \gamma)$$

$$(R-2r)\vartheta = (R-r)\varphi + r\gamma = 2r\gamma$$

$$\gamma = \frac{(R-2r)\vartheta}{2r} \quad \Rightarrow \quad \omega_P = \dot{\gamma} = \frac{(R-2r)\dot{\vartheta}}{2r}$$

The following is also the ratio of coordinates, which we have obtained before:

$$\gamma = \frac{(R-r)\varphi}{r} = \gamma = \frac{(R-2r)\vartheta}{2r} \quad \Rightarrow \quad \vartheta = 2\frac{(R-r)\varphi}{(R-2r)}$$

$$\vartheta = 2\frac{(R-r)\varphi}{(R-2r)}$$

$$\varphi = \frac{(R-2r)\vartheta}{2(R-r)}$$

If the shaft is held at a constant speed, it is estimated that:

$$\varphi = \frac{(R-2r)\Omega t}{2(R-r)}$$

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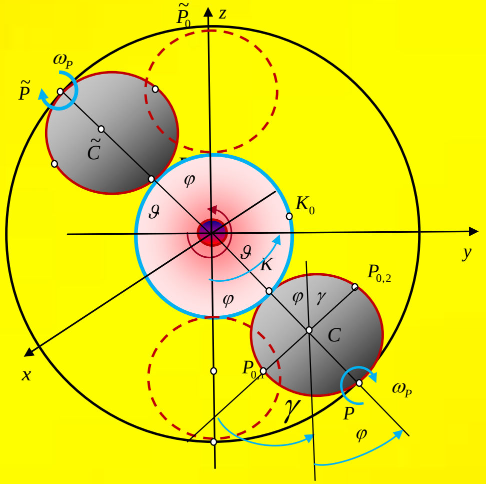
On the dynamics of rolling a pair of balls on a single diameter and on the appearance of nonlinear dynamics

Let us observe the kinetic and potential energy of a pair of balls on a single diameter from a roller bearing. Their mutual position can be defined by independent generalized angular coordinates that differ by an angle of 180 degrees, ie π : $\varphi_{n+1}=\varphi_n+\pi$: where: $\dot{\varphi}_{n+1}=\dot{\varphi}_n$.



$$\omega_{P,n} = \frac{(R-r)\dot{\varphi}_n}{r}$$

$$\omega_{P,n+1} = \frac{(R-r)\dot{\varphi}_{n+1}}{r} = \frac{(R-r)\dot{\varphi}_n}{r}$$

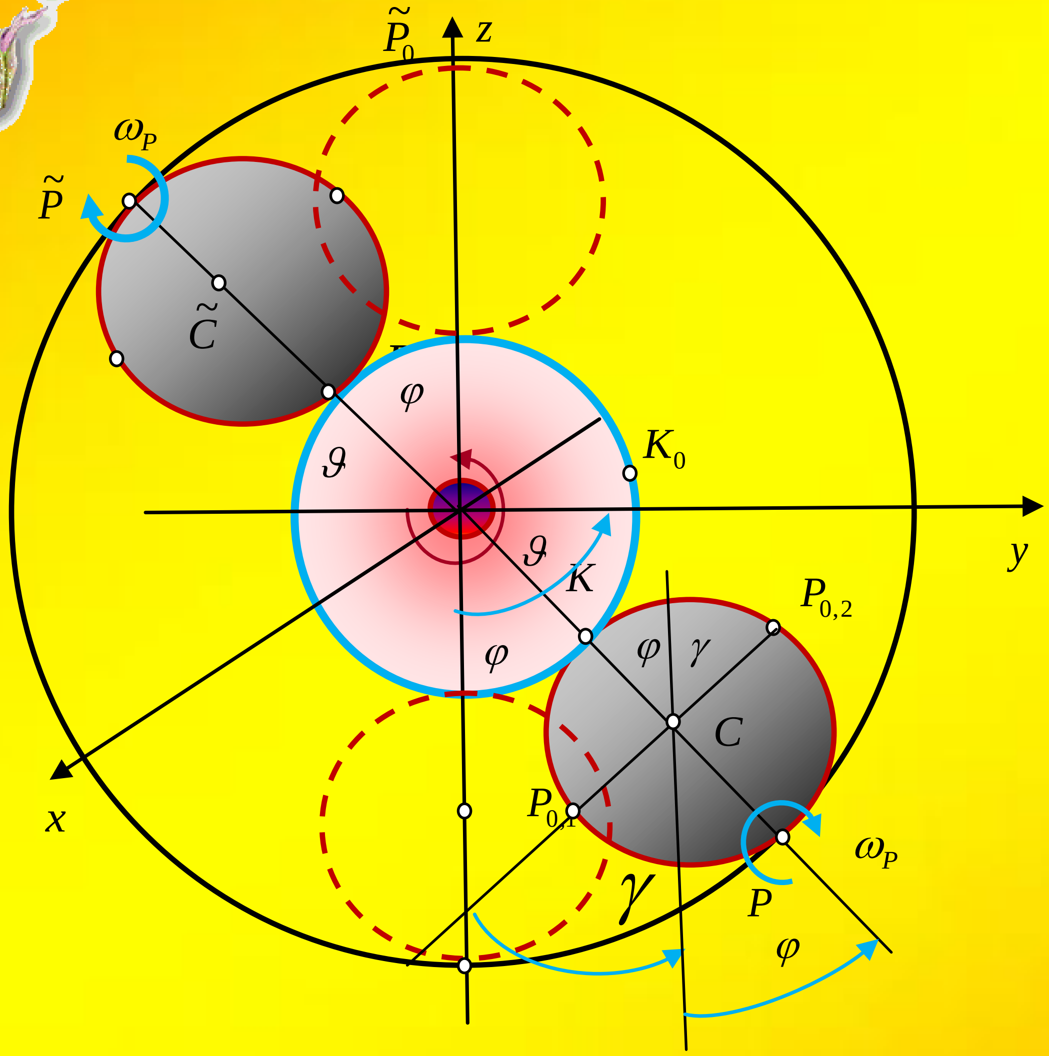


Figure 4. **On the dynamics of rolling a pair of balls on a single diameter and on the appearance of nonlinear dynamics - characteristic eometric elements, haracteristic points and arcs in correlation, non-slip, on fixed circular path and contact, non-slip, on circular contour of movable circular platform**

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If we now introduce the following labels:

* radii of inertia of ball masses for the current rolling axis:

$$\mathbf{i}_{\mathbf{P}}^{2}=\frac{\mathbf{J}_{\mathbf{P}}}{mr^{2}}$$

* Radius of polar inertia of mass for a movable rotating circular platform of radius relative to the mass of a spherical ball

$$\mathbf{i}_{0}^{2}=\frac{\mathbf{J}_{O}}{m(R-2r)^{2}}$$

* reduced eccentricity ε_n - dimensionless eccentricity in shape

$$\varepsilon_n=\frac{e_n}{(R-r)}$$

Then we can enter the following expressopm:

$$\frac{g}{\lambda}=\frac{g}{2(R-r)\left\{\frac{3\mathbf{J}_{\mathbf{P}}}{mr^2}+\frac{\mathbf{J}_O}{m(R-2r)^2}\right\}}=\frac{g}{2(R-r)\left\{3\mathbf{i}_{\mathbf{P}}^2+\mathbf{i}_0^2\right\}}$$

and $\frac{g}{\lambda}$ call it the coefficient of the roller bearing, while we will use the following expression

$$\lambda=2(R-r)\left\{\frac{3\mathbf{J}_{\mathbf{P}}}{mr^2}+\frac{\mathbf{J}_O}{m(R-2r)^2}\right\}=2(R-r)\left\{3\mathbf{i}_{\mathbf{P}}^2+\mathbf{i}_0^2\right\}$$

and λ call the reduced length of rolling in rotation in a ball bearing with twelve balls, with two balls in each of pairs of roller bearings.

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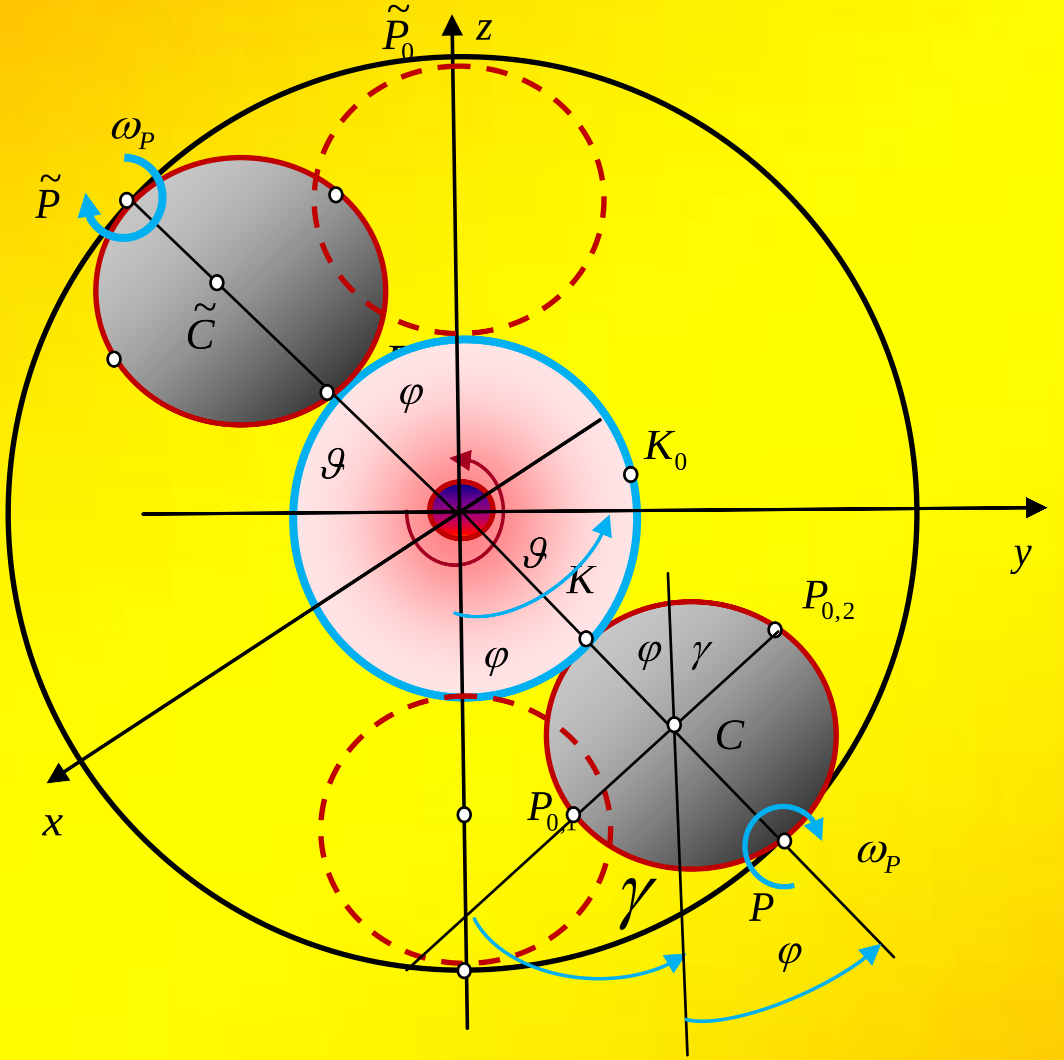

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Angular velocity $\dot{\vartheta}$ of rotation of the movable circular groove:

$$\dot{\vartheta} = 2\frac{(R-r)\dot{\varphi}}{(R-2r)}$$

The kinetic energy $\mathbf{E_k}$ of rolling a pair of balls on a single diameter and a movable circular groove of the axial moment of inertia of mass $\mathbf{J}_O = \frac{1}{2}\mathbf{M}(R-2r)^2$, $\mathbf{M} = \rho(R-2r)^2\pi$, for the shaft axis and the ball bearing is:

$$\mathbf{E_k} = 2\frac{1}{2}\mathbf{J_P}\left(\omega_{P,n}\right)^2 + \frac{1}{2}\mathbf{J}_O\left(\dot{\vartheta}\right)^2 = 2\frac{1}{2}\mathbf{J_P}\left[\frac{(R-r)\dot{\varphi}_n}{r}\right]^2 + \frac{1}{2}\mathbf{J}_O 4\left[\frac{(R-r)\dot{\varphi}_n}{(R-2r)}\right]^2$$

The potential energy $\mathbf{E}_p = const$ does not change because the center of mass of a pair of balls is in the center of the cross section of the shaft and the roller bearing.

$$\frac{d}{dt}\frac{\partial \mathbf{E_k}}{\partial \dot{\varphi}} - \frac{\partial \mathbf{E_k}}{\partial \varphi} + \frac{\partial \mathbf{E}_p}{\partial \varphi} = 0$$

The kinetic energy $\mathbf{E_k}$ of rolling a pair of balls on one diameter and movable platform with circulat grove is:

$$\mathbf{E_k} = 2\frac{1}{2}\mathbf{J_P}\left(\omega_{P,n}\right)^2 + \frac{1}{2}\mathbf{J}_O\left(\dot{\vartheta}\right)^2 = 2\frac{1}{2}\mathbf{J_P}\left[\frac{(R-r)\dot{\varphi}_n}{r}\right]^2 + \frac{1}{2}\mathbf{J}_O 4\left[\frac{(R-r)\dot{\varphi}_n}{(R-2r)}\right]^2$$

The kinetic energy $\mathbf{E_k}$ of rolling of all eight balls and a movable platform with a circular groove is:

$$\mathbf{E_k} = \sum_{1}^{4} \mathbf{J_P} \left(\omega_{P,n,i}\right)^2 + \frac{1}{2}\mathbf{J}_O 4\left[\frac{(R-r)\dot{\varphi}_n}{(R-2r)}\right]^2 = \mathbf{J_P} \sum_{n=1}^{n=4} \left[\frac{(R-r)\dot{\varphi}_n}{r}\right]^2 + \frac{1}{2}\mathbf{J}_O 4\left[\frac{(R-r)\dot{\varphi}_n}{(R-2r)}\right]^2$$

Bearing in mind that the center C of mass of a pair of balls is in the center C of circular, movable and immovable rolling grooves and kinematic contact K of balls in rolling, there is no change in the potential energy $\mathbf{E}_p = const$ of the ball bearing system in the rolling dynamics, so we can write the following system of ordinary differential rolling equations for each of the pairs of balls: We use Lagrange equations of the second kind:

$$\frac{d}{d}\left\{2\mathbf{J_P}\left[\frac{(R-r)\dot{\varphi}_n}{r}\right]\frac{(R-r)}{r} + \mathbf{J}_O 4\left[\frac{(R-r)\dot{\varphi}_n}{(R-2r)}\right]\frac{(R-r)\dot{\varphi}_n}{(R-2r)}\right\} = 0$$

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And the first integral of the previous differential equation is:

$$2\mathbf{J}_{\mathbf{P}}\left[\frac{(R-r)\dot{\varphi}_n}{r}\right]\frac{(R-r)}{r}+\mathbf{J}_O 4\left[\frac{(R-r)\dot{\varphi}_n}{(R-2r)}\right]\frac{(R-r)}{(R-2r)}=const$$

$$\dot{\varphi}_n=\dot{\varphi}_{n,0}=const$$

We see that in that case the motion - rolling of balls, without sliding, is a constant instanteneous angular velocity $\omega_{P,n}=\frac{(R-r)\dot{\varphi}_n}{r}$ of rolling, because the change of the independent generalized coordinate - the angle $\dot{\varphi}=\dot{\varphi}_{n,0}t$ is uniform with a constant angular speed $\dot{\varphi}_n=\dot{\varphi}_{n,0}=const$.

Then the angular velocity $\Omega = \dot{\vartheta} = 2\frac{(R-r)\dot{\varphi}}{(R-2r)} = const$ of shaft rotation is constant

$$\dot{\vartheta} = 2\frac{(R-r)\dot{\varphi}}{(R-2r)} = \Omega = const$$

The current angular velocity $\omega_{P,n+1}$ of rolling, withot slliping, of a pair of balls in a pair on one diameter is:

$$\omega_{P,n+1} = \frac{(R-r)\dot{\varphi}_{n+1}}{r} = \frac{(R-2r)\Omega}{2r} = const$$

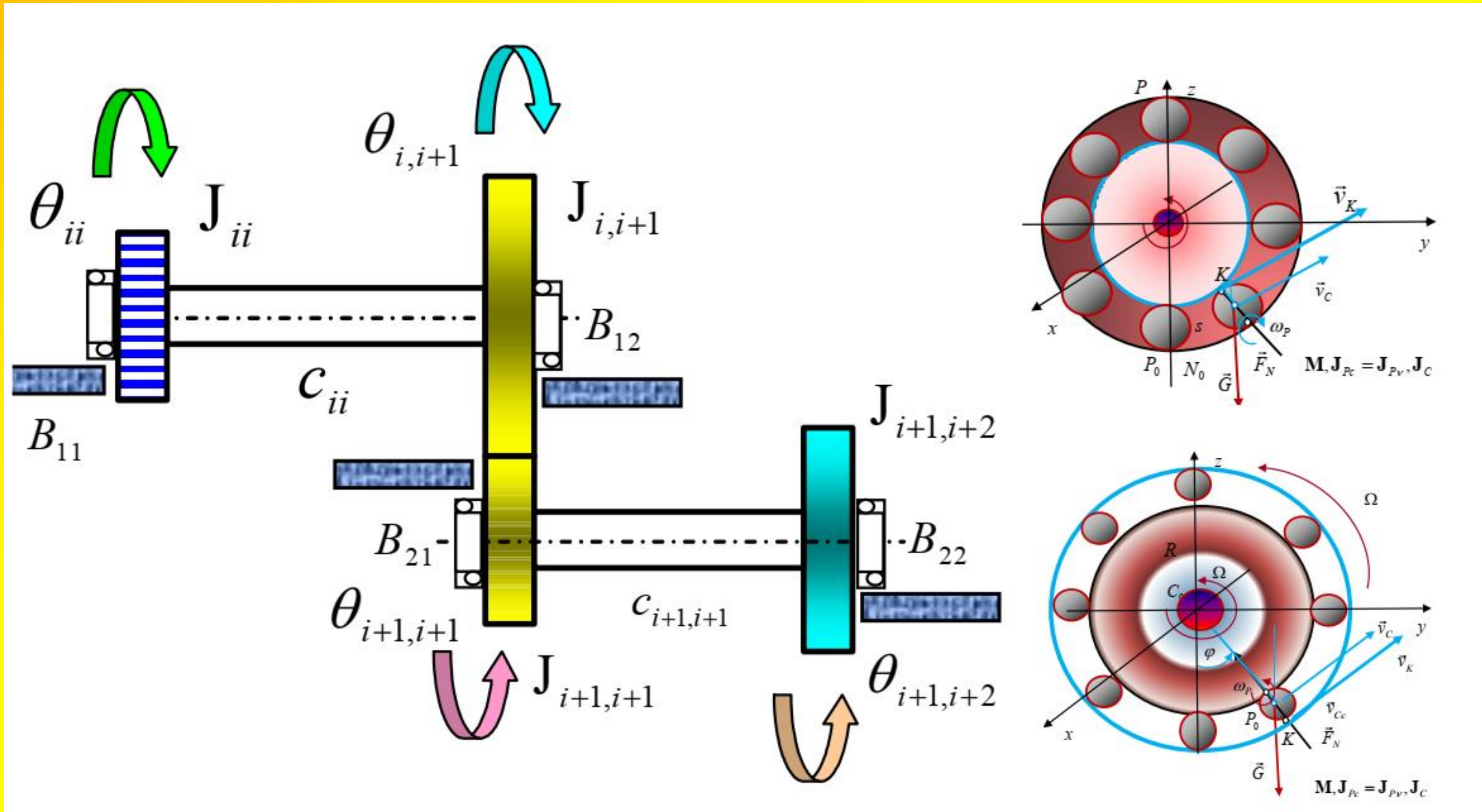

Figure 4. Configuration of radial ball bearings on the shafts of a two-stage gear transmission with unbalanced gears (with debalances in the form of eccentrically placed material points)

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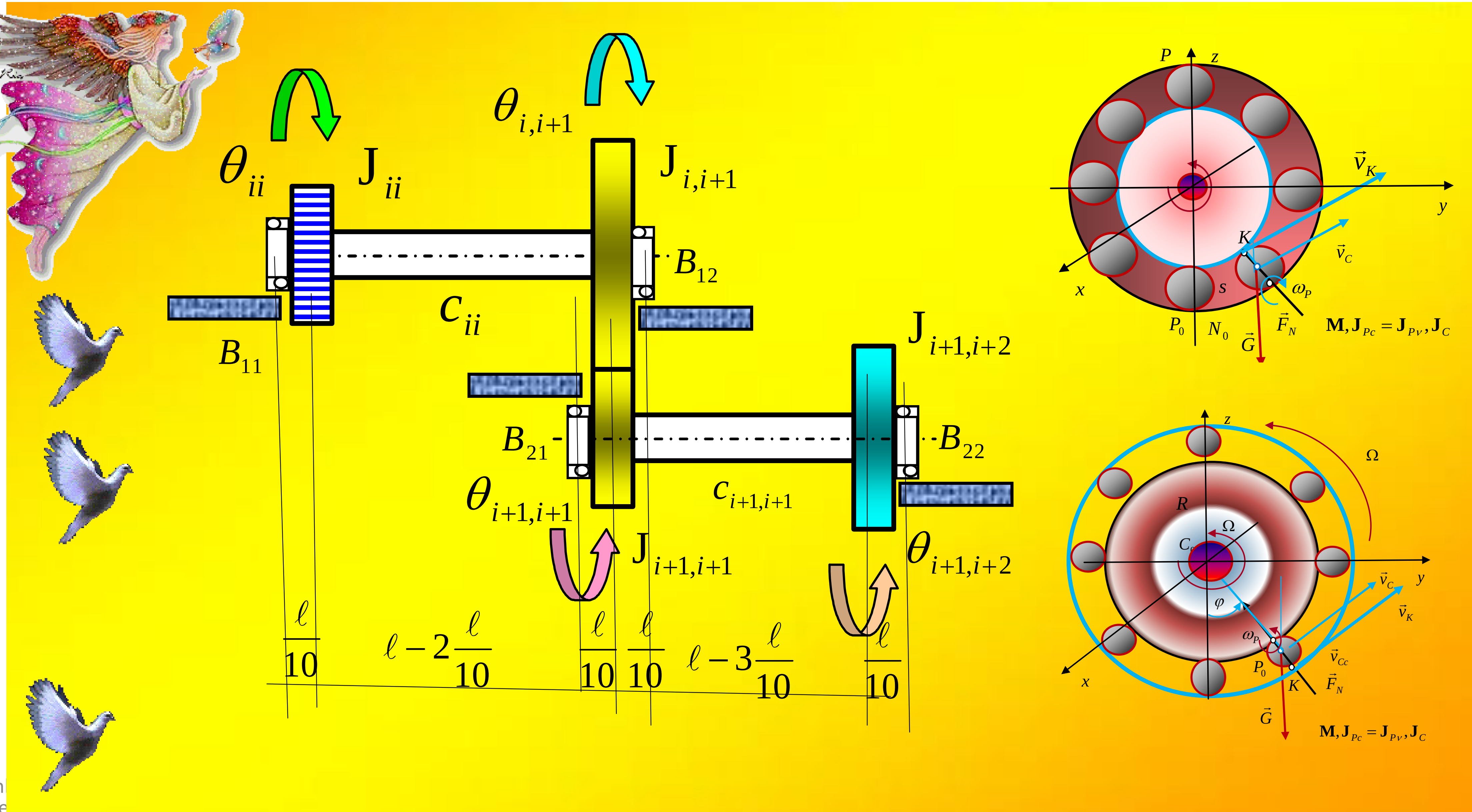

The eccentricity of the cent mass of the pair of balls is

$$e_n = \frac{1}{2}(R - r)\frac{(1 - p_n)}{(1 + p_n)}$$

Using these assumptions, the influence of the centrifugal forces of the two-stage gear transmission was studied and the real system was reduced to a fictitious model on the first shaft with several unbalanced gears and the contact forces in the radial ball bearings were analyzed.

