3rd CONFERENCE ON NONLINEARITY
5th Sep. 2023, Belgrade, Serbia

Predictions in Cartan F(R) Gravity

T. I., M. Taniguchi, Symmetry 14, 1830 (2022),
T. I., H. Sakamoto, M. Taniguchi, arXiv:2304.14769 [gr-qc] to be appear in JCAP.

Tomohiro Inagaki
Graduate School of
Advanced Science and Engineering
Hiroshima University

Outline

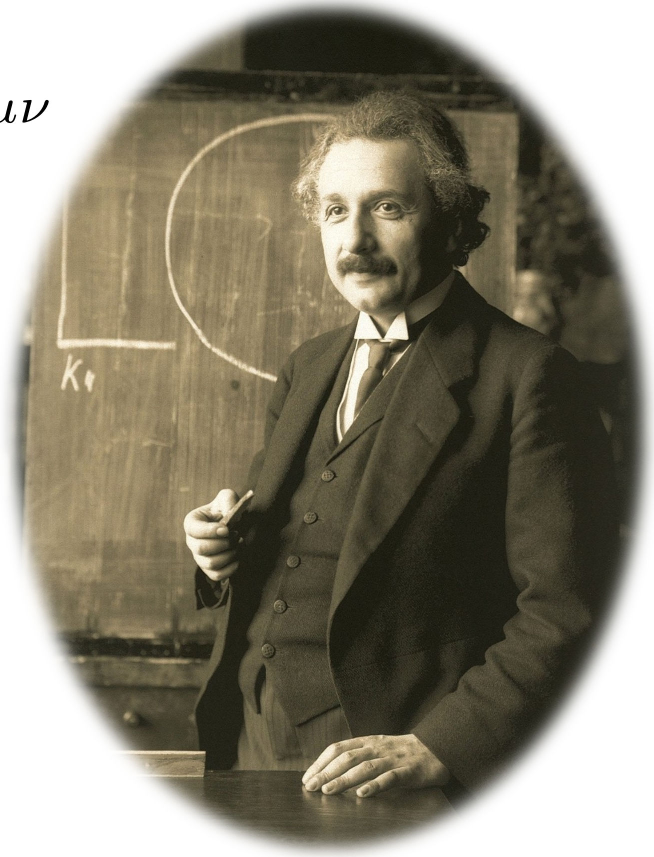
- Modified theory of gravity
- Cartan F(R) gravity
- Spacetime evolution in Cartan F(R) gravity
- Fluctuations in CMB
- Conclusion

Modified theory of gravity

Why?

Einstein's General Relativity

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

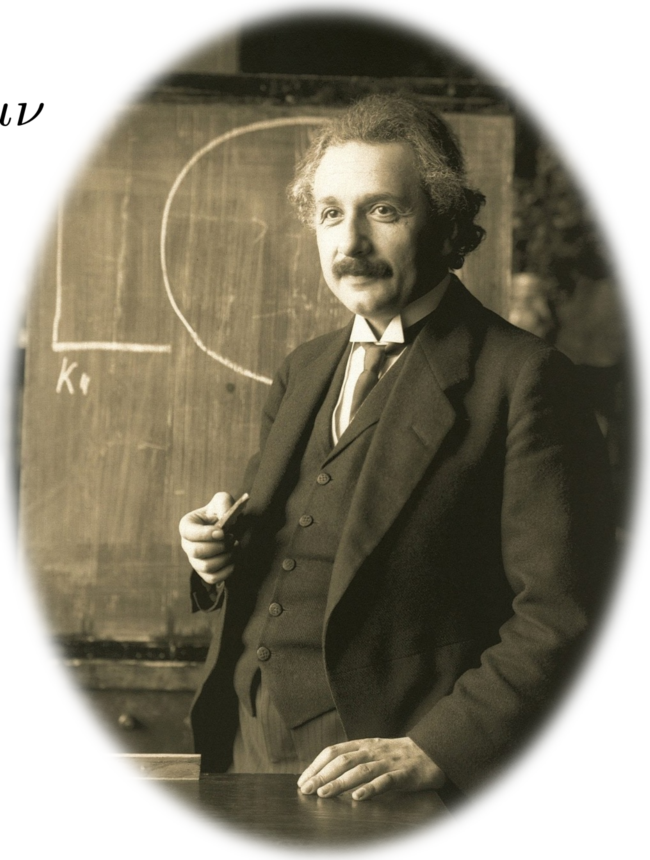


Why?