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The eccentricity of the center of mass of a pair of balls on one diameter is:

$$e_n = \frac{1}{2}(R - r)\frac{(1 - p_n)}{(1 + p_n)}$$

the p_n difference coefficient is the mass density of the balls in a pair on one diameter

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$$\tilde{\mathbf{J}}_0 = \frac{8}{5}\tilde{\mathbf{M}}R^2 \qquad \tilde{\mathbf{J}}_x = \tilde{\mathbf{J}}_y = \tilde{\mathbf{J}}_z = \frac{2}{5}p\mathbf{M}R^2$$

$$\tilde{\mathbf{M}} = \tilde{\rho}\frac{4}{8}R^3\pi \qquad \tilde{\mathbf{J}}_P = p\mathbf{J}_P = \tilde{\mathbf{J}}_y + p\mathbf{M}R^2 == \frac{7}{5}p\mathbf{M}R^2$$

$$\tilde{\mathbf{J}}_x = \tilde{\mathbf{J}}_y = \tilde{\mathbf{J}}_z = \frac{2}{5}\frac{1-\psi^5}{1-\psi^3}p\mathbf{M}R^2 \qquad \tilde{\mathbf{J}}_x = \tilde{\mathbf{J}}_y = \tilde{\mathbf{J}}_z \approx \frac{2}{3}p\mathbf{M}R^2$$

$$\tilde{\mathbf{M}} \approx 4p\rho R^2\pi\delta \qquad \tilde{\mathbf{J}}_P = \tilde{\mathbf{J}}_y + p\mathbf{M}R^2 == \frac{7}{3}p\mathbf{M}R^2$$

$$e = \frac{1}{2}(R-r)\frac{(1-p)}{(1+p)} \qquad e_n = \frac{1}{2}(R-r)\frac{(1-p_n)}{(1+p_n)}$$

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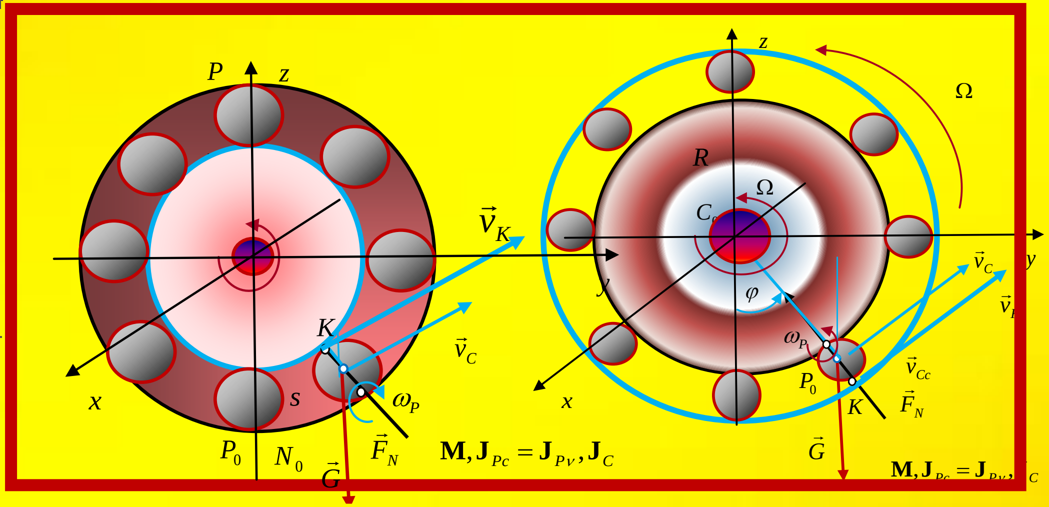
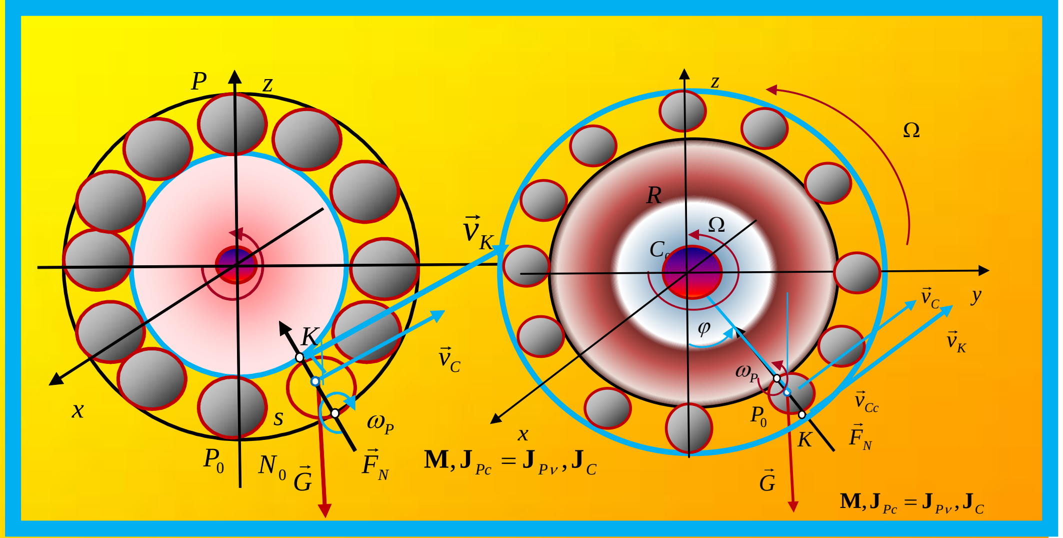
$$\mathbf{E_k} = \frac{1}{2}\mathbf{J_P}\left(7+p\right)\left[\frac{(R-r)\dot{\varphi}}{r}\right]^2 + 2\mathbf{J}_O\left[\frac{(R-r)\dot{\varphi}}{(R-2r)}\right]^2$$

$$\mathbf{E_k} = \frac{1}{2}\mathbf{J_P}\left(11+p_n\right)\left[\frac{(R-r)\dot{\varphi}}{r}\right]^2 + \frac{1}{2}\mathbf{J}_O 4\left[\frac{(R-r)\dot{\varphi}}{(R-2r)}\right]^2$$





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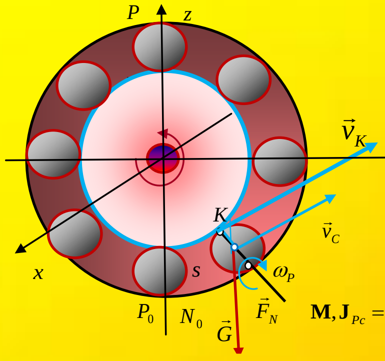
$$\frac{d}{dt}\frac{\partial \mathbf{E_k}}{\partial \dot{\varphi}} - \frac{\partial \mathbf{E_k}}{\partial \varphi} + \frac{\partial \mathbf{E}_p}{\partial \varphi} = 0 \qquad n = 1,2,3,4$$

$$\mathbf{E_k} = \frac{1}{2}\mathbf{J_P}(7+p)\left[\frac{(R-r)\dot{\varphi}}{r}\right]^2 + 2\mathbf{J}_O\left[\frac{(R-r)\dot{\varphi}}{(R-2r)}\right]^2$$

$$\mathbf{E_{p,1}} = \frac{1}{2}m(1-p)g(R-r)(1-cos\varphi)$$



$$\ddot{\varphi} + \frac{g}{2(R-r)\left[\frac{\mathbf{J_P}}{mr^2}\frac{(7+p)}{(1-p)} + \frac{2\mathbf{J}_O}{m(R-2r)^2(1-p)}\right]} sin\varphi = 0$$

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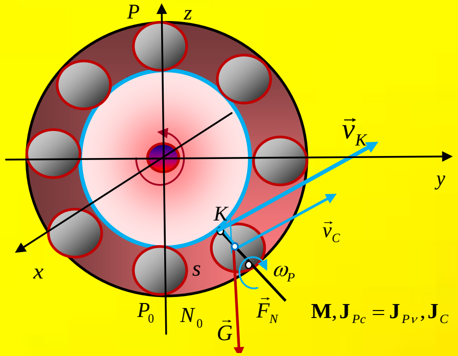
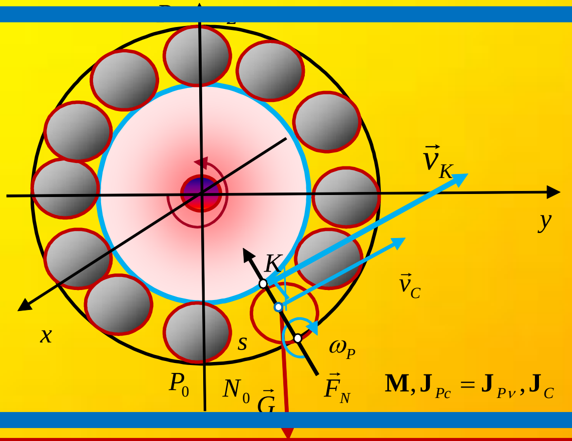
$$\varphi_k = \varphi + \frac{(k-1)\pi}{4} \qquad k = 1,2,3,4$$

$$\varphi_k = \varphi + \pi + \frac{(k-1)\pi}{4}$$



$$\varphi_k = \varphi + \frac{(k-1)\pi}{6} \qquad k = 1,2,3,4,5,6$$

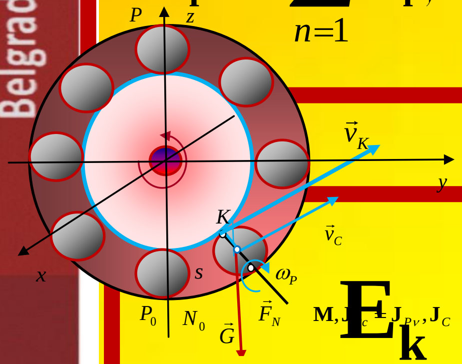
$$\varphi_k = \varphi + \pi + \frac{(k-1)\pi}{6}$$



$$\omega_{P,n} = \omega_P = \frac{(R-r)\dot{\varphi}}{r} \qquad \omega_{P,n} = \omega_P = \frac{(R-2r)\dot{\vartheta}}{2r}$$

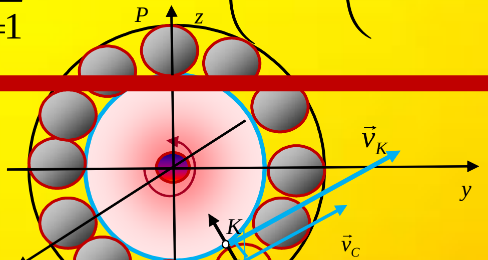
$$\mathbf{E_k} = \frac{1}{2}\mathbf{J_P}\left[\frac{(R-r)\dot{\varphi}}{r}\right]^2 \sum_{n=1}^{n=4}(1+p_n) + \frac{1}{2}\mathbf{J}_O 4\left[\frac{(R-r)\dot{\varphi}}{(R-2r)}\right]^2$$

$$\mathbf{E_p} = \sum_{n=1}^{n=4}\mathbf{E}_{\mathbf{p},n} = \frac{1}{2}mg(R-r)\sum_{n=1}^{n=4}(1-p_n)\left(\cos\left(\pi+\frac{(n-1)\pi}{4}\right) - coss\left(\varphi+\pi+\frac{(n-1)\pi}{4}\right)\right)$$



$$\mathbf{E_k} = \frac{1}{2}\mathbf{J_P}\left[\frac{(R-r)\dot{\varphi}}{r}\right]^2 \sum_{n=1}^{n=6}(1+p_n) + \frac{1}{2}\mathbf{J}_O 4\left[\frac{(R-r)\dot{\varphi}}{(R-2r)}\right]^2$$

$$\mathbf{E_p} = \sum_{n=1}^{n=6}\mathbf{E}_{\mathbf{p},n} = \frac{1}{2}mg(R-r)\sum_{n=1}^{n=6}(1-p_n)\left(\cos\left(\pi+\frac{(n-1)\pi}{6}\right) - \cos\left(\varphi+\pi+\frac{(n-1)\pi}{6}\right)\right)$$



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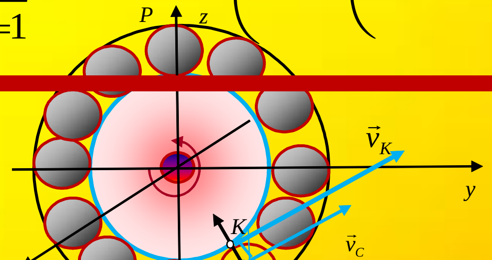
$$\mathbf{E_k} = \frac{1}{2}\mathbf{J_P}\left[\frac{(R-r)\dot{\varphi}}{r}\right]^2 \sum_{n=1}^{n=4}(1+p_n) + \frac{1}{2}\mathbf{J}_O 4\left[\frac{(R-r)\dot{\varphi}}{(R-2r)}\right]^2$$

$$\mathbf{E_p} = \sum_{n=1}^{n=4}\mathbf{E}_{\mathbf{p},n} = \frac{1}{2}mg(R-r)\sum_{n=1}^{n=4}(1-p_n)\left(cos\left(\pi+\frac{(n-1)\pi}{4}\right) - coss\left(\varphi+\pi+\frac{(n-1)\pi}{4}\right)\right)$$

$$\mathbf{E_k} = \frac{1}{2}\mathbf{J_P}\left[\frac{(R-r)\dot{\varphi}}{r}\right]^2 \sum_{n=1}^{n=6}(1+p_n) + \frac{1}{2}\mathbf{J}_O 4\left[\frac{(R-r)\dot{\varphi}}{(R-2r)}\right]^2$$

$$\mathbf{E_p} = \sum_{n=1}^{n=6}\mathbf{E}_{\mathbf{p},n} = \frac{1}{2}mg(R-r)\sum_{n=1}^{n=6}(1-p_n)\left(cos\left(\pi+\frac{(n-1)\pi}{6}\right) - cos\left(\varphi+\pi+\frac{(n-1)\pi}{6}\right)\right)$$



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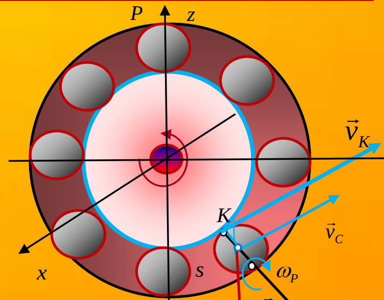
$$\mathbf{E_k} = \frac{1}{2}\mathbf{J_P}\left[\frac{(R-r)\dot{\varphi}}{r}\right]^2 \sum_{n=1}^{n=4}(1+p_n) + \frac{1}{2}\mathbf{J}_O 4\left[\frac{(R-r)\dot{\varphi}}{(R-2r)}\right]^2$$

$$\mathbf{E_p} = \sum_{n=1}^{n=4}\mathbf{E}_{\mathbf{p},n} = \frac{1}{2}mg(R-r)\sum_{n=1}^{n=4}(1-p_n)\left(cos\left(\pi + \frac{(n-1)\pi}{4}\right) - coss\left(\varphi + \pi + \frac{(n-1)\pi}{4}\right)\right)$$

$$\ddot{\varphi} + \frac{g}{2(R-r)\left[\frac{\mathbf{J_P}}{mr^2}\sum_{n=1}^{n=4}(1+p_n) + \frac{2\mathbf{J}_O}{m(R-2r)^2}\right]}\sum_{n=1}^{n=4}(1-p_n)sin\left(\varphi + \pi + \frac{(n-1)\pi}{4}\right) = 0$$



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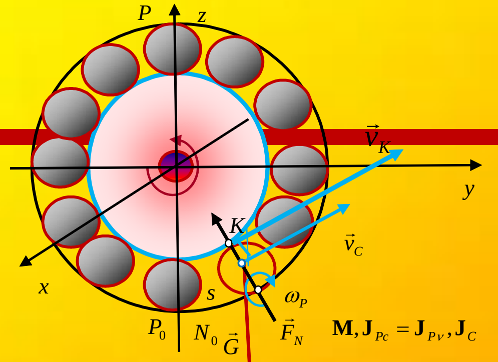
$$\mathbf{E_k} = \frac{1}{2}\mathbf{J_P}\left[\frac{(R-r)\dot{\varphi}}{r}\right]^2 \sum_{n=1}^{n=6}(1+p_n) + \frac{1}{2}\mathbf{J}_O 4\left[\frac{(R-r)\dot{\varphi}}{(R-2r)}\right]^2$$

$$\mathbf{E_p} = \sum_{n=1}^{n=6}\mathbf{E}_{\mathbf{p},n} = \frac{1}{2}mg(R-r)\sum_{n=1}^{n=6}(1-p_n)\left(\cos\left(\pi+\frac{(n-1)\pi}{6}\right) - \cos\left(\varphi+\pi+\frac{(n-1)\pi}{6}\right)\right)$$

$$\ddot{\varphi} + \frac{g}{2(R-r)\left[\frac{\mathbf{J_P}}{mr^2}\sum_{n=1}^{n=6}(1+p_n) + \frac{2\mathbf{J}_O}{m(R-2r)^2}\right]}\sum_{n=1}^{n=6}(1-p_n)\sin\left(\varphi+\pi+\frac{(n-1)\pi}{6}\right) = 0$$



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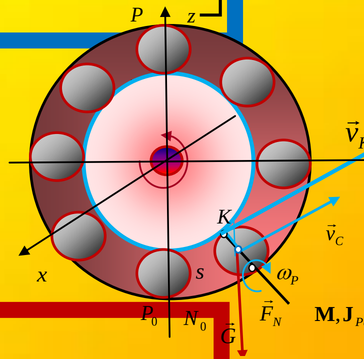
$$\frac{g}{\lambda} = \frac{g}{2(R-r)\left[\frac{\mathbf{J_P}}{mr^2}\sum_{n=1}^{n=4}(1+p_n) + \frac{2\mathbf{J}_O}{m(R-2r)^2}\right]} = \frac{g}{2(R-r)\left[\mathbf{i}_\mathbf{P}^2\sum_{n=1}^{n=4}(1+p_n) + 2\mathbf{i}_0^2\right]}$$

$$\lambda = 2(R-r)\left[\frac{\mathbf{J_P}}{mr^2}\sum_{n=1}^{n=4}(1+p_n) + \frac{2\mathbf{J}_O}{m(R-2r)^2}\right] = 2(R-r)\left[\mathbf{i}_\mathbf{P}^2\sum_{n=1}^{n=4}(1+p_n) + 2\mathbf{i}_0^2\right]$$

$$\kappa = \left[\mathbf{i}_\mathbf{P}^2\sum_{n=1}^{n=4}(1+p_n) + 2\mathbf{i}_0^2\right]$$



$$\ddot{\varphi} + \frac{g}{\lambda}\sum_{n=1}^{n=4}(1-p_n)\sin\left(\varphi + \frac{(n-1)\pi}{4}\right) = 0$$

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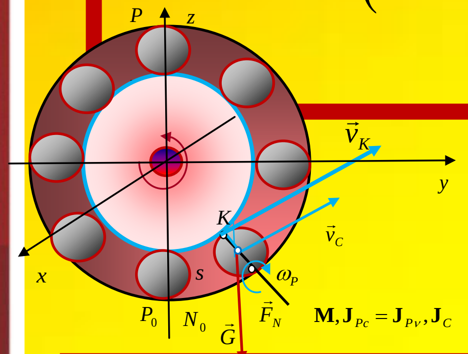
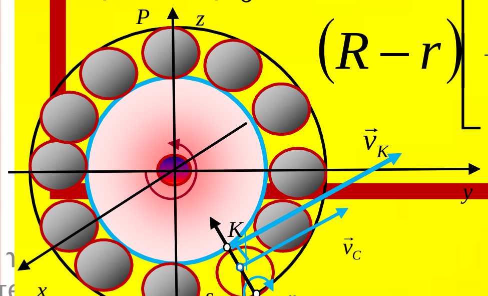
$$\mathbf{E} = \mathbf{E_k} + \mathbf{E_p} = \mathbf{E}_0 - const$$

$$\dot{\varphi}^2 = \dot{\varphi}_0^2 - \frac{2g}{(R-r)\left[\frac{\mathbf{J_P}}{mr^2}\sum_{n=1}^{n=4}(1+p_n)+\frac{2\mathbf{J}_O}{m(R-2r)^2}\right]}\sum_{n=1}^{n=4}\frac{e_n(1-p_n)}{(R-r)}\left(\cos\frac{(n-1)\pi}{4}-\cos\left\langle\frac{(n-1)\pi}{4}+\varphi\right\rangle\right)$$



$$\dot{\varphi}^2 = \dot{\varphi}_0^2 - \frac{2g}{(R-r)\left[\frac{\mathbf{J_P}}{mr^2}\sum_{n=1}^{n=6}(1+p_n)+\frac{2\mathbf{J}_O}{m(R-2r)^2}\right]}\sum_{n=1}^{n=6}\frac{e_n(1-p_n)}{(R-r)}\left(\cos\frac{(n-1)\pi}{6}-\cos\left\langle\frac{(n-1)\pi}{6}+\varphi\right\rangle\right)$$



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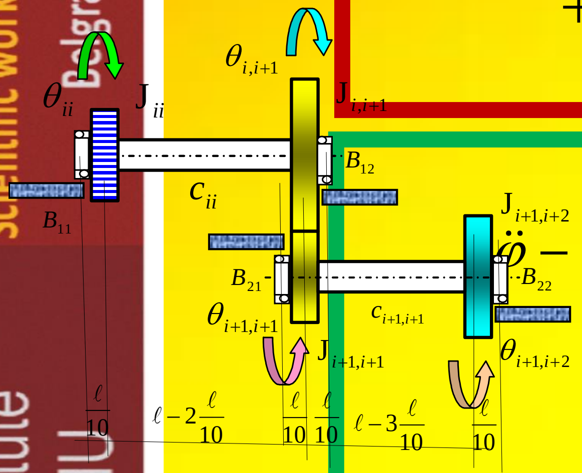
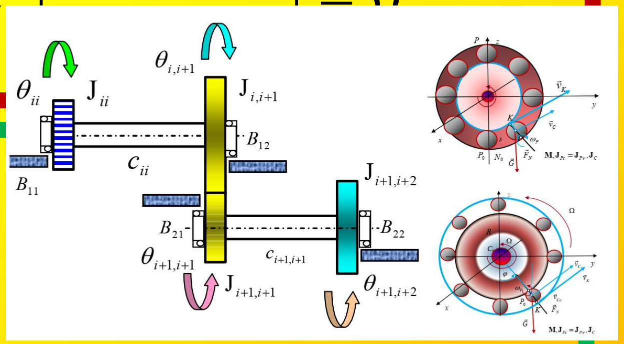
$$\ddot{\vartheta} - \frac{g}{(R-2r)\kappa_{deb1}}\sin\vartheta + \frac{g}{(R-2r)\kappa_{deb2}}\sin i_{12}\vartheta + \frac{g}{(R-2r)\kappa_m}\sum_{n=1}^{n=4}(1-p_{1,n})\sin\left(\frac{(R-2r)\vartheta}{2(R-r)}+\pi+\frac{(n-1)\pi}{4}\right) + \frac{g i_{12}}{(R-2r)\kappa_m}\sum_{n=1}^{n=4}(1-p_{1,n})\sin\left(i_{12}\frac{(R-2r)\vartheta}{2(R-r)}+\pi+\frac{(n-1)\pi}{4}\right)=0$$





$$\ddot{\varphi} - \frac{(R-2r)}{2(R-r)}\frac{(m_{11}R_{11}+m_{12}R_{12})g}{(\mathbf{J}_e+\mathbf{J}_{bearing,e})}\sin 2\frac{(R-r)\varphi}{(R-2r)} + i_{12}\frac{(R-2r)}{2(R-r)}\frac{(m_{23}R_{23}+m_{24}R_{24})g}{(\mathbf{J}_e+\mathbf{J}_{bearing,e})}\sin 2i_{12}\frac{(R-r)\varphi}{(R-2r)} + \frac{mg}{(\mathbf{J}_e+\mathbf{J}_{bearing,e})}\frac{(R-2r)^2}{8(R-r)}\sum_{n=1}^{n=4}(1-p_{1,n})\sin\left(\varphi+\pi+\frac{(n-1)\pi}{4}\right) + \frac{mg i_{12}}{(\mathbf{J}_e+\mathbf{J}_{bearing,e})}\frac{(R-2r)^2}{8(R-r)}\sum_{n=1}^{n=4}(1-p_{2,n})\sin\left(i_{12}\varphi+\pi+\frac{(n-1)\pi}{4}\right)=0$$

$$\frac{g}{(R-2r)\kappa_{deb1}} = \frac{g}{(R-2r)\dfrac{\left(\mathbf{J}_e + \mathbf{J}_{bearing,e}\right)}{(m_{11}R_{11} + m_{12}R_{12})(R-2r)}}$$

$$\kappa_{deb1} = \frac{\left(\mathbf{J}_e + \mathbf{J}_{bearing,e}\right)}{(m_{11}R_{11} + m_{12}R_{12})(R-2r)}$$

$$\frac{g}{(R-2r)\kappa_{deb2}} = \frac{g}{(R-2r)\dfrac{\left(\mathbf{J}_e + \mathbf{J}_{bearing,e}\right)}{(m_{23}R_{23} + m_{24}R_{24})(R-2r)}}$$

$$\kappa_{deb2} = \frac{\left(\mathbf{J}_e + \mathbf{J}_{bearing,e}\right)}{(m_{23}R_{23} + m_{24}R_{24})(R-2r)}$$

$$\frac{g}{(R-2r)\kappa_m} = \frac{g}{(R-2r)\left(\dfrac{\mathbf{J}_e + \mathbf{J}_{bearing,e}}{m(R-r)^2}\right)}$$

$$\kappa_m = \frac{\mathbf{J}_e + \mathbf{J}_{bearing,e}}{m(R-r)^2}$$

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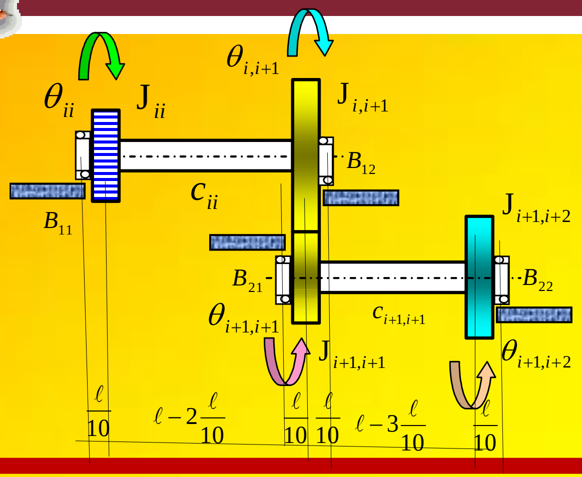

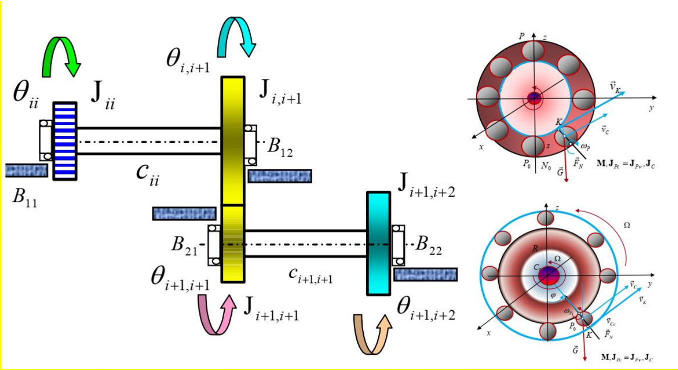

$$\dot{\varphi}^2-\dot{\varphi}_0^2=\frac{(R-2r)^2}{4(R-r)^2}\left\langle\frac{2m_{11}gR_{11}}{\left(\mathbf{J}_e+\mathbf{J}_{bearing,e}\right)}\left(\cos\alpha_{11}-\cos2\frac{(R-r)\varphi}{(R-2r)}\right)+\frac{2m_{12}gR_{12}}{\left(\mathbf{J}_e+\mathbf{J}_{bearing,e}\right)}\left(\cos\alpha_{12}-\cos2\frac{(R-r)\varphi}{(R-2r)}\right)\right\rangle-$$

$$-\frac{(R-2r)^2}{4(R-r)^2}\left\langle\frac{2m_{23}gR_{23}}{\left(\mathbf{J}_e+\mathbf{J}_{bearing,e}\right)}\left(\cos\alpha_{23}-\cos i_{12}2i_{12}\frac{(R-r)\varphi}{(R-2r)}\right)+\frac{2m_{24}gR_{24}}{\left(\mathbf{J}_e+\mathbf{J}_{bearing,e}\right)}\left(\cos\alpha_{24}-\cos2i_{12}\frac{(R-r)\varphi}{(R-2r)}\right)\right\rangle+$$

$$-\frac{(R-2r)^2}{4(R-r)^2}\frac{mg(R-r)}{\left(\mathbf{J}_e+\mathbf{J}_{bearing,e}\right)}\sum_{n=1}^{n=4}\left(1-p_{1,n}\right)\left(\cos\sigma\left(\pi+\frac{(n-1)\pi}{4}\right)-\cos\left(\varphi+\pi+\frac{(n-1)\pi}{4}\right)\right)+$$

$$-\frac{(R-2r)^2}{4(R-r)^2}\frac{mg(R-r)}{\left(\mathbf{J}_e+\mathbf{J}_{bearing,e}\right)}\sum_{n=1}^{n=4}\left(1-p_{2,n}\right)\left(\cos\sigma\left(\pi+\frac{(n-1)\pi}{4}\right)-\cos\left(i_{12}\varphi+\pi+\frac{(n-1)\pi}{4}\right)\right)$$


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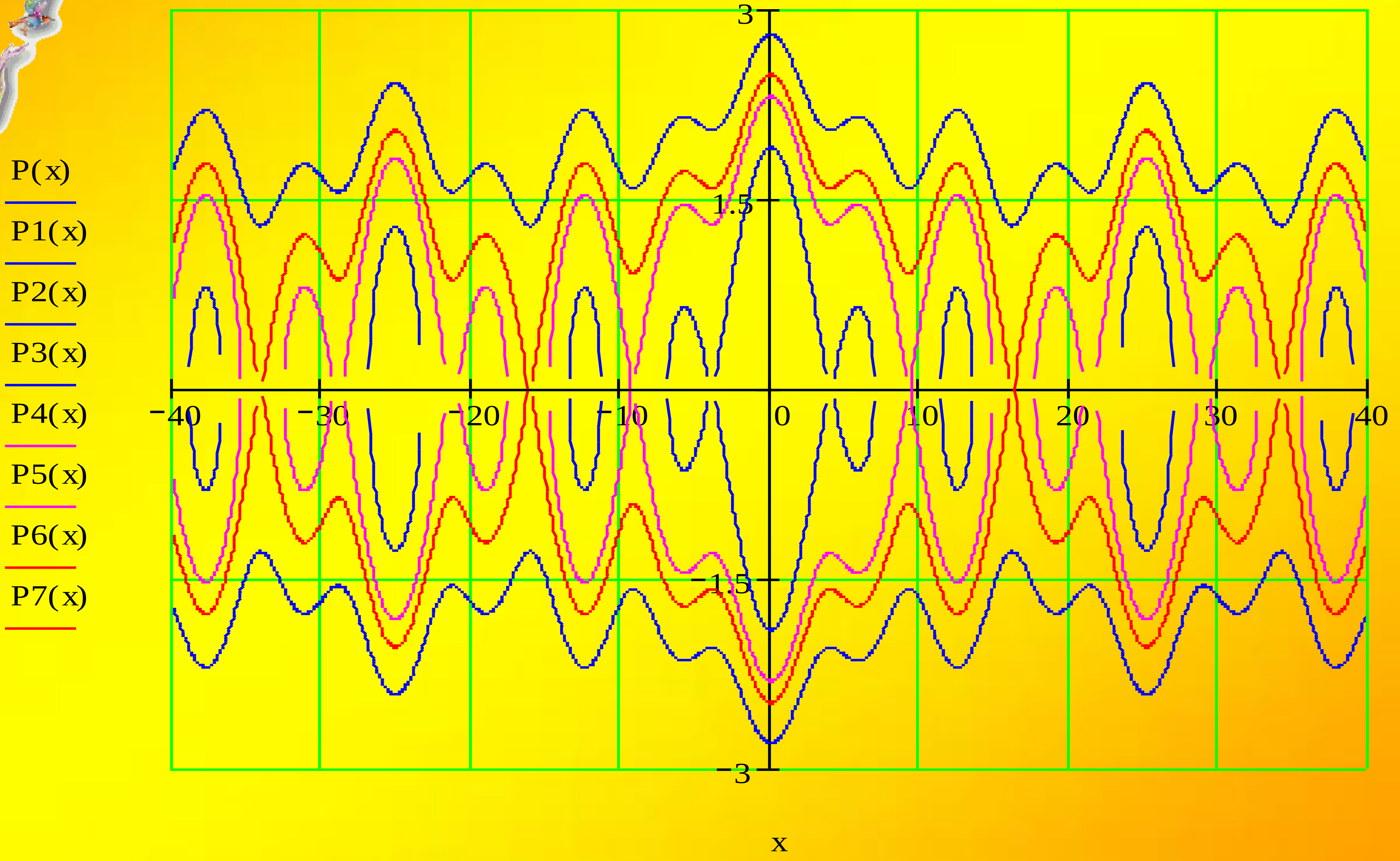
P(x)
P1(x)
P2(x)
P3(x)
P4(x)
P5(x)
P6(x)
P7(x)
3
1.5
−1.5
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−40
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−20
−10
0
10
20
30
40
x

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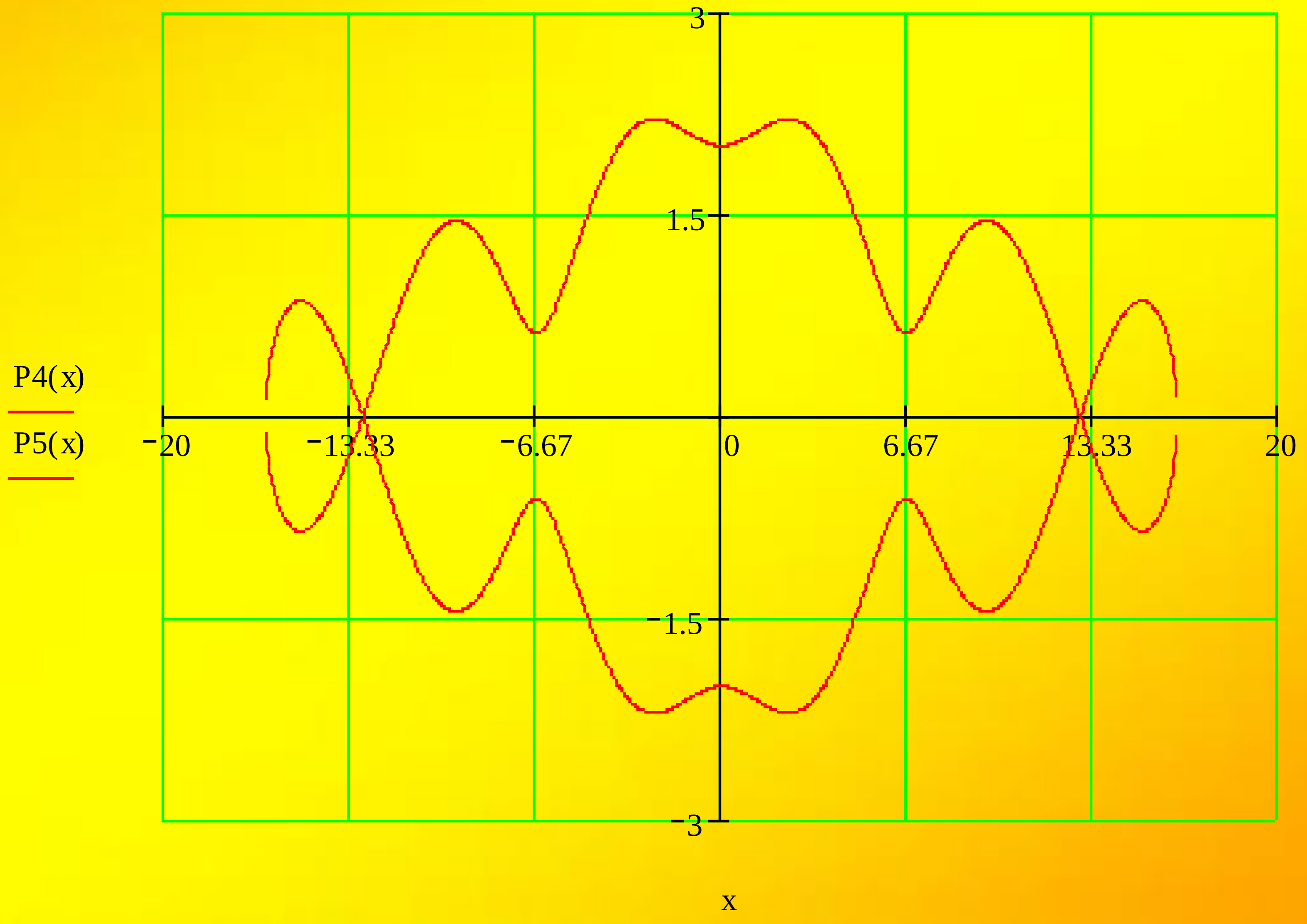

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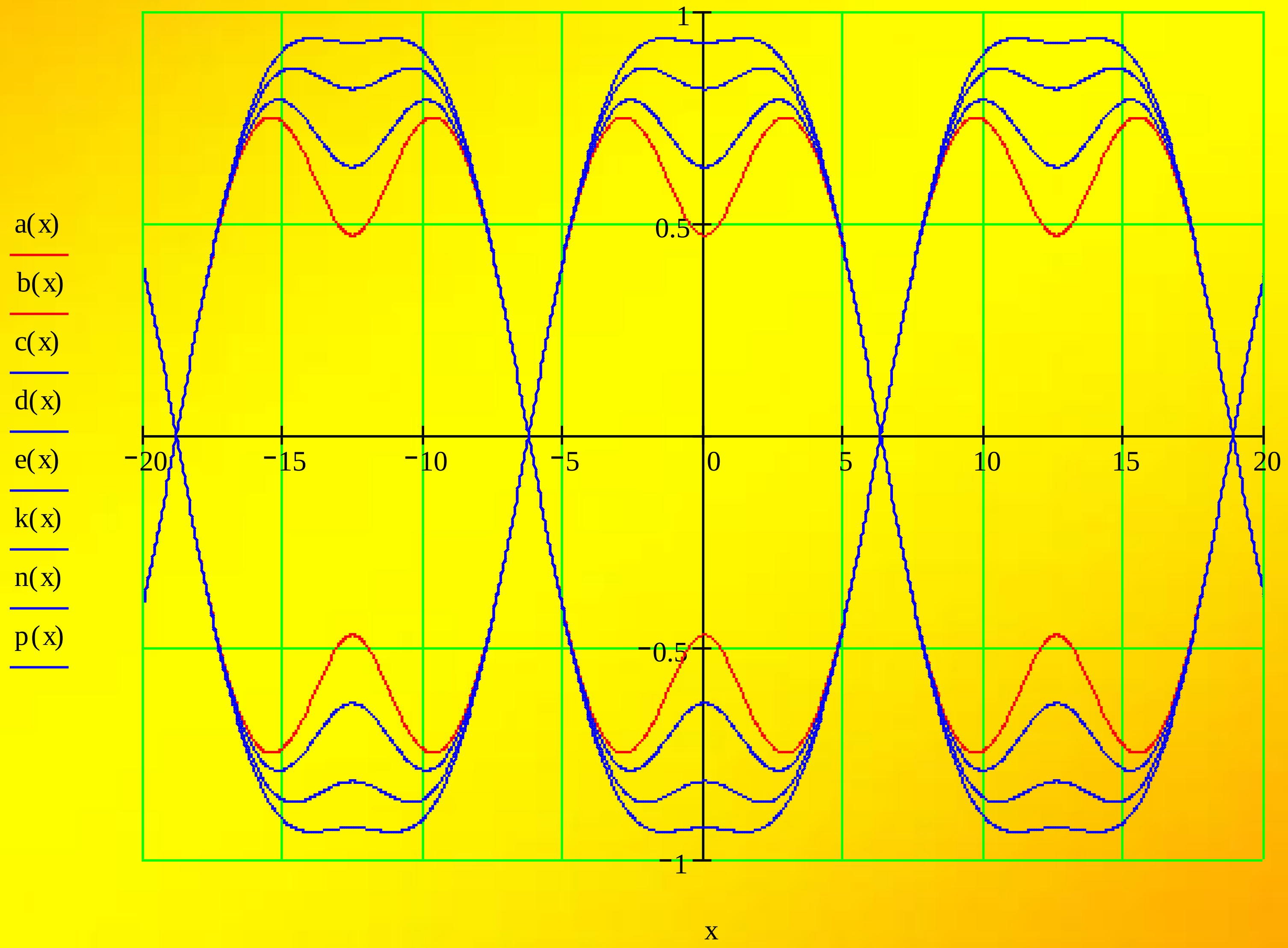

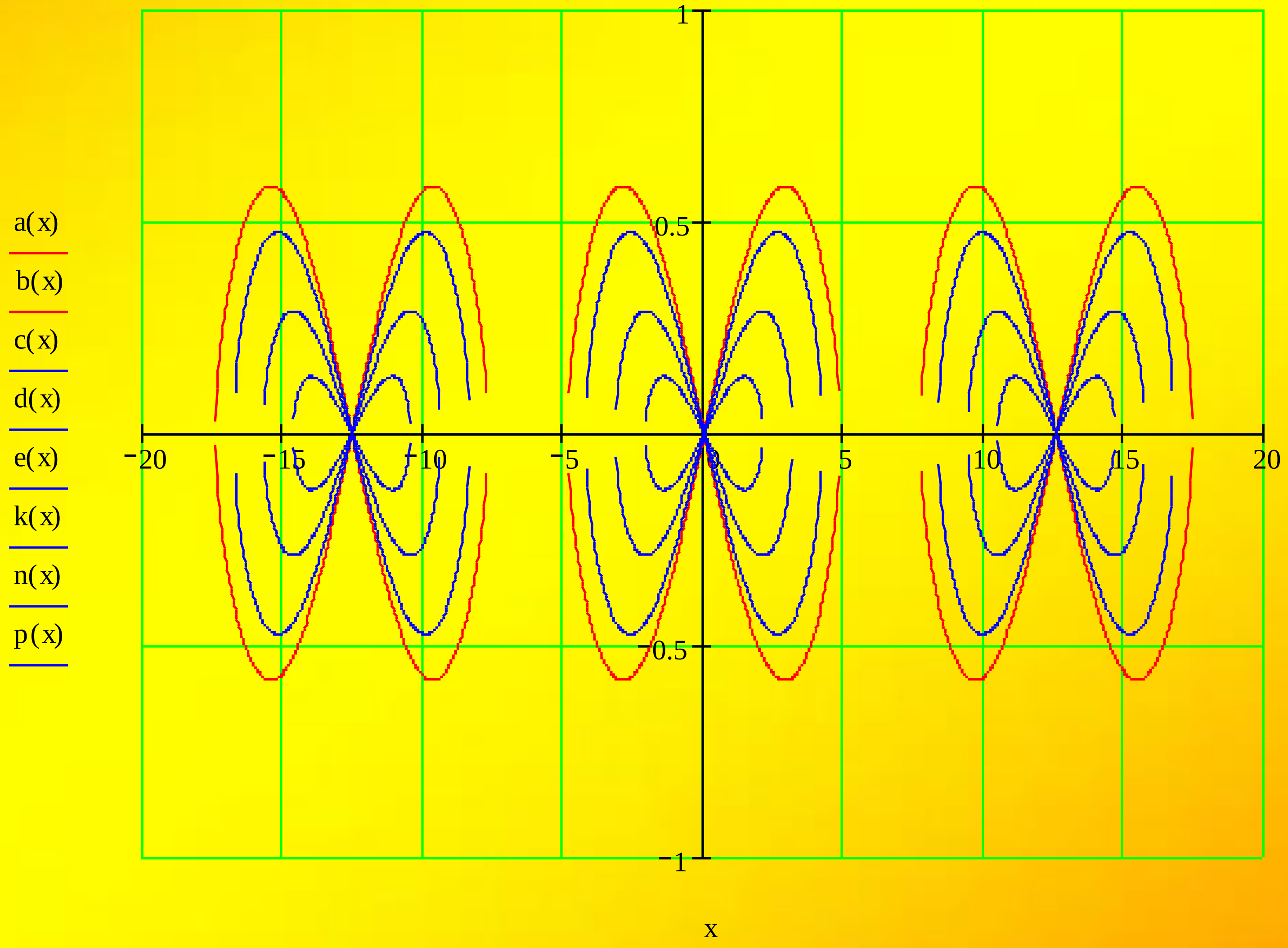
a(x)
b(x)
c(x)
d(x)
e(x)
k(x)
n(x)
p(x)
1
0.5
−0.5
−1
−20
−15
−10
−5
0
5
10
15
20
x