Einstein's General Relativity

Non-linear equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$



Why?

Einstein's General Relativity

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- Phenomenological consequences
  Apsidal precession of the planet Mercury,
  Dense stars, Black holes,
  Gravitational lens,
  Gravitational wave,
  Expansion of the universe,
  …

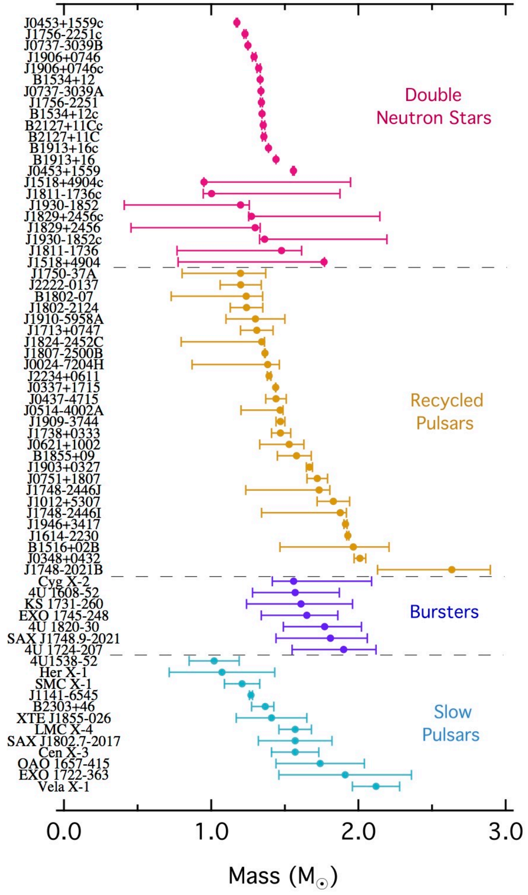
Why?

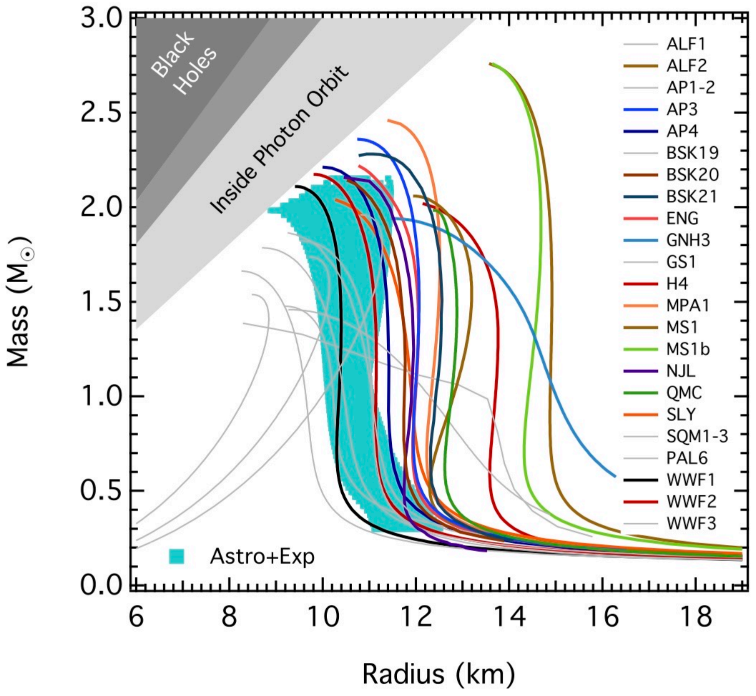
Einstein's General Relativity

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- Phenomenological consequences
  Apsidal precession of the planet Mercury,
  Dense stars, Black holes, ←Heavy neutron stars
  Gravitational lens, ←Dark matter
  Gravitational wave, ←Just started
  Expansion of the universe, ←Accelerated expansion
  ⋯ ←Small scale $<$ 0.01mm