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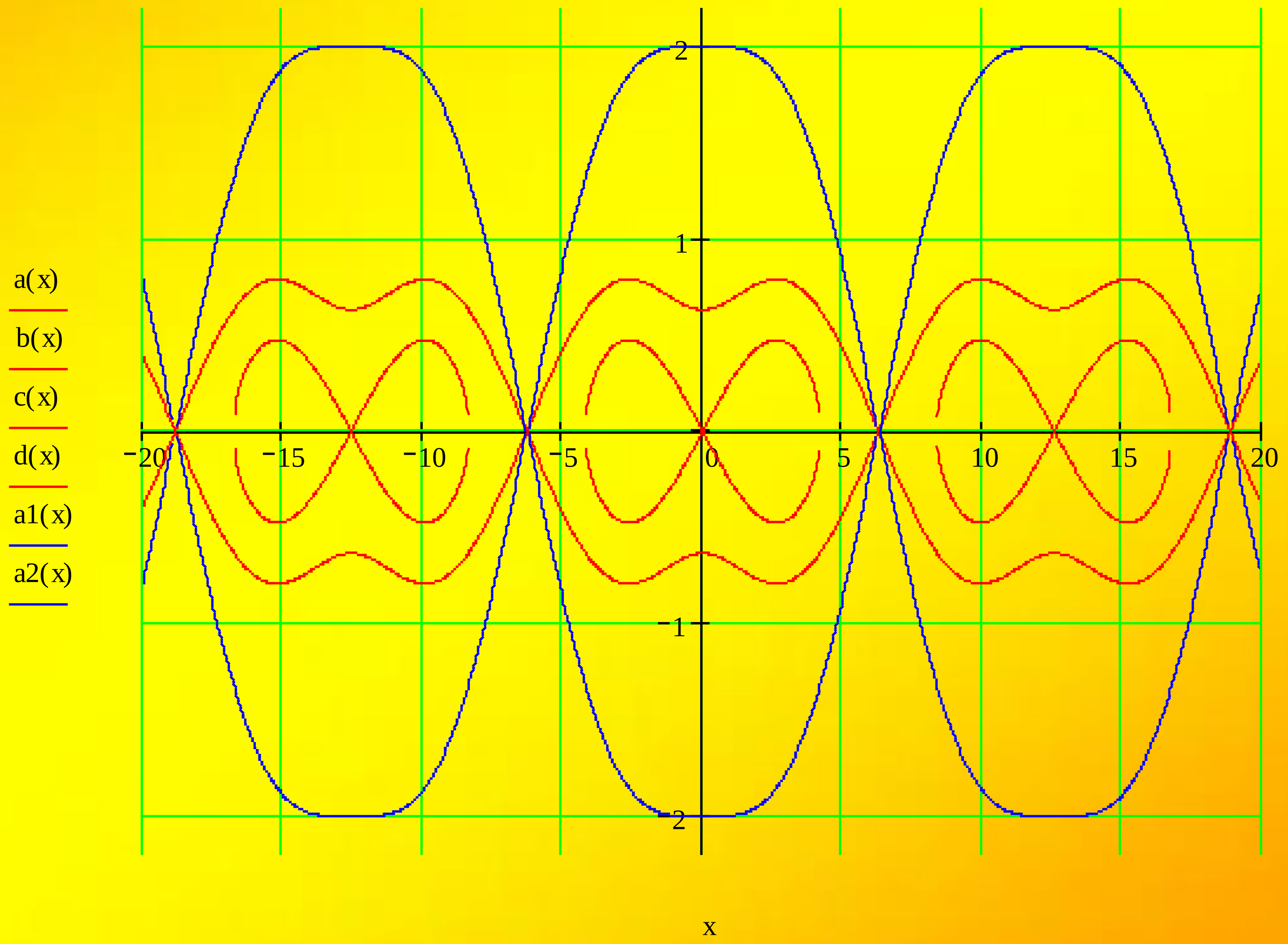

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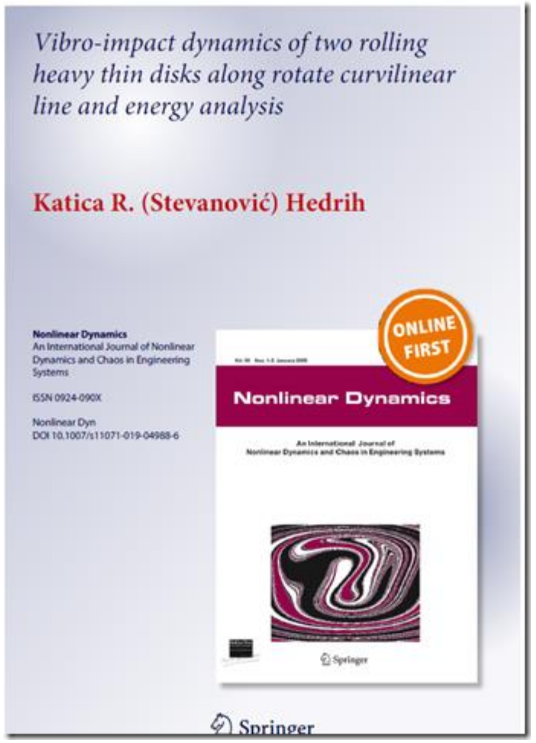

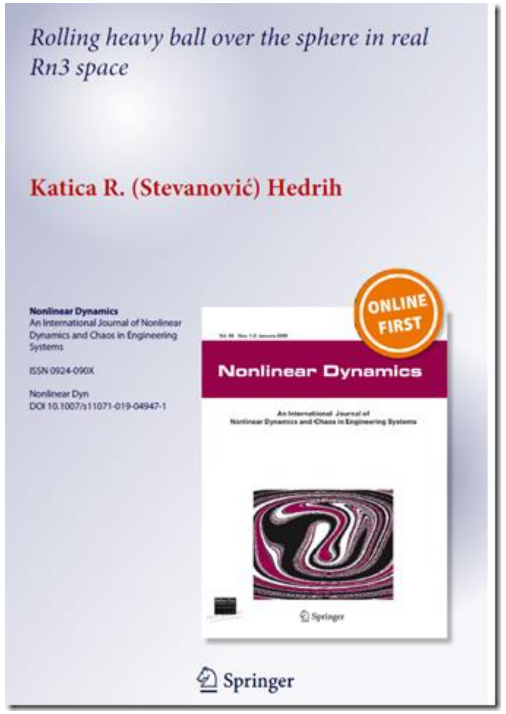

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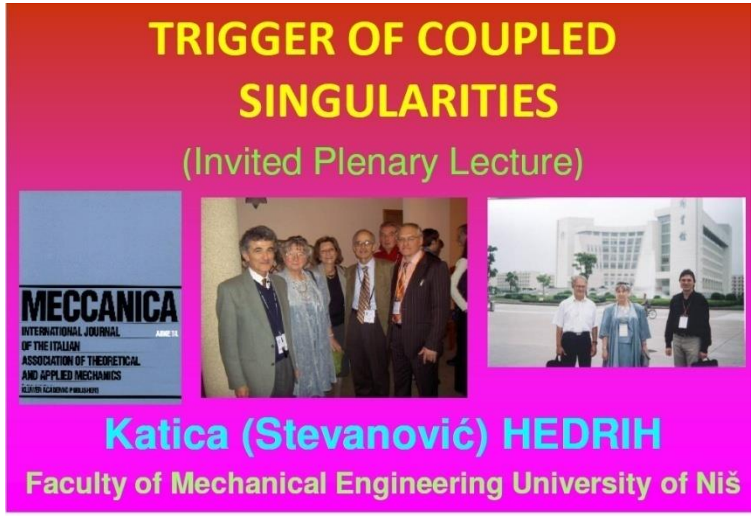
Hedrih, Katica R. (Stevanovic)
Vibro-impact dynamics of two rolling heavy thin disks along rotate curvilinear line and energy analysis
Nonlinear Dynamics. 2019



Theorem: *In the system whose dynamics can be described with the use of non-linear differential equation in the form:*

$$\ddot{x} + g[k, F(x)]f(x) = 0 \qquad (1)$$

and whose potential energy is in the form:

$$\mathbf{E} = m\int_0^x g[k, F(x)]f(x)dx = \mathbf{G}[k, F(x)] \qquad (2)$$

in which the functions $f(x)$ *and* $g(x)$ *are:*

$$F(x) = \int_0^x f(x)dx \quad \text{and} \quad G(k,x) = \int_0^x g(k,x)dx \qquad (3)$$

and satisfy the following conditions:

and satisfy the following conditions:

$$f(-x) = -f(x) \qquad g(k,-x) = g(k,x)$$

$$f(x + nT_0) = f(x) \qquad g(k, x + nT_0) = g(k,x) \qquad (4)$$

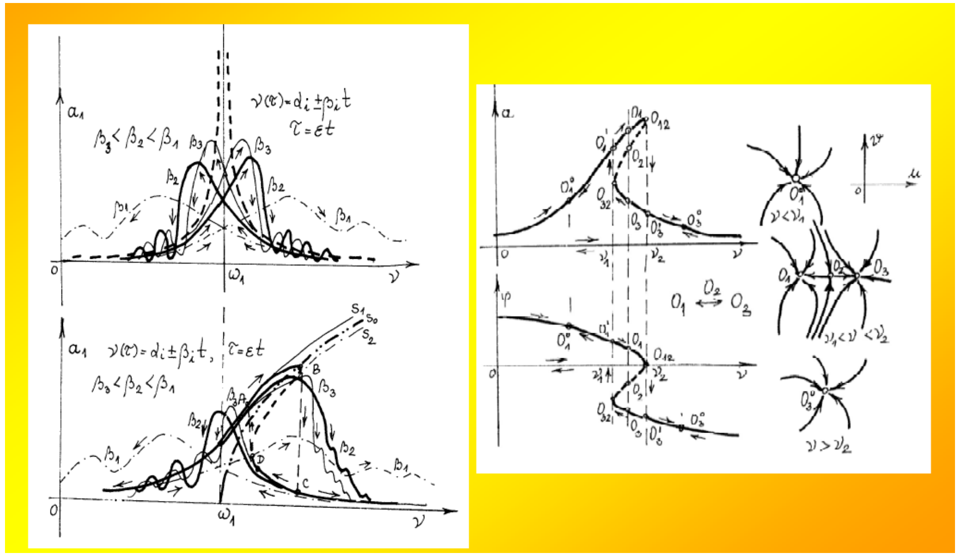
$$f(0) = 0 \qquad g[k, F(x_r)] = 0, \ for\ k \in (k_1,k_2)\cup(k_2,k_3)....$$

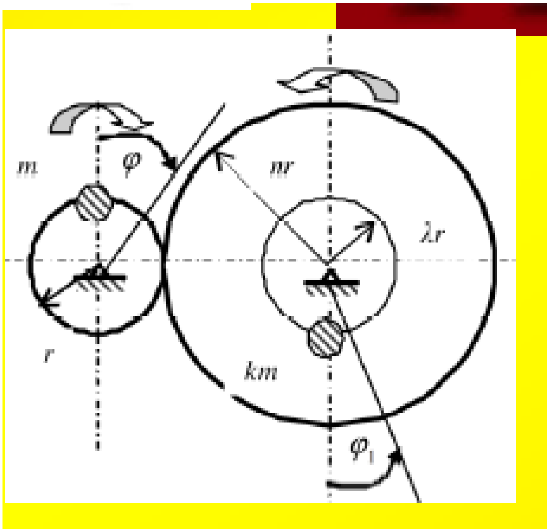
$$f(x_s) = 0 \qquad r = 0,1,2,3,4,.... \quad x_r = \pm x_0 \pm rT_0 \quad |x_0| < \frac{T_0}{2}$$

$$x_s = sT_0 \qquad g[k, F(x)] \neq 0, \ for\ k \notin (k_1,k_2)\cup(k_2,k_3)....$$

$$s = 1,2,3,4,....$$

and both functions $f(x)$ *and* $g(x)$ *have one maximum or minimum in the interval between two zero roots:*





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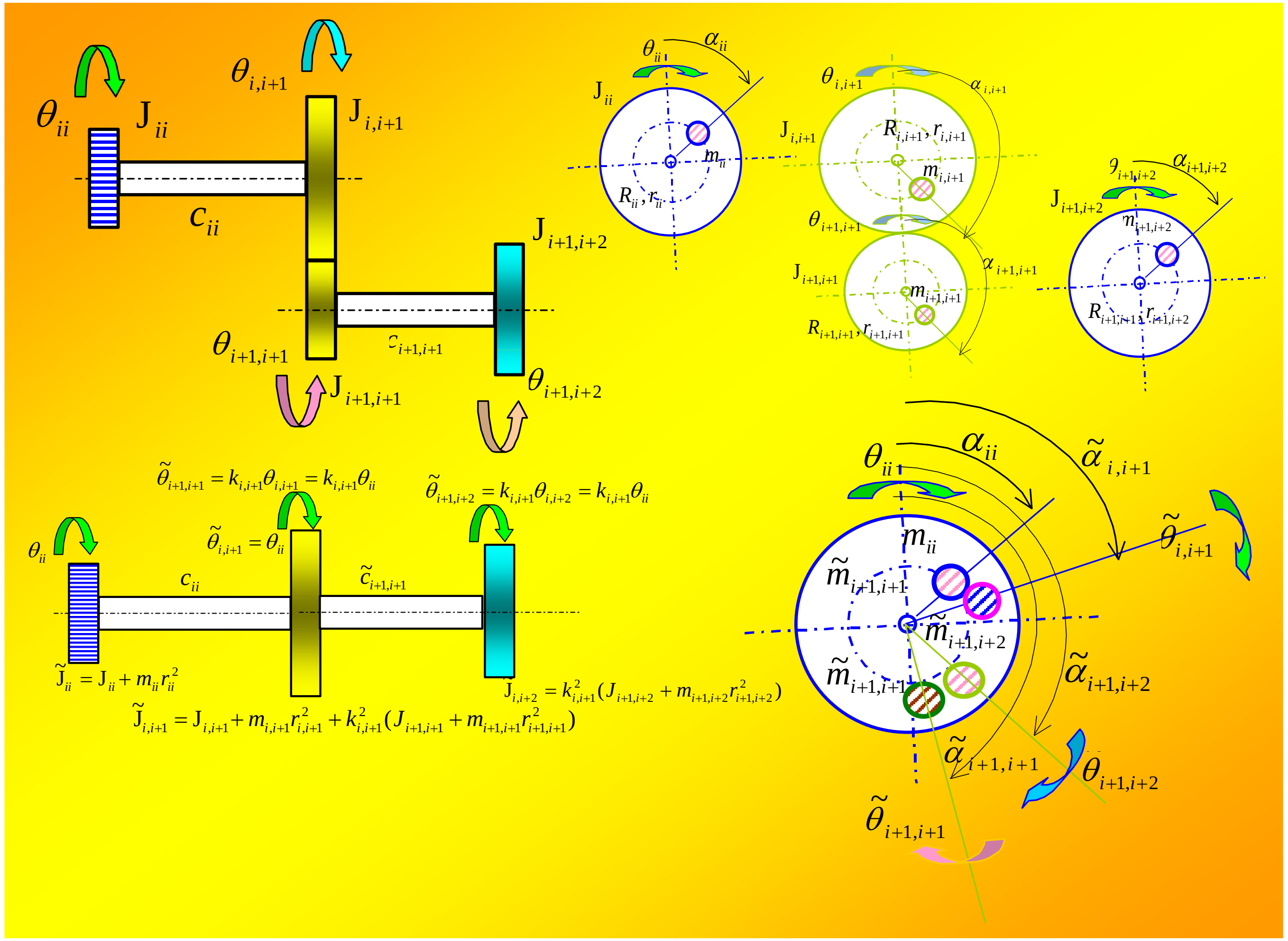

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On kinetic contact forces on the balls of radial ball bearings

Katica (Stevanović) Hedrih

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ON CENTRIFUGAL FORCES IN A MULTI-STAGE ROTATOR TRABSMISSION AND ON KINETIC CONTACT FORCES IN RADIAL BALL BEARINGS

Katica (Stevanović) Hedrih

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ABSTRACT

In the paper, the kinetic forces on the balls of the radial ball bearings of the multi-stage gear reducers, i.e. the multiplier of the revolutions of the main shaft, were determined. The kinetic contact forces of balls and circular guides, stationary and moving radial ball bearing due to the occurrence of:

a* centrifugal forces of unbalanced gears fixed on the shafts

and

b* when eccentricity of the center of mass of a pair of balls with one diameter occurs in a radial ball bearing, due to the difference in their mass density, at equal radii. **of mass of the balanced part of the gear.**

The number of revolutions of the balls in rolling, without sliding, and the change of contact points in which kinetic contact forces occur, for one revolution of each of the shafts and reduced to the main shaft, were determined. It is determined for the cases of radial ball bearings with four pairs of balls and with six pairs of balls in radial ball bearings.

The centrifugal force, which occurs due to the eccentricity of the corresponding material point, is equal to the product of the mass of the material point and its normal deflection due to the angular speed of rotation vratial,

$$\mathbf{F}_{c,m_{ik}} = -m_{ik} a_{N,m_{ik}} = -m_{ik} R_{ik} \omega_i^2 = -m_{ik} R_{ik} \dot{\vartheta}_i^2$$

It acts in the radial direction and rotates together with the shaft, with the angular velocity of the shaft.

Due to the appearance of these centrifugal forces, kinetic pressures appear on the radial balls of the shaft bearing. Those kinetic pressures on the radial ball bearings lead to the appearance of contact forces between the balls rolling on the stationary circular groove, on which they roll, and in the dynamical contact points of the movable circular groove, which rotates at the angular velocity of the shaft to which it is rigidly connected, assuming that the shaft rotates at a constant angular velocity.

Another source of centrifugal forces is the eccentricity of the center of mass of one or more pairs of balls in a radial ball bearing. The centripetal force of a pair of balls on one diameter is equal to the product of the sunn masses of the two balls and the normal acceleration of its center of mass rotating at the angular velocity of the shaft, assuming that the shaft rotates at a constant angular speed.

We have assumed that all the balls of the radial ball bearing are all with the ельуал spherical contour surfaces of equal radii. But we also introduced the assumption that in one or more pairs of balls, there are balls with different mass densities. That difference in the mass densities of the balls in a pair on one diameter is the cause of the eccentricity of the center of mass of one pair on one diameter. Now we can write that the centrifugal force that occurs due to the eccentricity of the center mass of the ball pair on one diameter:

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$$\mathbf{F}_{c,m_i} = -m_{ik}(1+p_n)a_{N,m_{ik}} = -m_{ik}(1+p_n)e_{ik}\omega_i^2 = -m_{ik}(1+p_n)e_{ik}\dot{\vartheta}_i^2 = -\frac{1}{2}(R-r)m_{ik}(1-p_n)\dot{\vartheta}_i^2$$

Where p_n **is the coefficient of inequality of the mass of the balls in the pair.**

The eccentricity of the cent mass of the pair of balls is

$$e_n = \frac{1}{2}\left(R - r\right)\frac{\left(1 - p_n\right)}{\left(1 + p_n\right)}$$

Using these assumptions, the influence of the centrifugal forces of the two-stage gear transmission was studied and the real system was reduced to a fictitious model on the first shaft with several unbalanced gears and the contact forces in the radial ball bearings were analyzed.

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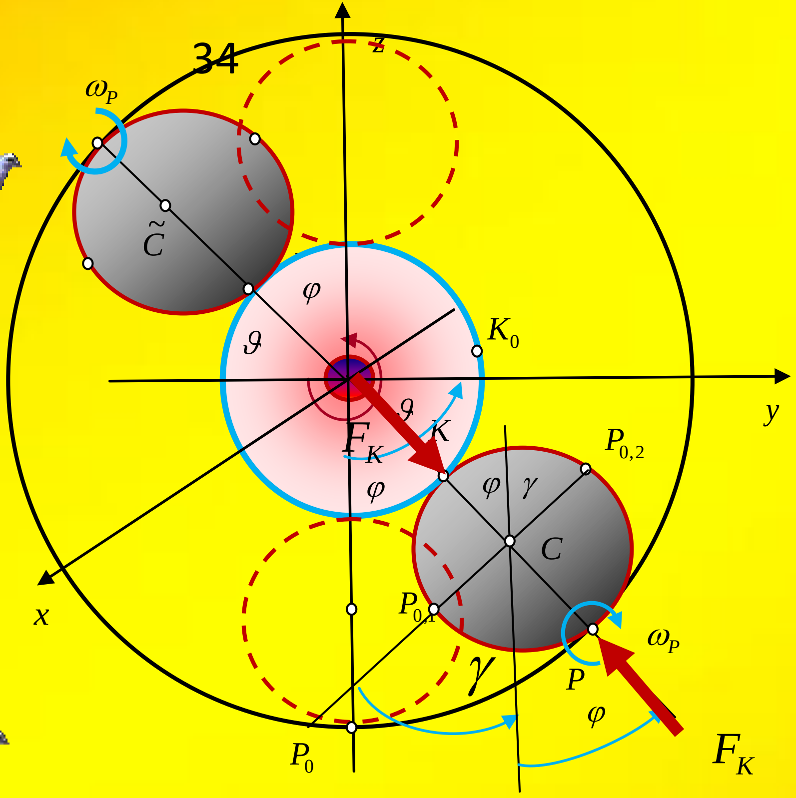
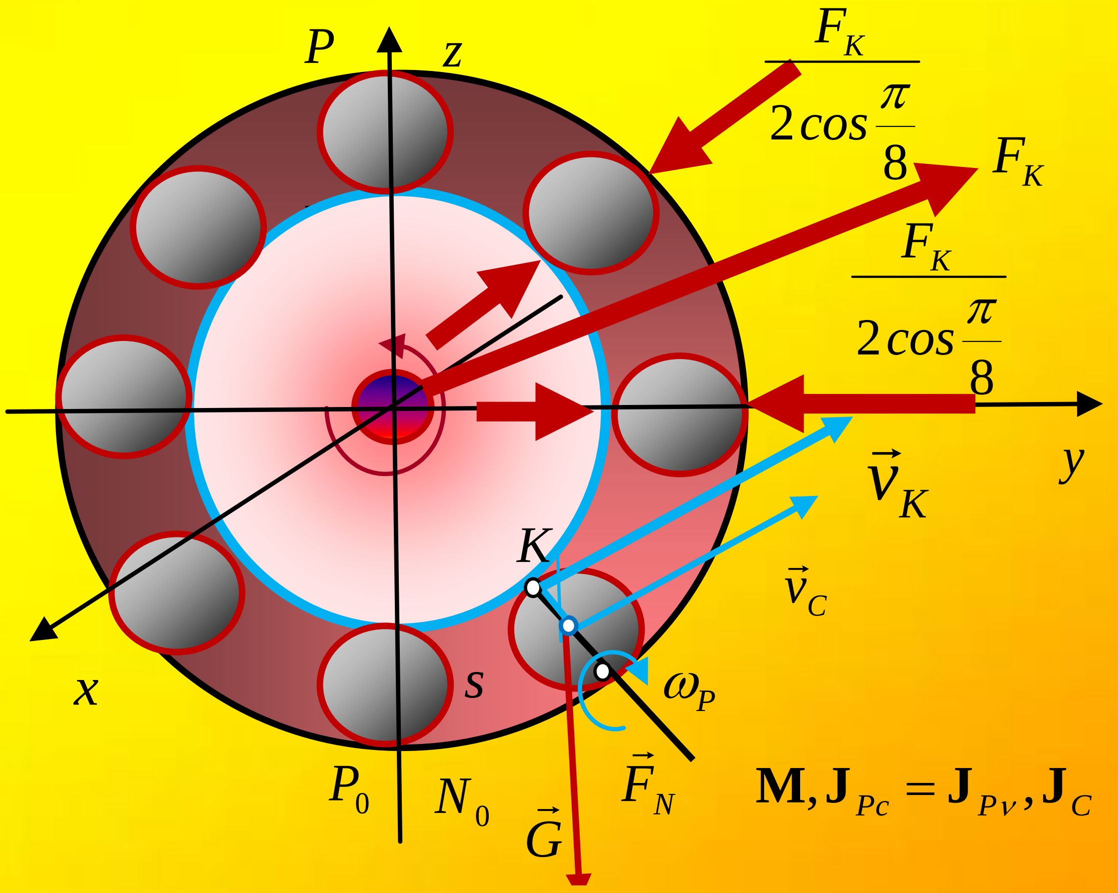
First, we observe a light, homogeneous isotropic shaft, with a circular cross-section, with two mounted unbalanced gears, which rotates at a constant angular velocity $\omega_{1=const}$ and is supported on two radial ball bearings, both at distances $\frac{\ell}{10}$ to the left and right of the first and second gears, respectively. see Figure 1).