Heavy neutron stars

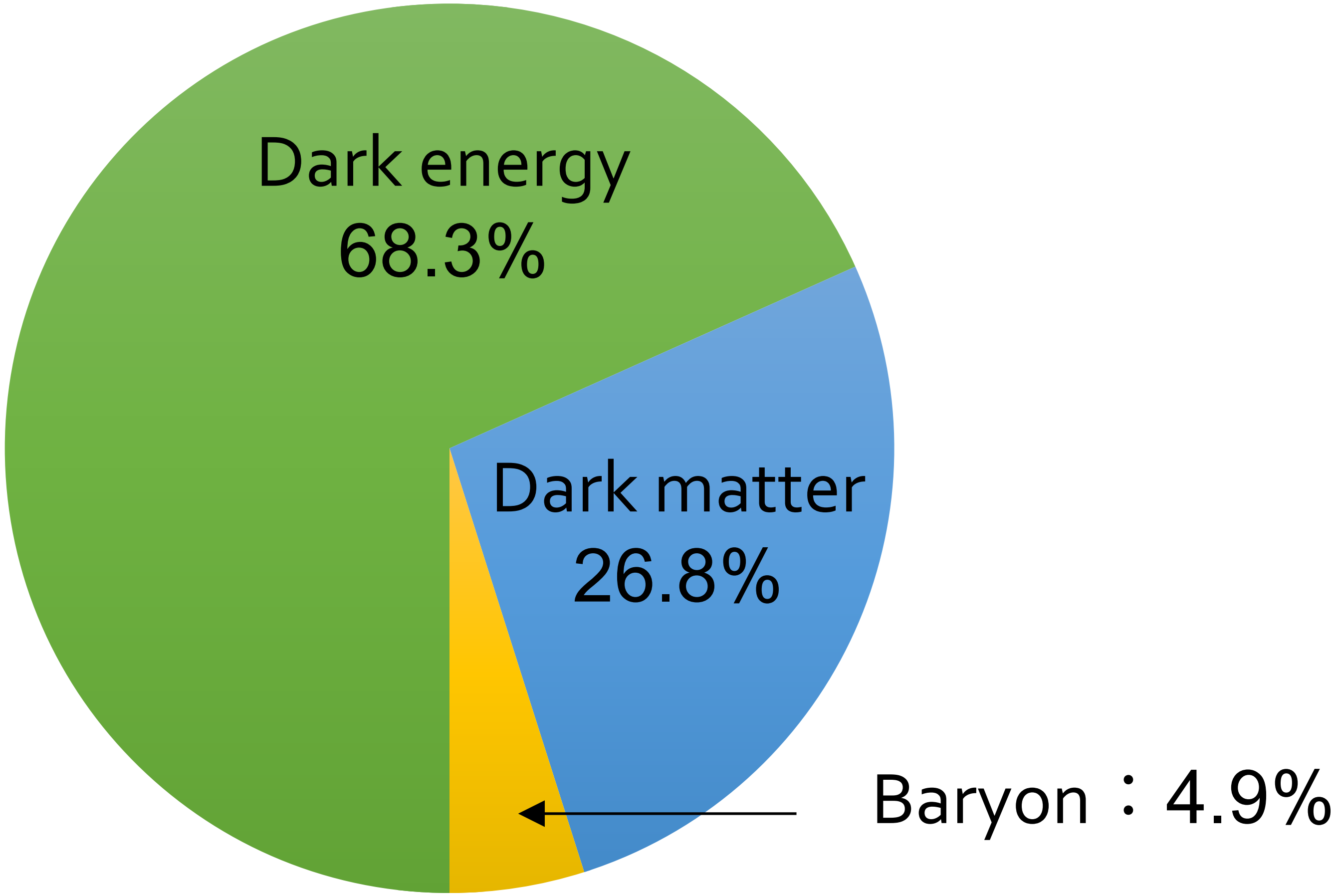




Ozel & Freire 2016

http://xtreme.as.arizona.edu/NeutronStars/

Dark matter and dark energy



How?