Figure 2. Sketch of the contact forces on the balls of a radial ball bearing when: a) the direction of kinetic presssure from the centrifugal force coincides with the diameter of a pair of balls in the rolling of a radial ball bearing; c) when is the direction of kinetic pressure due to centrifugal forces between pairs of balls on two adjacent diameters of one radial ball bearing.





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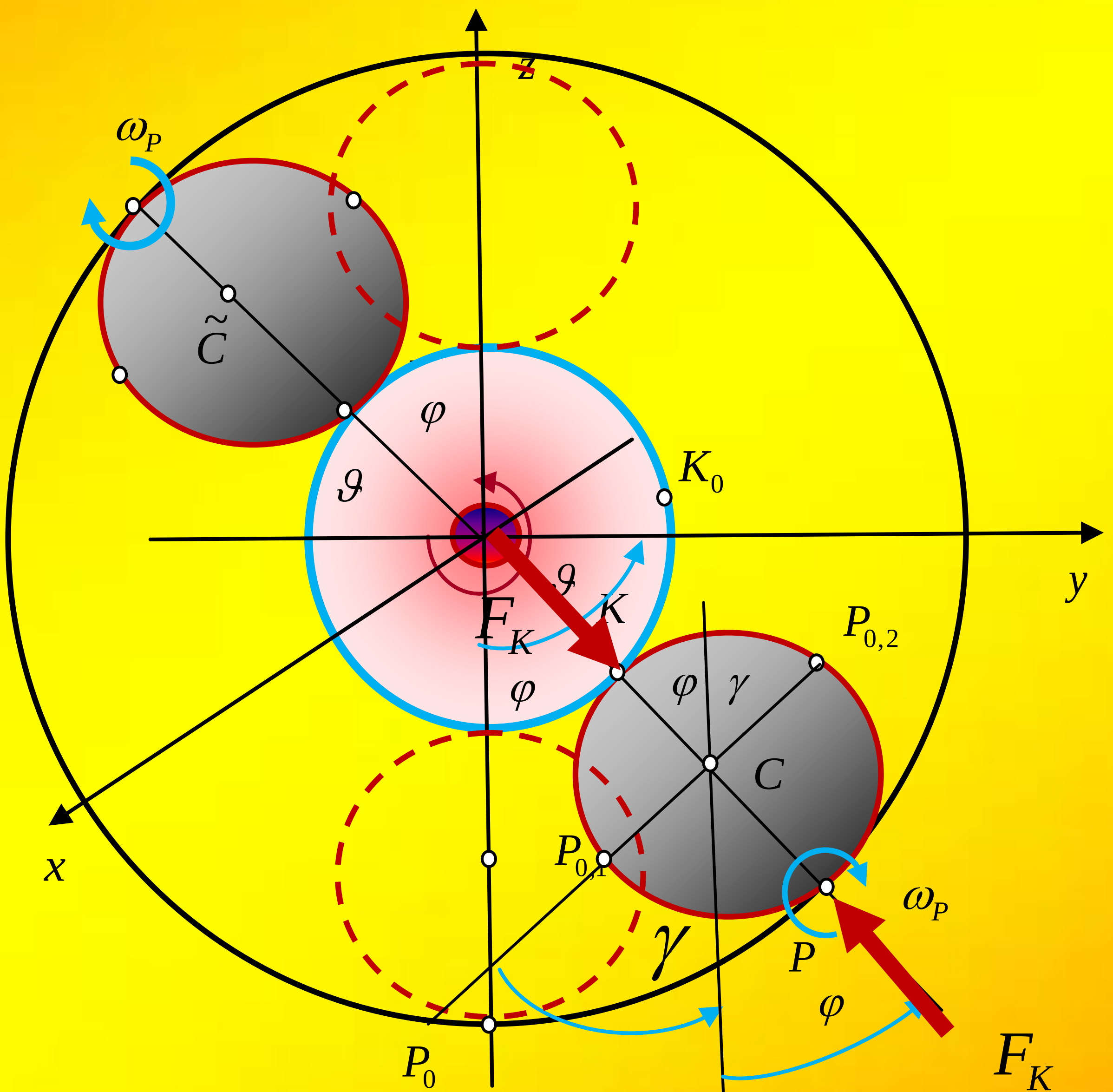

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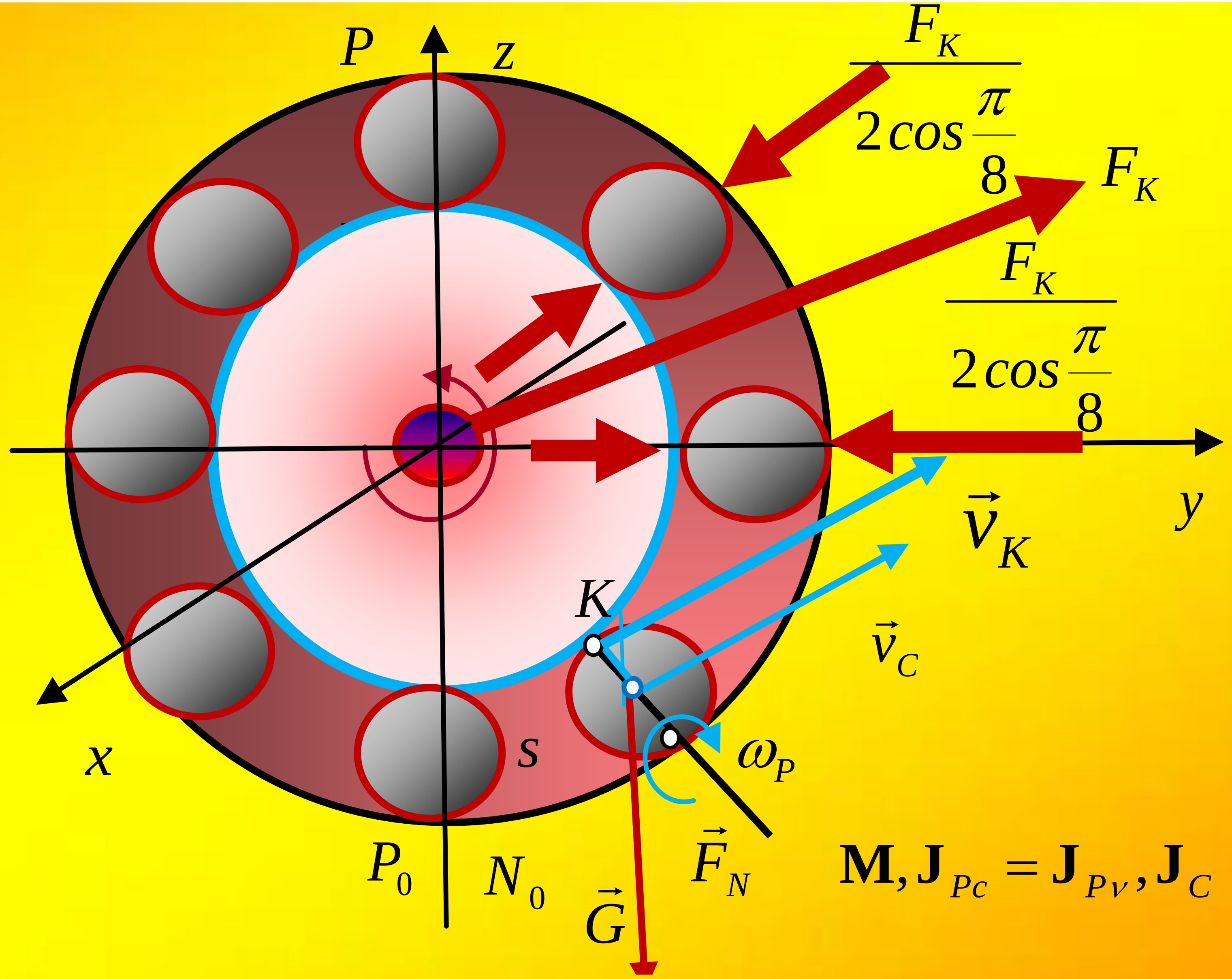

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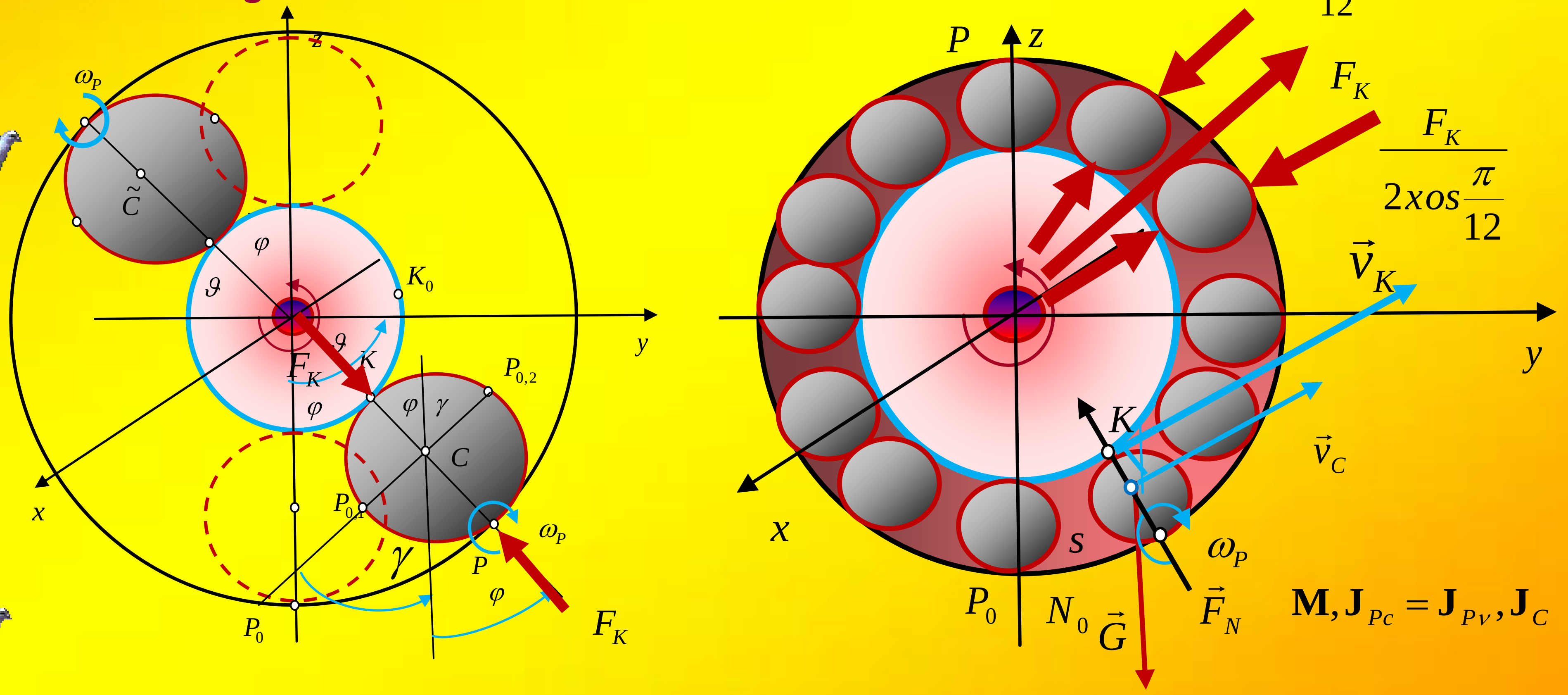
We can conclude that in the case of a radial ball bearing with four pairs of balls (eight balls) and when the direction of the resultant kinetic pressure on the bearing from centrifugal forces is on the half of the directions of the two adjacent diameters on which the adjacent pairs of balls are located, the kinetic pressures from the centrifugal forces is decomposed on two components that make angles of $\frac{\pi}{8}$ with the direction of the kinetic resultant pressure between adjacent balls. This means that all ball-adjacent pairs of two adjacent pairs experience kinetic contact pressures from the centrifugal forces of the gear imbalance on that shaft. The kinetic contact forces of the balls of a radial ball bearing are in the radial directions relative to the shaft and the diameters on which the balls are in pairs.

Figure 2. Sketch of the contact forces on the balls of a radial ball bearing when: a) the direction of kinetic presssure from the centrifugal force coincides with the diameter of a pair of balls in the rolling of a radial ball bearing; c) when is the direction of kinetic pressure due to centrifugal forces between pairs of balls on two adjacent diameters of one radial ball bearing.



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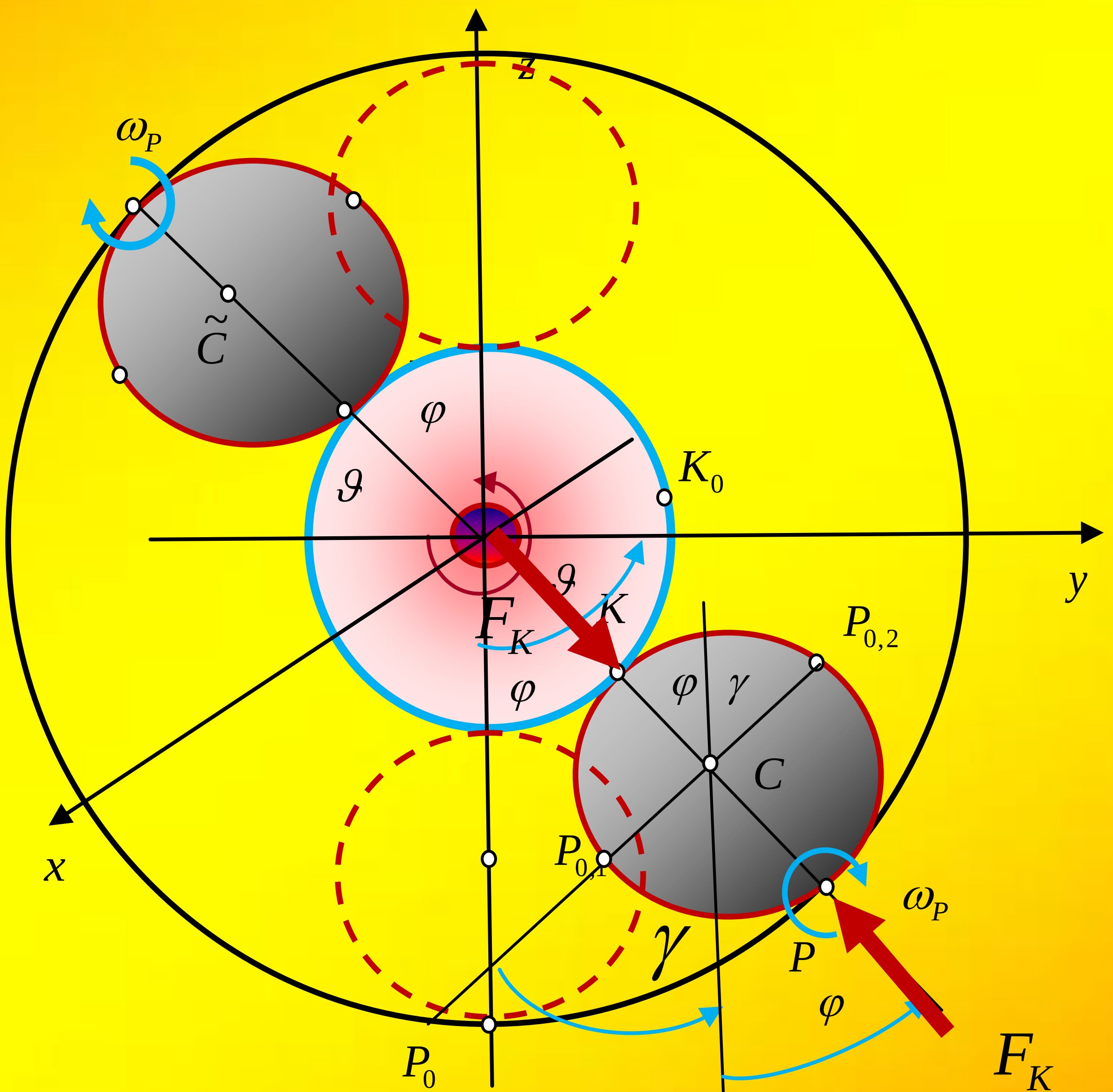

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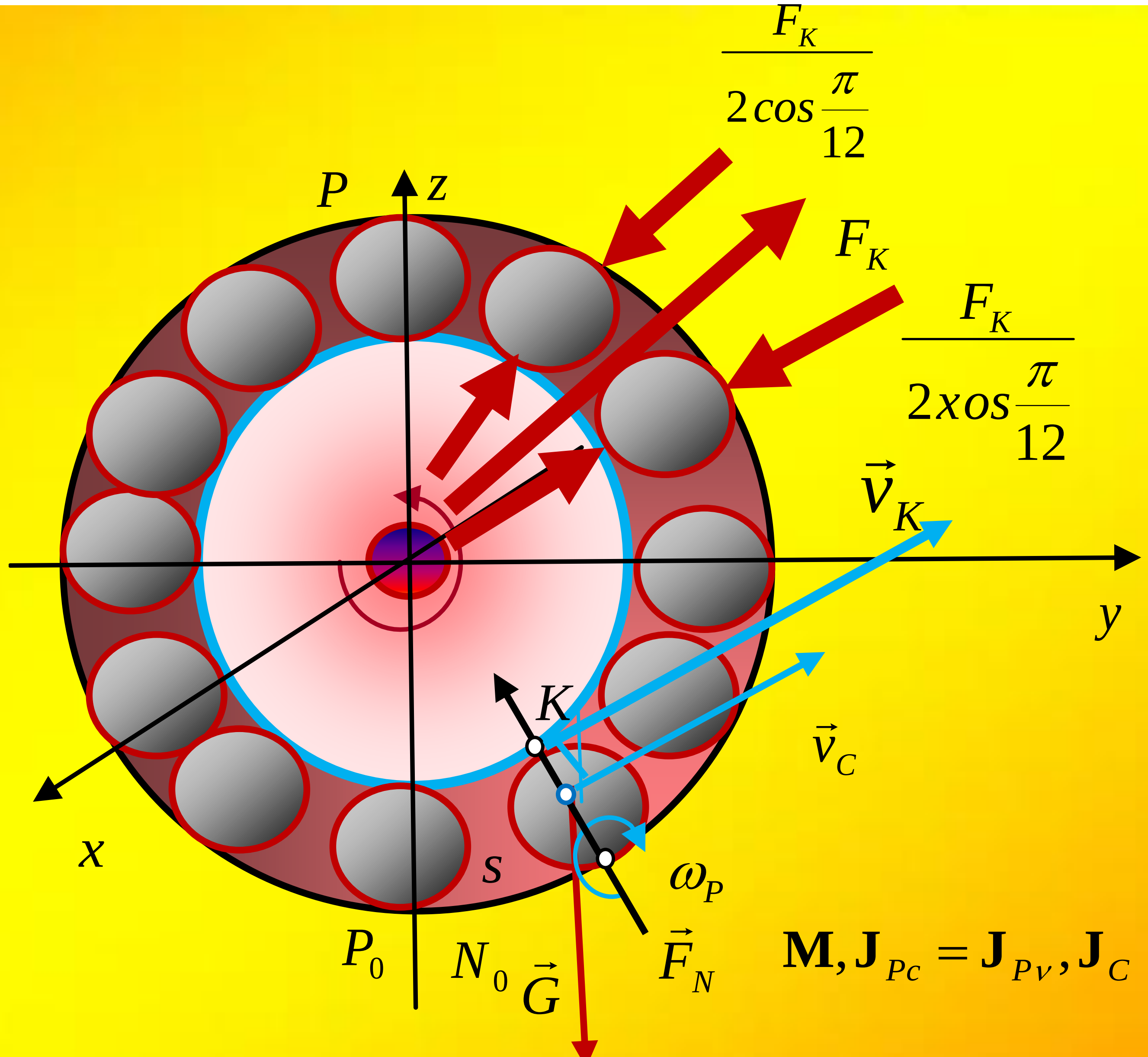

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The relationship between the angular velocity of the rolling ball and the angular velocity of the shaft should be taken into account. Here is the link:

$$\omega_{P,n} = \frac{(R-r)\dot{\varphi}_n}{r} \qquad \dot{\vartheta} = \omega_1 = 2\frac{(R-r)\dot{\varphi}_n}{(R-2r)}$$

$$\omega_{P,n} = \omega_P = \frac{(R-r)\dot{\varphi}}{r} \qquad \omega_{P,n} = \omega_P = \frac{(R-2r)\dot{\vartheta}}{2r}$$

The number of revolutions of the ball for one revolution of the shaft is:

$$N_1 = \frac{(R-2r)2\pi}{2r} = \frac{(R-2r)\pi}{r}$$

And, depends on the radius of the radial ball bearing and the radius r of the ball. It means that the ball turns several times for one revolution of the shaft, and therefore every contact point of the ball changes so much, in rolling without slipping, and exposed to the forces of kinetic pressure due to the kinetic effect of the centrifugal forces of the imbalance of the gears, for one revolution of the shaft. This number $N_1 = \frac{(R-2r)\pi}{r}$ **does not depend on the number of balls in the radial ball bearing, but only on the dimensions of the radial ball bearing.**

$N_1 = \frac{(R-2r)\pi}{r}$

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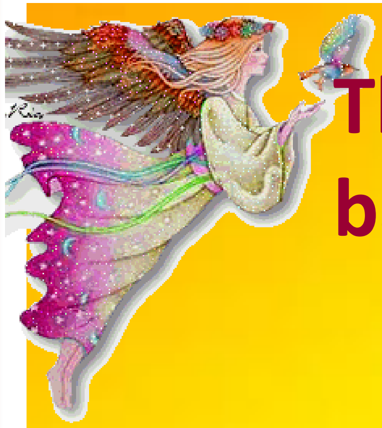
The second case is when the gears are balanced, but in a radial ball bearing, the eccentricity of the center of mass of a pair of balls on one diameter appears, and due to the difference in the mass density of those two balls on one diameter, even though they have the equal radius.