Einstein equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Action

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}(R - 2\Lambda) + S_{\mathrm{matter}}$$

How?

Einstein equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Action

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}(R - 2\Lambda) + S_{\text{matter}}$$

Modified Gravity

- Higher order terms
- Non-local terms
- Gauss-Bonnet
- Torsion
- …

How?

Einstein equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Action

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}(R - 2\Lambda) + S_{\mathrm{matter}}$$

Particle physics

- Neutrino
- Axion
- Super partners
- …

How?

Einstein equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Action

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}(R - 2\Lambda) + S_{\text{matter}}$$

Spacetime

- Extra dimensions
- D-brane
- $\cdots$

M. Montesinos, R. Romero and D. Gonzalez, Class. Quant. Grav. 37 (2020) 045008.
T.P. Sotiriou and S. Liberati, J. Phys. Conf. Ser. 68 (2007) 012022.
T.P. Sotiriou and S. Liberati, Annals Phys. 322 (2007) 935.
D. Iosifidis, A.C. Petkou and C.G. Tsagas, Gen. Rel. Grav. 51 (2019) 66.
S. Capozziello, R. Cianci, C. Stornaiolo and S. Vignolo, Class. Quant. Grav. 24 (2007) 6417.
S. Capozziello, R. Cianci, C. Stornaiolo and S. Vignolo, Int. J. Geom. Meth. Mod. Phys. 5 (2008) 765.
T.P. Sotiriou, Class. Quant. Grav. 26 (2009) 152001.
S. Capozziello and S. Vignolo, Annalen Phys. 19 (2010) 238.
G.J. Olmo, Int. J. Mod. Phys. D 20 (2011) 413.
…

Cartan F(R) gravity

T. I., M. Taniguchi, Symmetry 14, 1830 (2022).

Modified gravity in Cartan formalism

- Action

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}(R - 2\Lambda) + S_{\mathrm{matter}} \qquad g = \det g_{\mu\nu}$$

$$\Downarrow$$

$$S = \frac{1}{2\kappa}\int d^4x eF(R) + S_{\mathrm{matter}} \qquad e = \det e^i{}_\mu$$

  - Einstein-Cartan geometry
  - Modified Lagrangian density

Cartan formalism E. Cartan (1923), T. W. B. Kibble (1961), D. W. Sciama(1962)

- Vierbein

$e^{i}{}_{\mu}$

i: Flat (local Lorentz frame)

μ: Curved (general coordinate frame)

$$g_{\mu\nu} = \eta_{ij} e^{i}{}_{\mu} e^{j}{}_{\nu}$$

$g_{\mu\nu}$: curved, η_{ij}: flat

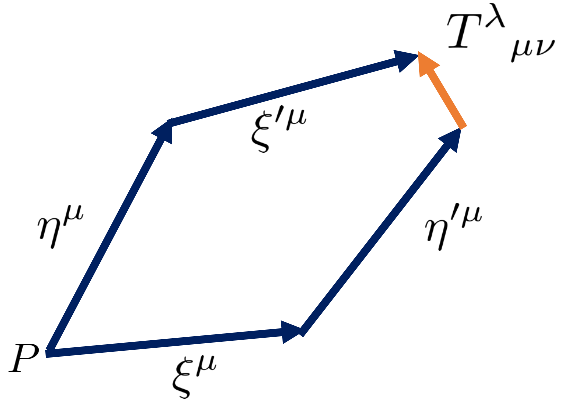
- Covariant Derivative

$$\nabla_{\nu} e^{k}{}_{\mu} = \partial_{\nu} e^{k}{}_{\mu} + \omega^{k}{}_{l\nu} e^{l}{}_{\mu} - \Gamma^{\lambda}{}_{\mu\nu} e^{k}{}_{\lambda} = 0.$$

$\omega^{k}{}_{l\nu}$: Spin connection, $\Gamma^{\lambda}{}_{\mu\nu}$: Affine connection

- Torsion

$$T^{\lambda}{}_{\mu\nu} = \Gamma^{\lambda}{}_{\mu\nu} - \Gamma^{\lambda}{}_{\nu\mu}$$



Nontorsion and contorsion parts

- Affine connection

$$\Gamma^\lambda{}_{\mu\nu} = (\Gamma_E)^\lambda{}_{\mu\nu} + K^\lambda{}_{\mu\nu}$$

Nontorsion   Contorsion

$$(\Gamma_E)^\lambda{}_{\mu\nu} \equiv \frac{1}{2} g^{\lambda\rho}(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu})$$

$$K^\lambda{}_{\mu\nu} = \frac{1}{2}\left(T^\lambda{}_{\mu\nu} + T_{\mu\nu}{}^\lambda + T_{\nu\mu}{}^\lambda\right)$$

Field equations

- Variations of action

$\delta S/\delta e^{i}{}_{\mu} = 0$ ➡ Modified Einstein equation

$$F' R^{i}{}_{\mu} - \frac{1}{2} e^{i}{}_{\mu} F(R) = M_{\rm Pl}{}^{-2} \Sigma^{i}{}_{\mu} \rightarrow R(\Sigma)$$

EM from matter

$\delta S/\delta w^{i}{}_{j\mu} = 0$ ➡ Cartan equation

Here, we assume that the matter Lagrangian does not depend on the spinor connection.

$$T^{\mu}{}_{kl} - e_l{}^{\mu} T_k + e_k{}^{\mu} T_l + (e_k{}^{\alpha} e_l{}^{\mu} - e_k{}^{\mu} e_l{}^{\alpha}) \partial_\alpha \ln F'(R) = 0$$

➡ $$T^{k}{}_{ij} = \frac{1}{2} (\delta^{k}{}_{j} e_i{}^{\lambda} - \delta^{k}{}_{i} e_j{}^{\lambda}) \partial_\lambda \ln F'(R(\Sigma))$$

Scalar Tensor theory

T. I., M. Taniguchi, Symmetry 14, 1830 (2022)

- We introduce a scalaron field

$$\phi \equiv -\sqrt{\frac{3}{2}} M_{\mathrm{Pl}} \ln F'(R)$$

and rewrite the action (without any conformal transformation)

$$S = \int d^4x e \left(\frac{{M_{\mathrm{Pl}}}^2}{2} R_E - \frac{1}{2}\partial_\lambda \phi \partial^\lambda \phi - V(\phi) \right)$$