The eccentricity of the center of mass of a pair of balls on one diameter is:

$$e_n = \frac{1}{2}(R-r)\frac{(1-p_n)}{(1+p_n)}$$

the p_n difference coefficient is the mass density of the balls in a pair on one diameter

Centrifugal force due to the eccentricity of the center of the mass of a pair of balls of different mass densities and equal radii, but on one diameter, is:

$$\varphi = \frac{(R-2r)}{2(R-r)}\vartheta$$

$$\mathbf{F}_{c,m_n} = -m(1+p_n)a_{N,m_n} = -m(1+p_n)e_n\omega_n^2 = -m(1+p_n)e_{in}\dot{\varphi}^2$$

$$\mathbf{F}_{c,m_n} = -m(1+p_n)e_{in}\dot{\varphi}^2 = -\frac{1}{8}m(1-p_n)\frac{(R-2r)^2}{(R-r)}\dot{\vartheta}^2$$

This centrifugal force acts on a pair of balls on the same diameter, which have different mass densities and equal radii. The number of changes in contact points in which kinetic contact forces occur due to the appearance of centrifugal forces caused by the eccentricity of the center of mass of a pair of balls on one diameter of different mass densities and equal radii is $N_1 = \frac{(R-2r)\pi}{r}$ **times for one revolution of the shaft, and thus each contact point of the ball changes so much, in rolling without sliding, exposed to kinetic pressure forces due to the kinetic action of the centrifugal forces of the eccentricity of the center of mass of a pair of balls in a radial ball bearing.**

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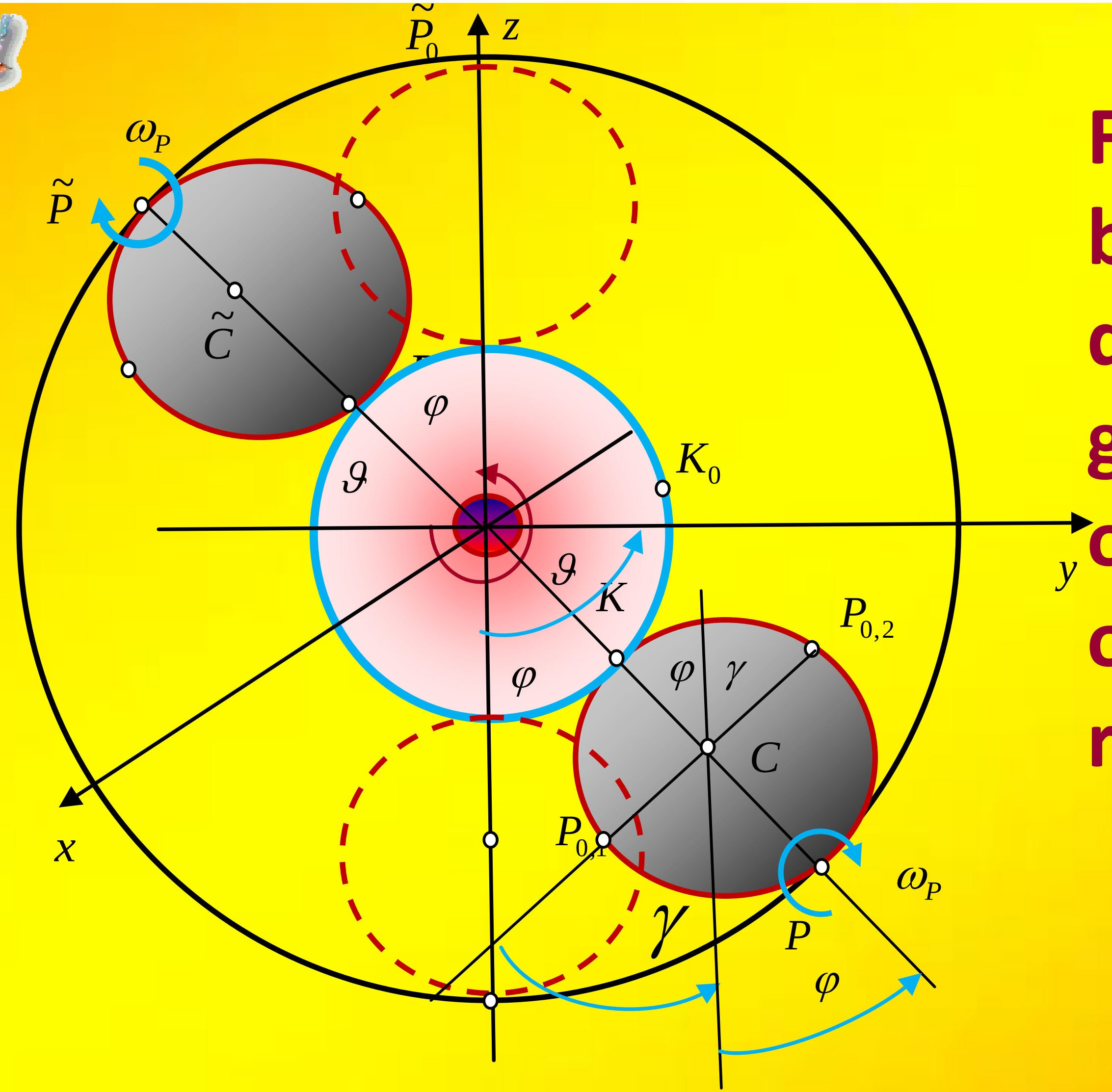

Figure 7. A pair of balls on one diameter and the geometric relations of ball dynamics in one model of a radial ball bearing

Figure 8. Four models of radial ball bearings with pairs of balls on one diameter: models with: with four pairs of balls a* and b*, radial bearing with a total of eight balls in rolling without sliding; and with six pairs of balls c* and d*, a radial bearing with a total of twelve balls in rolling, without sliding, inside or outside of fixed circle rolling path and in dynamical contact with movable circle path outside or insiide

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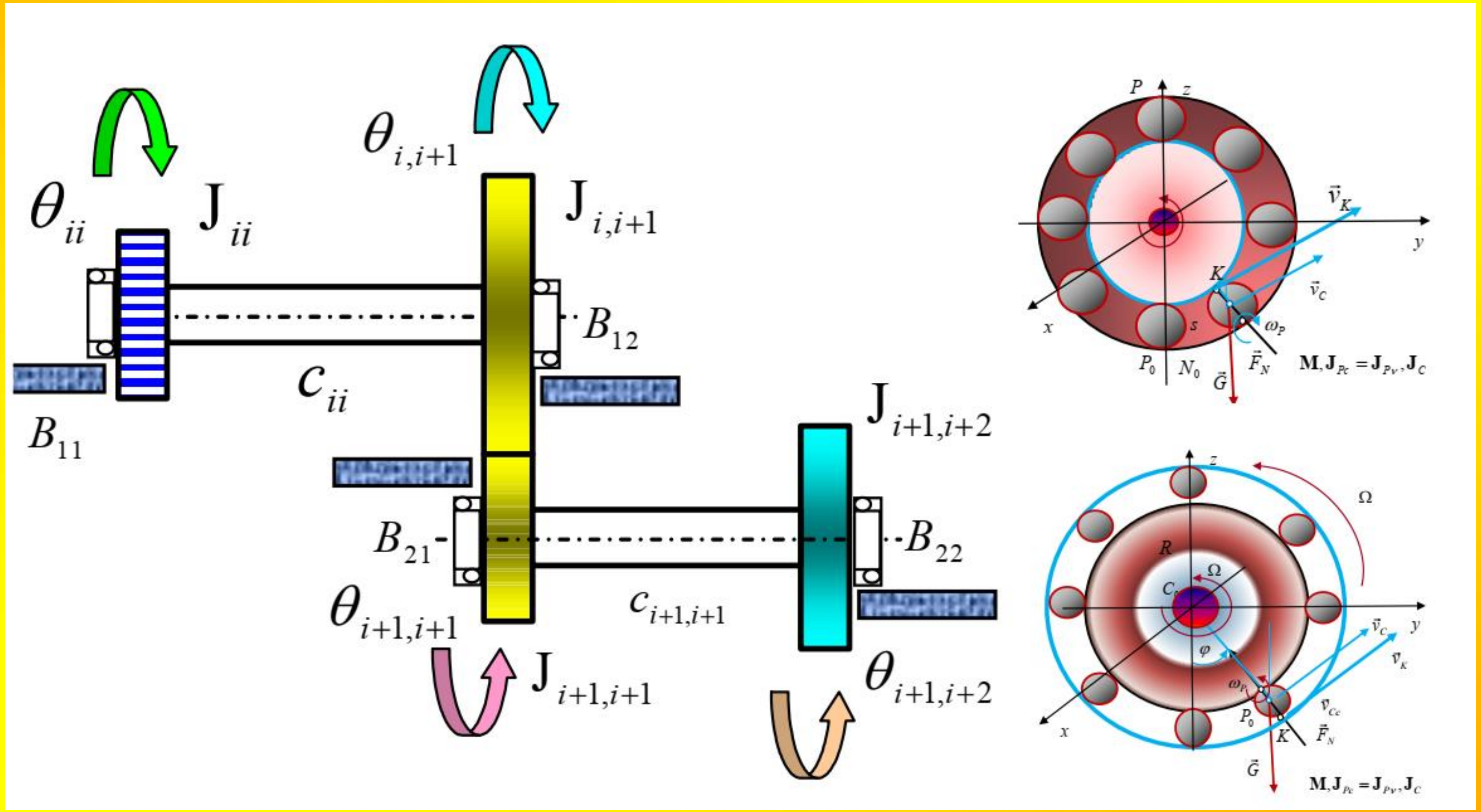
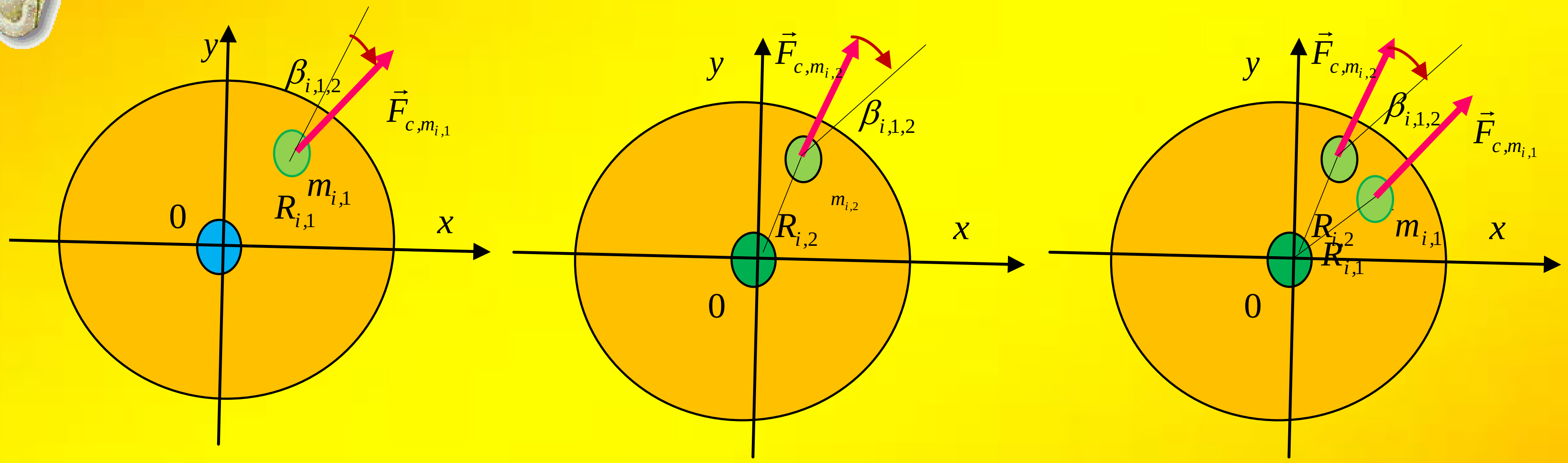


Figure 4. Configuration of radial ball bearings on the shafts of a two-stage gear transmission with unbalanced gears (with debalances in the form of eccentrically placed material points)


y
x
0
$\beta_{i,1,2}$
$\vec{F}_{c,m_{i,1}}$
$m_{i,1}$
$R_{i,1}$
$\vec{F}_{c,m_{i,2}}$
$m_{i,2}$
$R_{i,2}$

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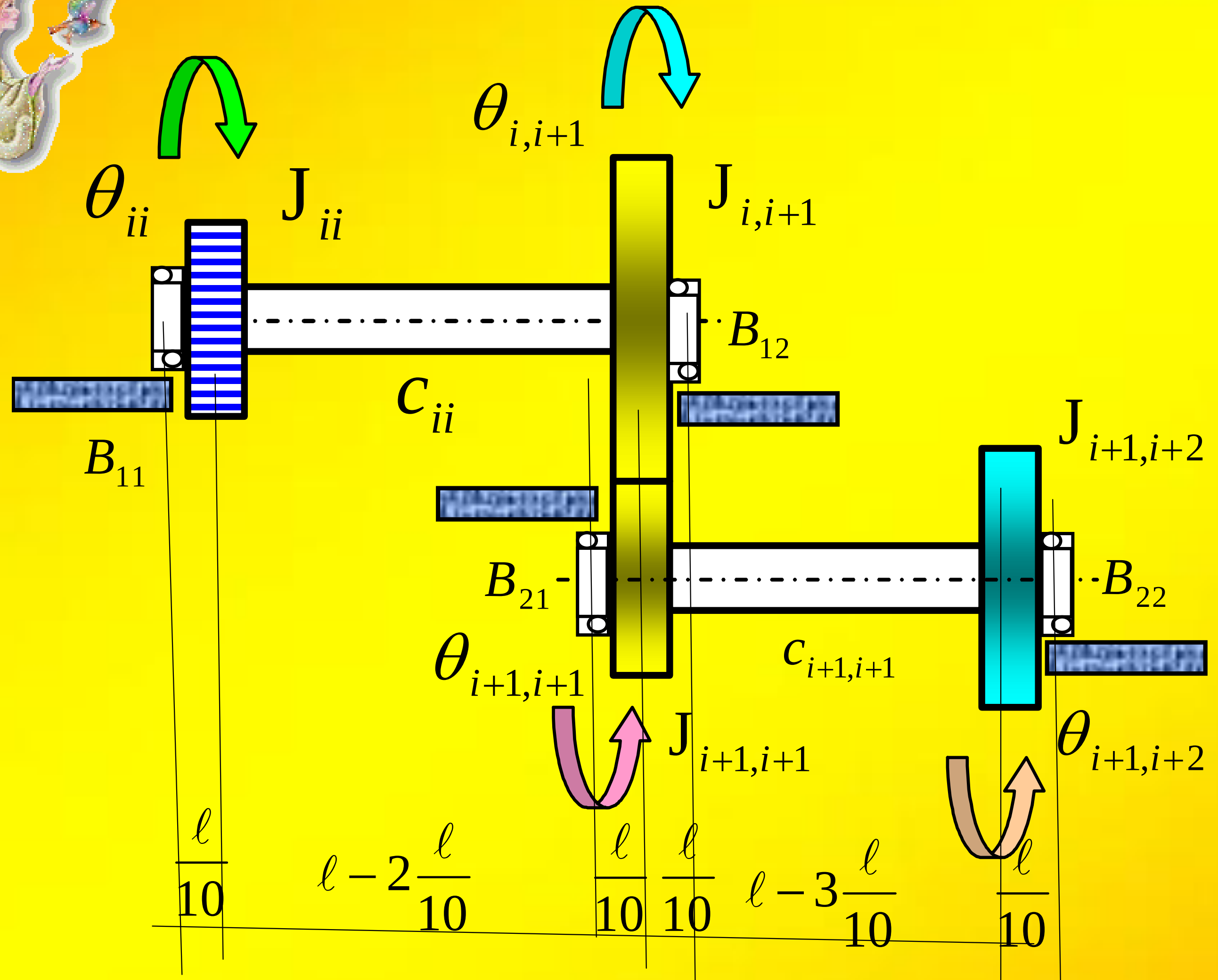

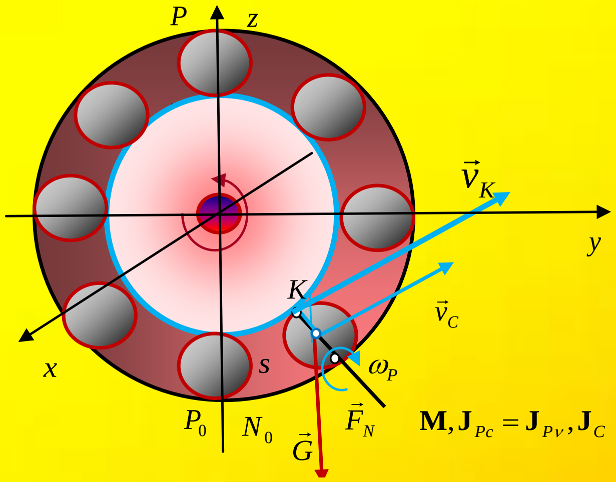

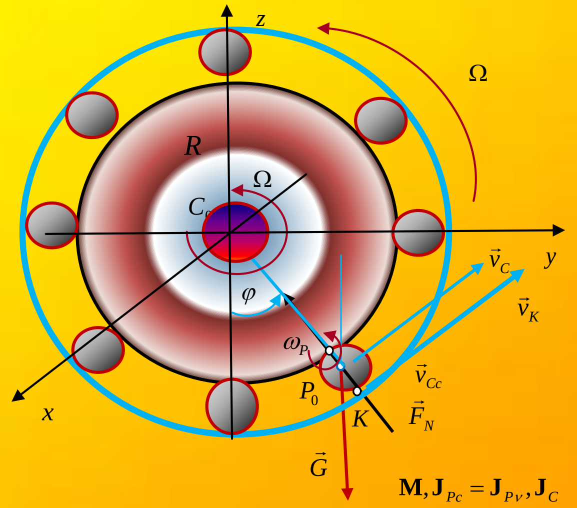

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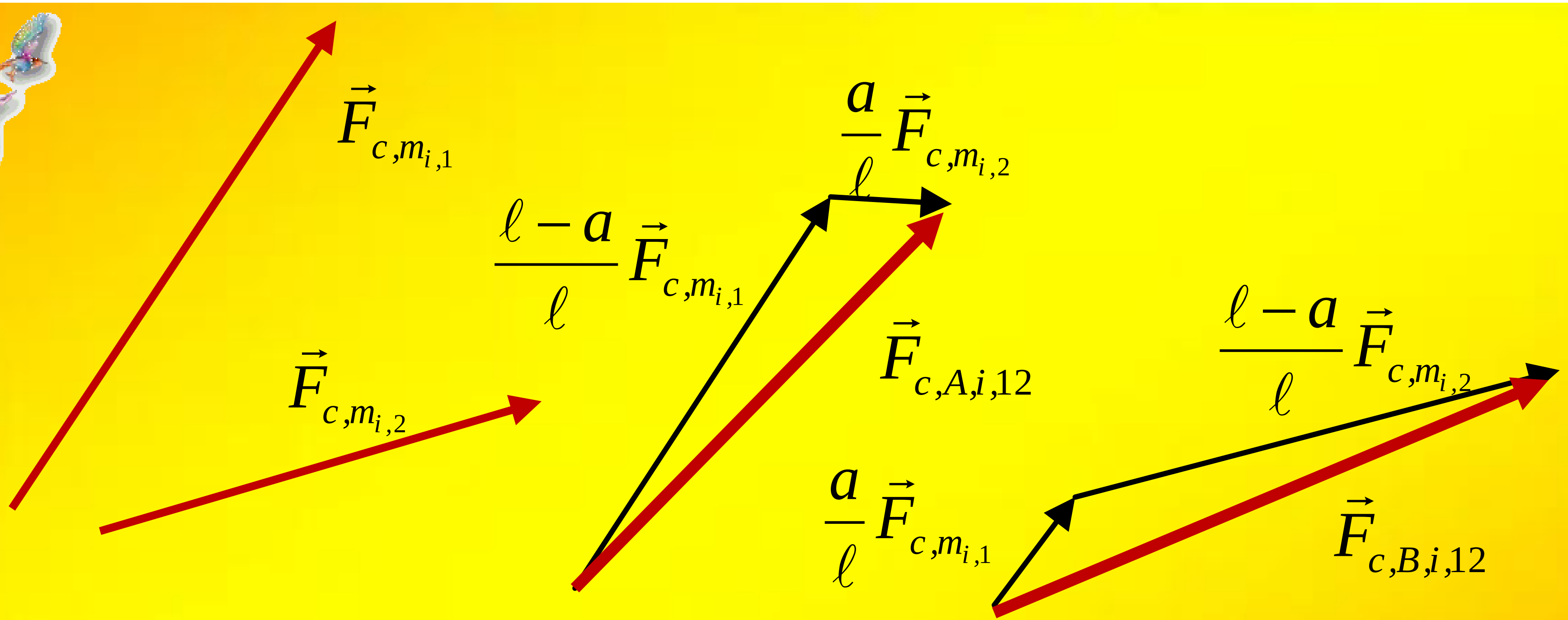

$$\vec{F}_{c,A,i,12} = \frac{\ell-a}{\ell}\vec{F}_{c,m_i,1} + \frac{a}{\ell}\vec{F}_{c,m_i,2}$$

$$\vec{F}_{c,B,i,12} = \frac{a}{\ell}\vec{F}_{c,m_i,1} + \frac{\ell-a}{\ell}\vec{F}_{c,m_i,2}$$

$$\left|\vec{F}_{c,A,i,12}\right| = F_{c,A,i,12} = \sqrt{\left(\frac{\ell - a}{\ell}\right)^2 \left(\vec{F}_{c,m_{i,1}}\right)^2 + \left(\frac{a}{\ell}\right)^2 \left(\vec{F}_{c,m_{i,2}}\right)^2 + 2\left(\frac{\ell - a}{\ell}\right)\left(\frac{a}{\ell}\right) F_{c,m_{i,1,2}} F_{c,m_{i,2}} \cos\beta_i}$$

$$\left|\vec{F}_{c,B,i,12}\right| = F_{c,B,i,12} = \sqrt{\left(\frac{a}{\ell}\right)^2 \left(\vec{F}_{c,m_{i,12}}\right)^2 + \left(\frac{\ell - a}{\ell}\right)^2 \left(\vec{F}_{c,m_{i,2}}\right)^2 + 2\left(\frac{\ell - a}{\ell}\right)\left(\frac{a}{\ell}\right) F_{c,m_{i,12}} F_{c,m_{i,2}} \cos\beta_i}$$

$$\phi = \arccos \frac{\left(\vec{F}_{c,A,i,2}, \vec{F}_{c,B,i,12}\right)}{F_{c,A,i,2} F_{c,B,i,12}}$$

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$$N_b = \frac{2\pi(R-r)}{r}$$

$$N_1 = \frac{(R-2r)2\pi}{2r} = \frac{(R-2r)\pi}{r}$$

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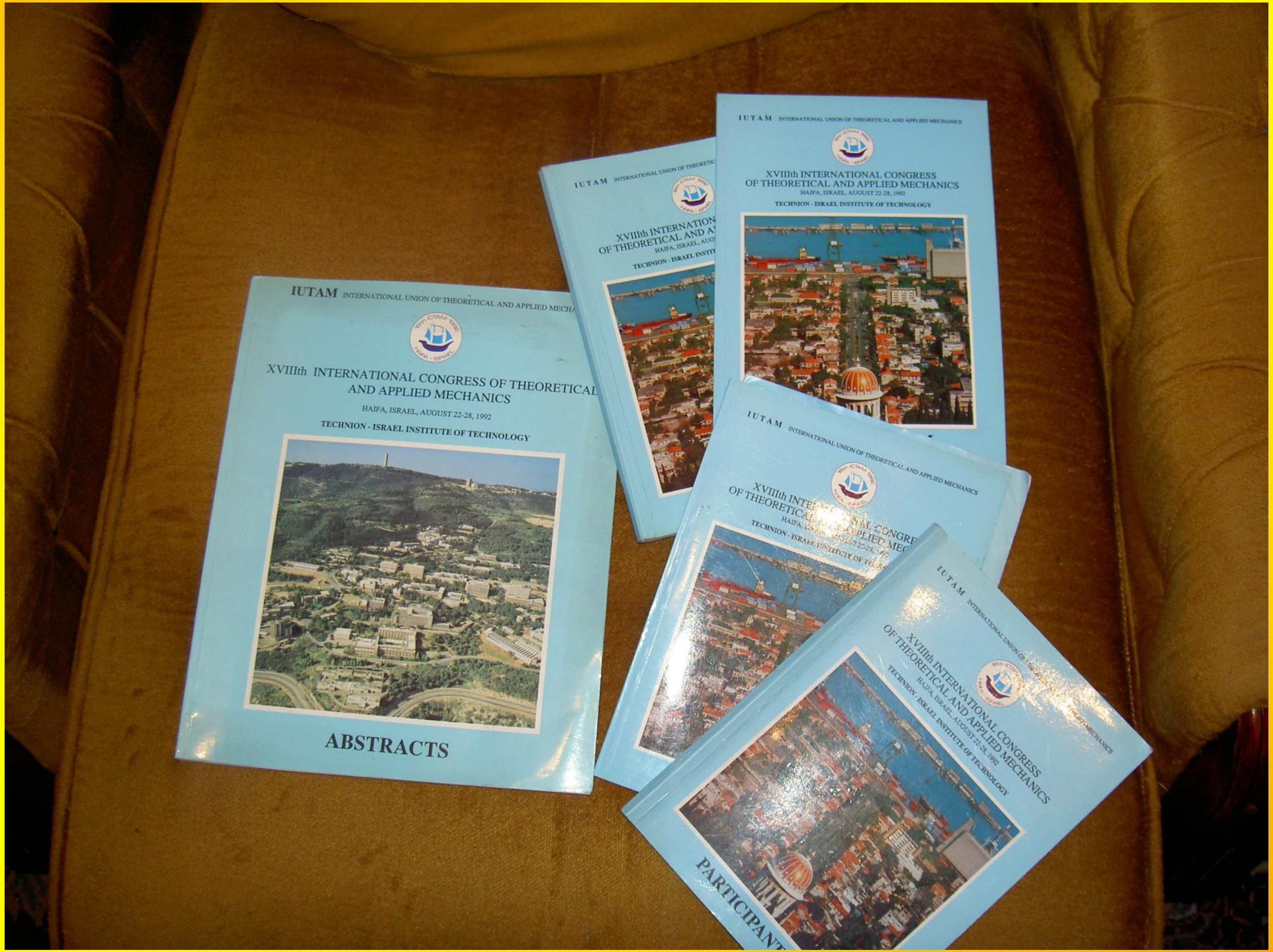

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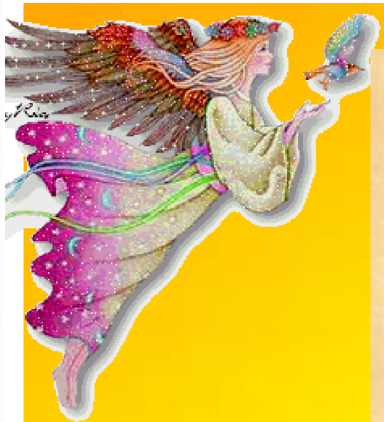

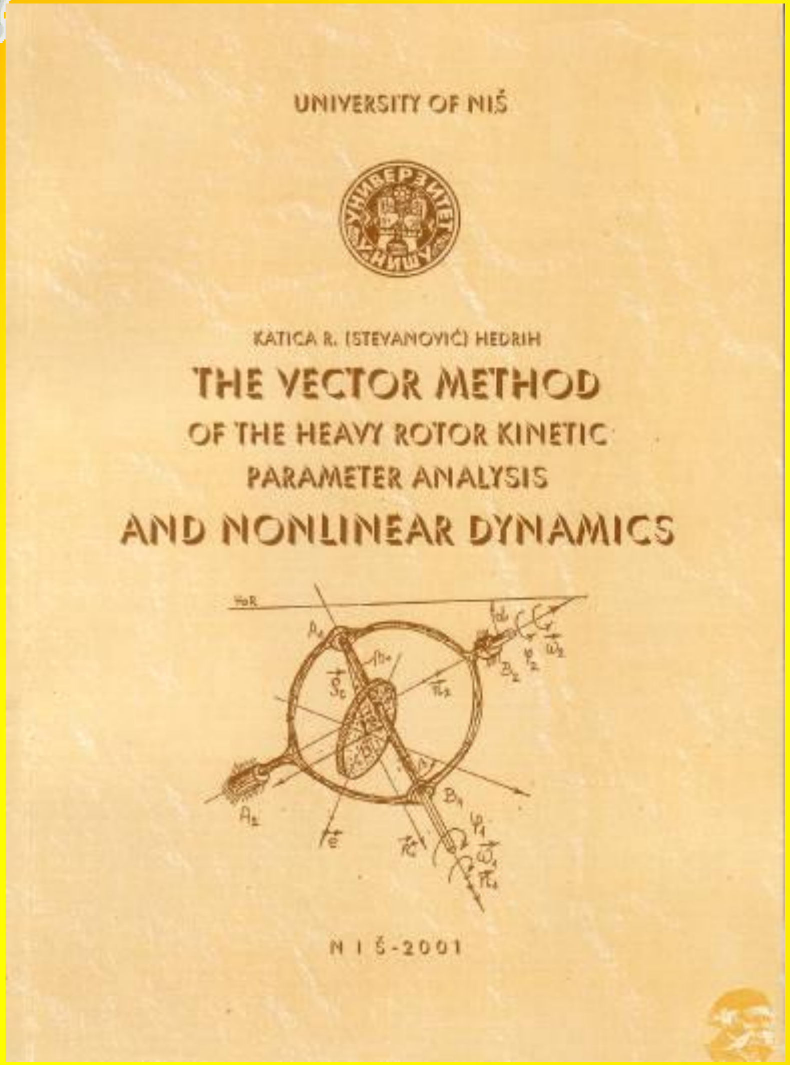

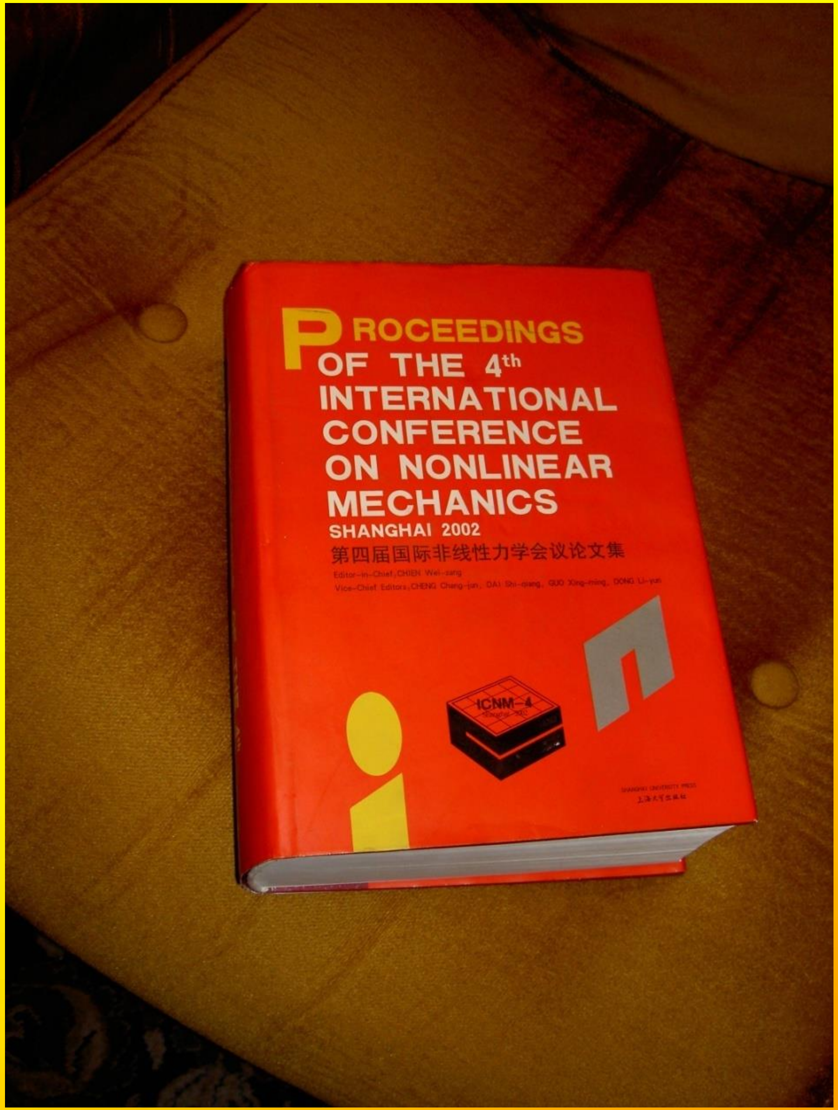

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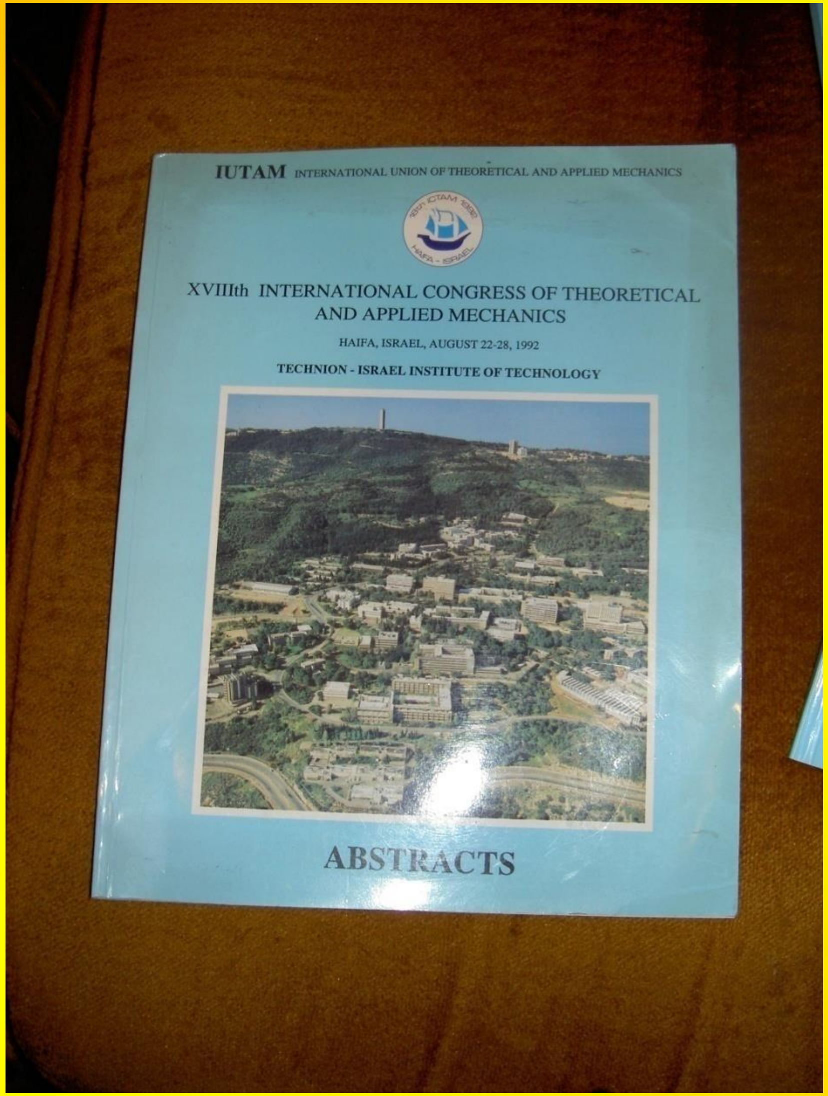

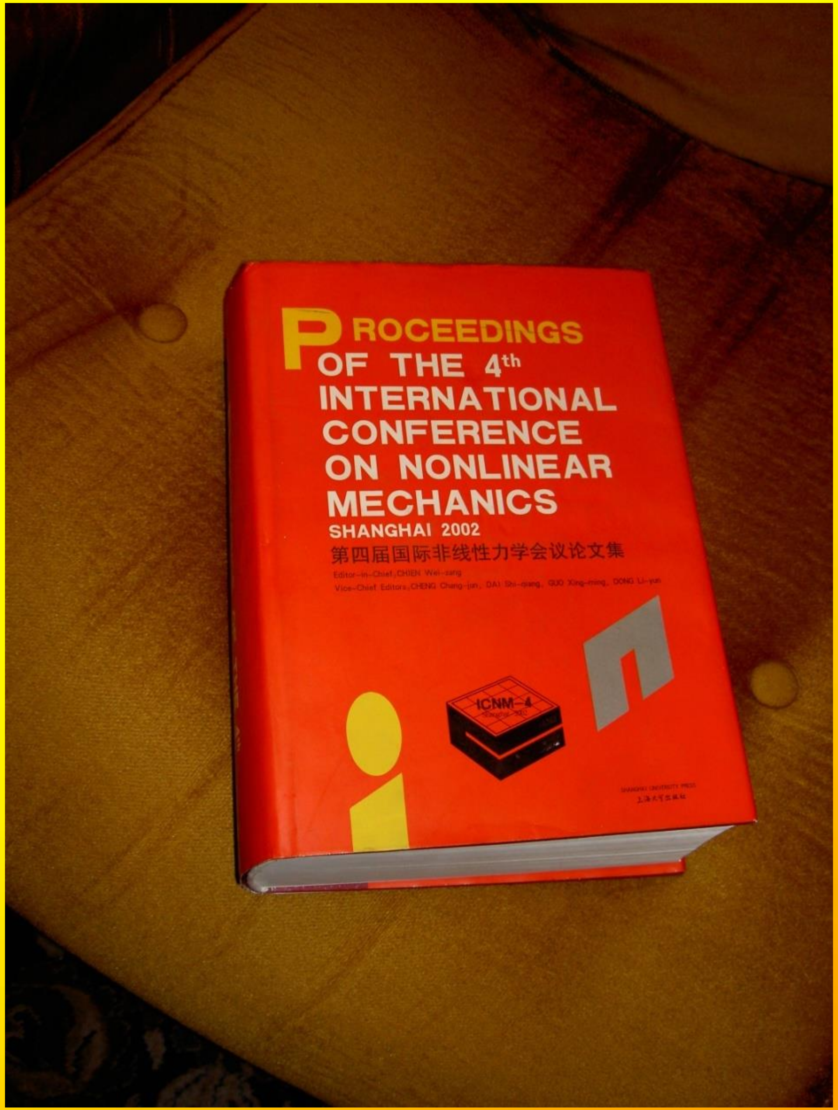

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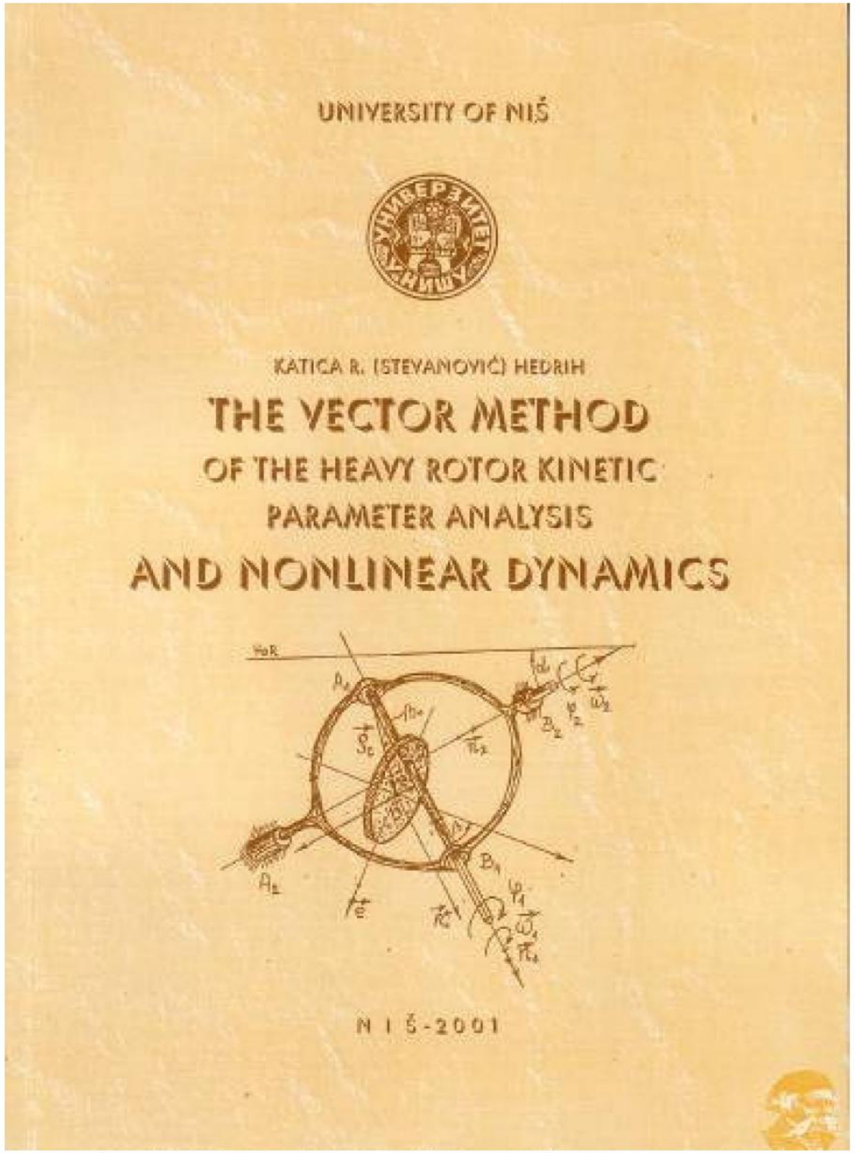
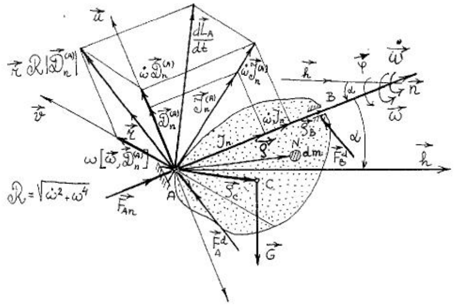


$$\vec{\mathfrak{J}}_{\vec n}^{(O)} \overset{def}{=} \iiint_V [\vec\rho,[\vec n,\vec\rho]]dm$$

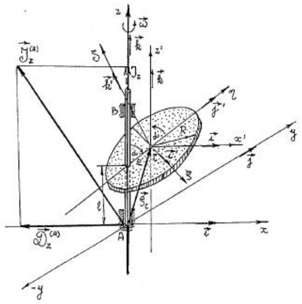
$$\vec{\mathfrak{J}}_{\vec n}^{(O)} = \vec{\mathfrak{J}}_{\vec n}^{(C)} + [\vec\rho_C,[\vec n,\vec\rho_C]]M$$

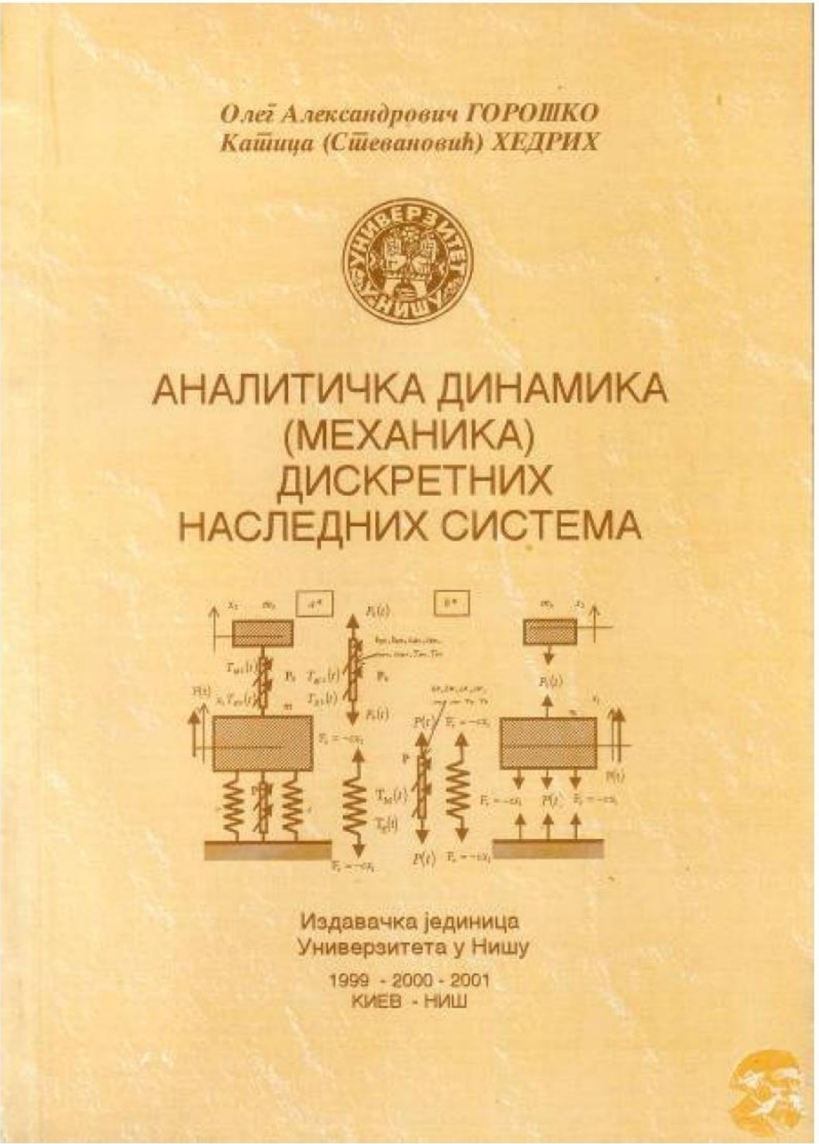


$$\frac{d\vec{\mathfrak{K}}}{dt} = \vec{\mathfrak{R}}_1 \left|\vec{\mathfrak{S}}_{\vec n}^{(A)}\right| = \sum_{k=1}^{k=N}\vec{F}_k + \vec{F}_A + \vec{F}_B$$

$$\frac{d\vec{\mathfrak{L}}_A}{dt} = \dot\omega J_{\vec n}^{(A)} + \dot\omega \vec{\mathfrak{D}}_{\vec n}^{(A)} + \omega\left[\vec\omega, \vec{\mathfrak{D}}_{\vec n}^{(A)}\right] =$$

$$= \dot\omega J_{\vec n}^{(A)} + \left|\vec{\mathfrak{D}}_{\vec n}^{(A)}\right| \vec{\mathfrak{R}}_2 = \sum_{k=1}^{k=N}\left[\vec\rho_k, \vec{F}_k\right] + \left[\vec\rho_B, \vec{F}_B\right]$$





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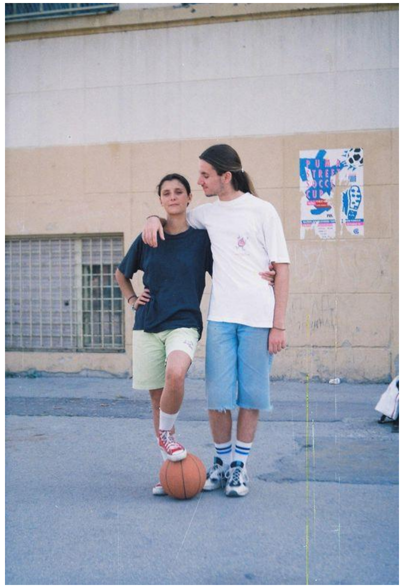

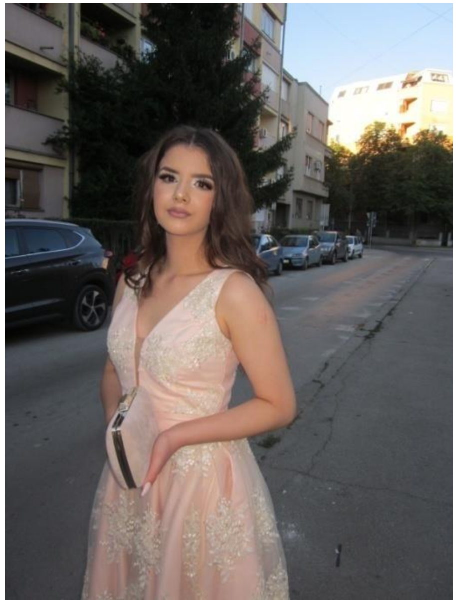

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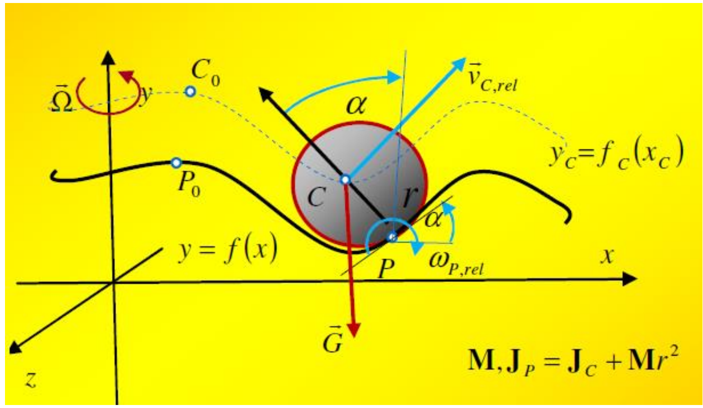

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Research and Authentic Author Scientific Results

Katica R. (Stevanović) Hedrih

in the period longer than half a century (from 1963-2019)

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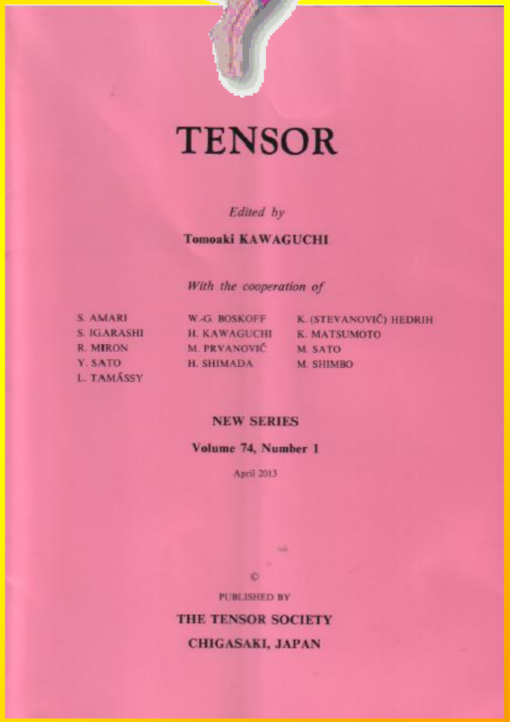
THE 13th INTERNATIONAL CONFERENCE OF TENSOR SOCIETY
ON DIFFERENTIAL GEOMETRY AND ITS APPLICATIONS, AND INFORMATICS BESIDES
The 86th Anniversary of Radu MIRON's birthday
テンソル学会
(TENSOR SOCIETY)
Faculty of Mathematics
Alexandru Ioan Cuza
University of Iaşi
Institute of Mathematics
"Octav Mayer"
Romanian Academy
September 3-7, 2013, Iaşi, Romania

Plemaru Levture at Centre of Romanian Academy of Sciences in IASHI 2015 Tensor Society Conference: Chairman Academivian Radu Miron


TENSOR

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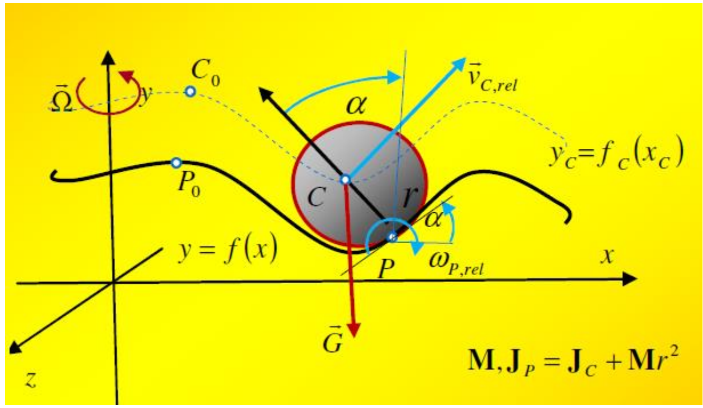
C_0
$\vec{v}_{C,rel}$
$\vec{\Omega}$
α
$y_C = f_C(x_C)$
P_0
C
r
α
$y = f(x)$
P
$\omega_{P,rel}$
x
z
$\vec{G}$
$\mathbf{M}, \mathbf{J}_P = \mathbf{J}_C + \mathbf{M}r^2$

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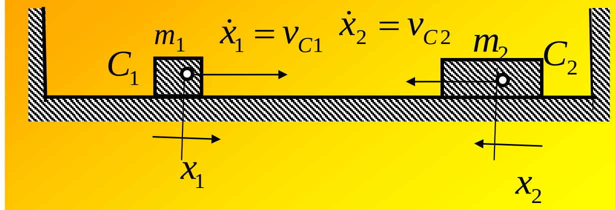
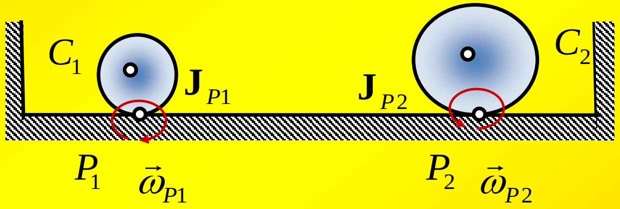
Translator motion of two bodies



Rolling motion of two balls



Differential equation of motion

$$m_k \dot{v}_{Ck} = F_k \qquad k = 1,2$$

$$J_{Pk} \dot{\omega}_{Pk} = \mathfrak{M}_{Pk} \qquad k = 1,2$$

Linear momentum

$$m_1\vec{v}_1(t_0)+m_2\vec{v}_2(t_0)=m_1\vec{v}_1(t_0+\tau)+m_2\vec{v}_2(t_0+\tau)$$

$$k=\frac{v_r(t_0+\tau)}{v_r(t_0)}=\frac{v_2(t_0+\tau)-v_1(t_0+\tau)}{v_1(t_0)-v_2(t_0)}$$

$$v_1(t_0+\tau)=v_1(t_0)-\frac{1+k}{1+\frac{m_1}{m_2}}(v_1(t_0)-v_2(t_0))$$

$$v_2(t_0+\tau)=v_2(t_0)+\frac{1+k}{1+\frac{m_2}{m_1}}(v_1(t_0)-v_2(t_0))$$

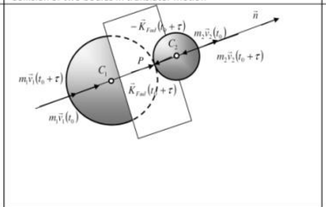
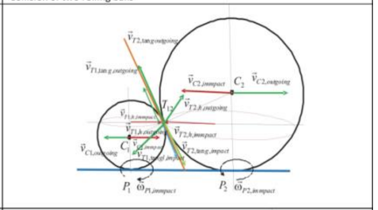
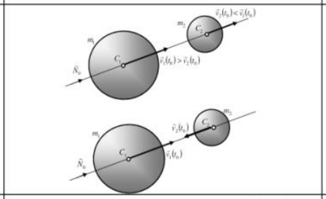
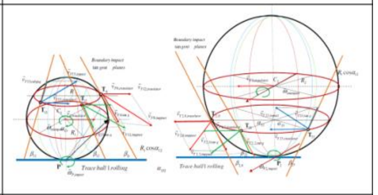
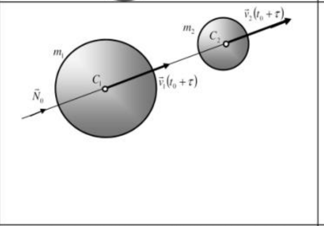
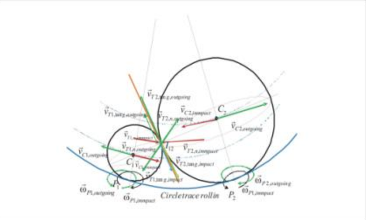
Angular momentum

$$\mathbf{J}_{P1}\vec{\omega}_{P1}(t_0)+\mathbf{J}_{P2}\vec{\omega}_{P2}(t_0)=\mathbf{J}_{P1}\vec{\omega}_{P1}(t_0+\tau)+\mathbf{J}_{P2}\vec{\omega}_{P2}(t_0+\tau)$$

$$k=\frac{\omega_r(t_0+\tau)}{\omega_r(t_0)}=\frac{\omega_{P2}(t_0+\tau)-\omega_{P1}(t_0+\tau)}{\omega_{P1}(t_0)-\omega_{P2}(t_0)}$$

$$\omega_{P1}(t_0+\tau)=\omega_{P1}(t_0)-\frac{1+k}{1+\frac{\mathbf{J}_{P1}}{\mathbf{J}_{P2}}}(\omega_{P1}(t_0)-\omega_{P2}(t_0))$$

$$\omega_{P2}(t_0+\tau)=\omega_{P2}(t_0)+\frac{1+k}{1+\frac{\mathbf{J}_{P2}}{\mathbf{J}_{P1}}}(\omega_{P1}(t_0)-\omega_{P2}(t_0))$$

	Collision of two bodies in translator motion	Collision of two rolling balls
Configuration of the systems in collision state and plans f velocities and tanfent plane of bodies collisions		
		
		
Analogous theorms of conservation of linear momentum (impulse) or angular momentum	Theorm of conservation of linear momentum (impulse) in collision of two bodies in translator motion $m_1\vec{v}_1(t_0)+m_2\vec{v}_2(t_0)=m_1\vec{v}_1(t_0+\tau)+m_2\vec{v}_2(t_0+\tau)$	Theorm of conservation of angular momentum (kinetic moment) in collision of two rolling balls $\mathbf{J}_{P1}\vec{\omega}_{P1}(t_0)+\mathbf{J}_{P2}\vec{\omega}_{P2}(t_0)=\mathbf{J}_{P1}\vec{\omega}_{P1}(t_0+\tau)+\mathbf{J}_{P2}\vec{\omega}_{P2}(t_0+\tau)$
Coefficient of the restitution of two body collision	Coefficient of the restitution in collision of two bodies in translator motion $k=\frac{v_r(t_0+\tau)}{v_r(t_0)}=\frac{v_2(t_0+\tau)-v_1(t_0+\tau)}{v_1(t_0)-v_2(t_0)}$	Coefficient of the restitution in collision of two rolling balls $k=\frac{\omega_r(t_0+\tau)}{\omega_r(t_0)}=\frac{\omega_{P2}(t_0+\tau)-\omega_{P1}(t_0+\tau)}{\omega_{P1}(t_0)-\omega_{P2}(t_0)}$
Outgoing velocities of thwo bodies at post-collision moment	Outgoing velocities of the of two bodies in translator motion at post-collision moment $v_1(t_0+\tau)=v_1(t_0)-\frac{1+k}{1+\frac{m_1}{m_2}}(v_1(t_0)-v_2(t_0))$ $v_2(t_0+\tau)=v_2(t_0)+\frac{1+k}{1+\frac{m_1}{m_2}}(v_1(t_0)-v_2(t_0))$	Outgoing angular velocities of the rolling balls at post-collision moment $\omega_{P1}(t_0+\tau)=\omega_{P1}(t_0)-\frac{1+k}{1+\frac{\mathbf{J}_{P1}}{\mathbf{J}_{P2}}}(\omega_{P1}(t_0)-\omega_{P2}(t_0))$ $\omega_{P2}(t_0+\tau)=\omega_{P2}(t_0)+\frac{1+k}{1+\frac{\mathbf{J}_{P1}}{\mathbf{J}_{P2}}}(\omega_{P1}(t_0)-\omega_{P2}(t_0))$
Impuls (linear momentum) of collision	*Impuls (linear momentum) of collision of impact forces* $K_{Fud}=m_1(v_1(t_0+\tau)-v_1(t_0))=-\frac{m_1m_2}{m_1+m_2}(1+k)(v_1(t_0)-v_2(t_0))$	*Moment of impuls (linear momentum) of collision of impact couple (moment of impact forces)* $\mathfrak{L}_{\mathfrak{M},impact}=\mathbf{J}_{P1}(\omega_{P1}(t_0+\tau)-\omega_{P1}(t_0))=-\frac{\mathbf{J}_{P1}\mathbf{J}_{P2}}{\mathbf{J}_{P1}+\mathbf{J}_{P2}}(1+k)(\omega_{P1}(t_0)-\omega_{P2}(t_0))$
Kinetic energy change fromprecollison to postcollision kinetic state	$\Delta E_{k,plast}=E_k(t_0+\tau)-E_k(t_0)=\frac{m_1m_2}{2(m_1+m_2)}(v_1(t_0)-v_2(t_0))^2$	$\Delta E_k=E_k(t_0+\tau)-E_k(t_0)=\frac{\mathbf{J}_{P1}\mathbf{J}_{P2}}{2(\mathbf{J}_{P1}+\mathbf{J}_{P2})}(1-k^2)(\omega_{P1}(t_0)-\omega_{P2}(t_0))^2$

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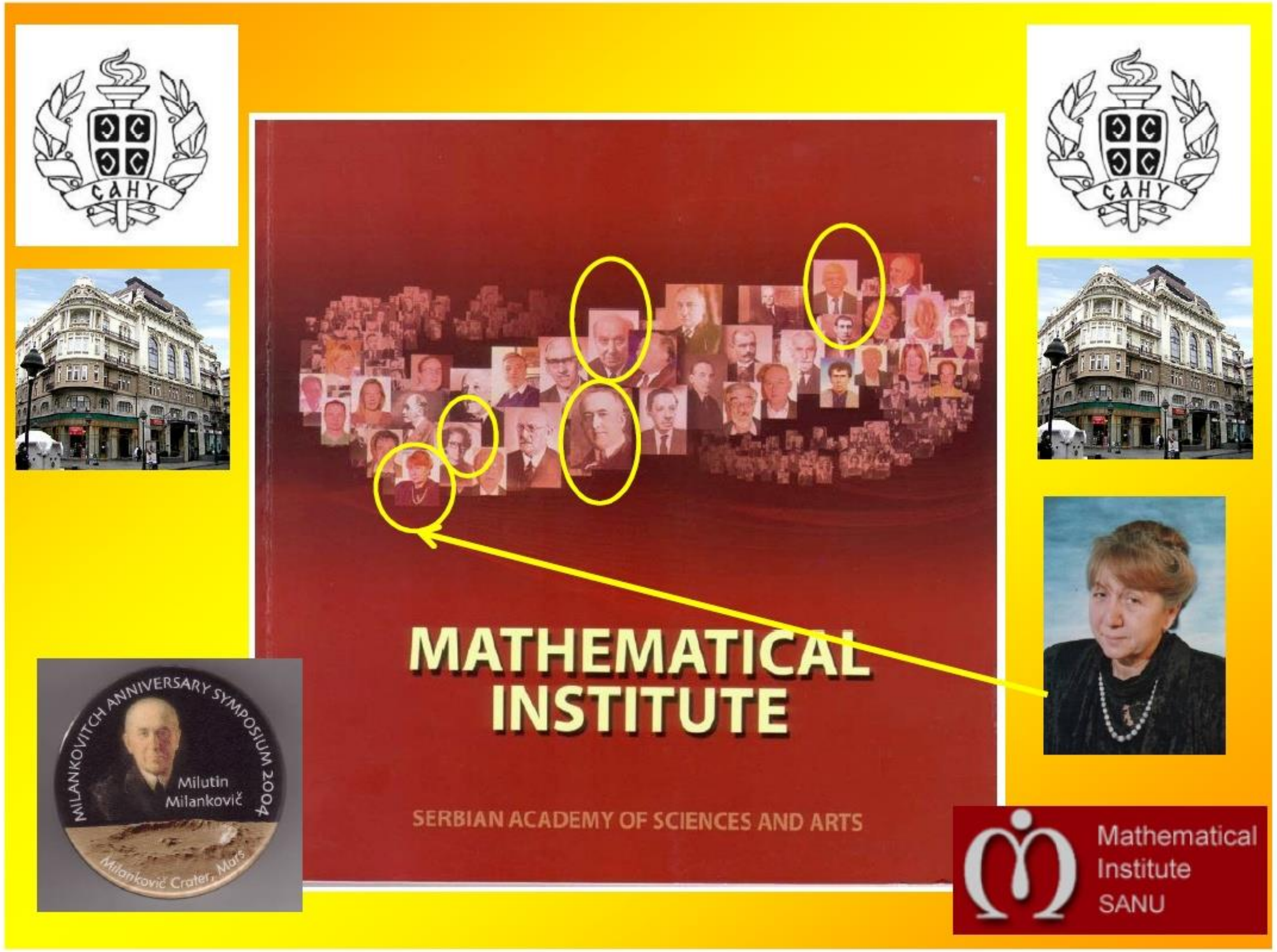

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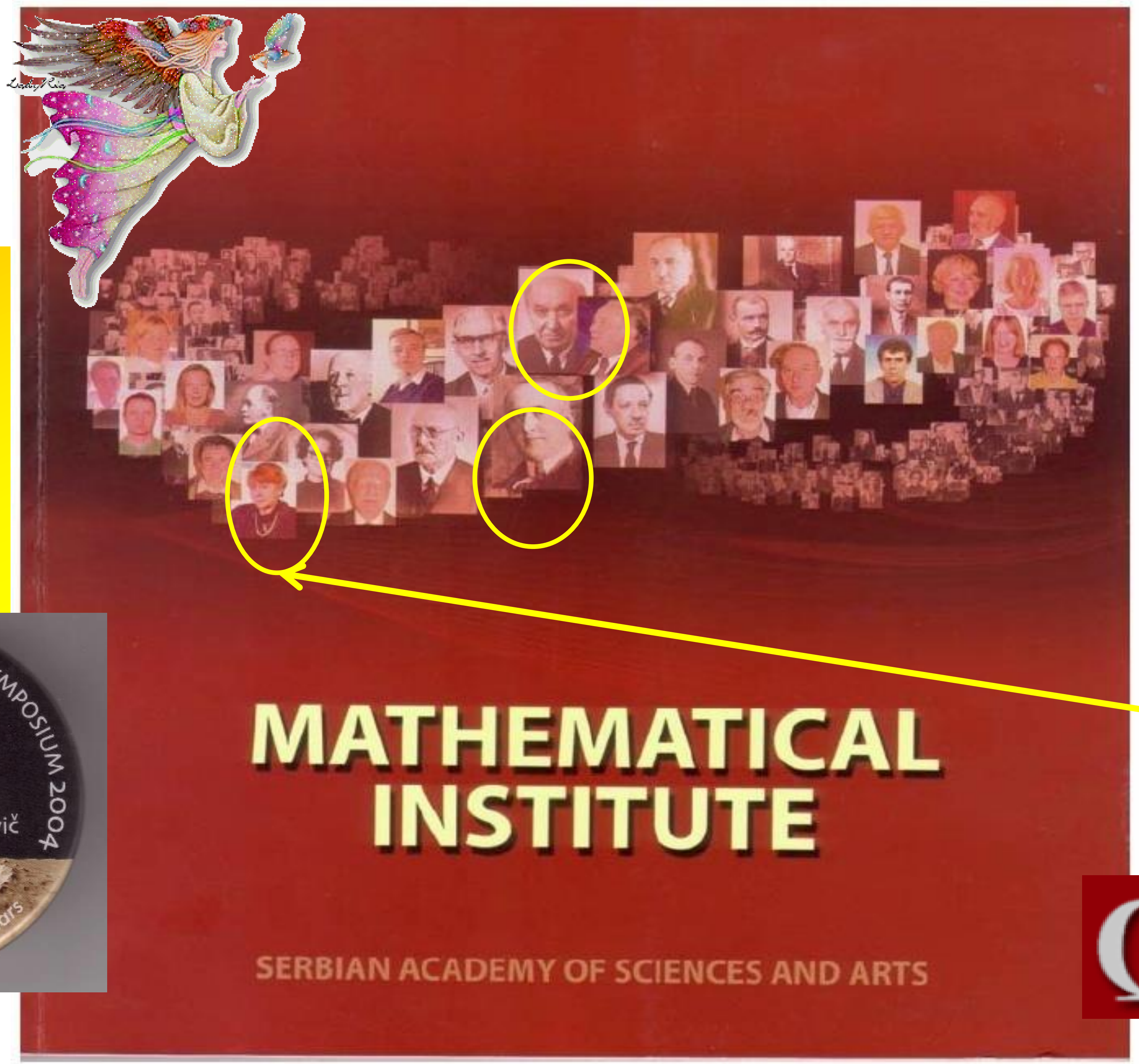

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