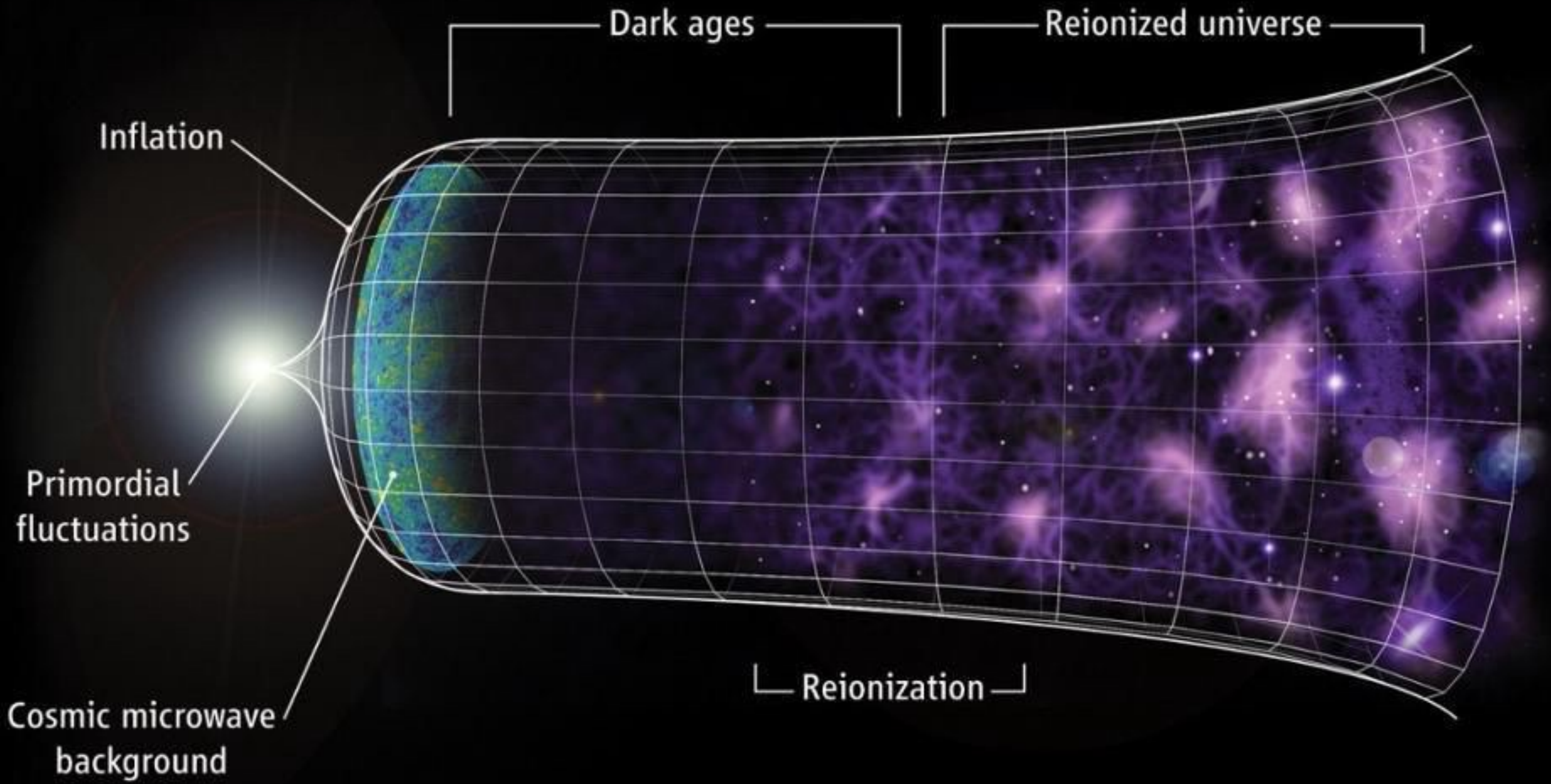
Nontorsion

$$V(\phi) = -\frac{{M_{\mathrm{Pl}}}^2}{2}(F(R) - R)\Big|_{R=R(\phi)}$$

We drop a total derivative term.

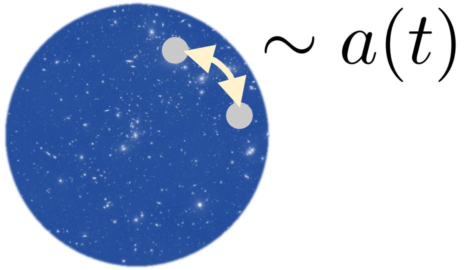
Spacetime evolution in Cartan F(R) gravity

Expanding Universe



C. Faucher-Giguère, A. Lidz, and L. Hernquist, Science 319, 5859 (47)

Energy source for accelerated expansion



- Homogeneous and isotropic spacetime

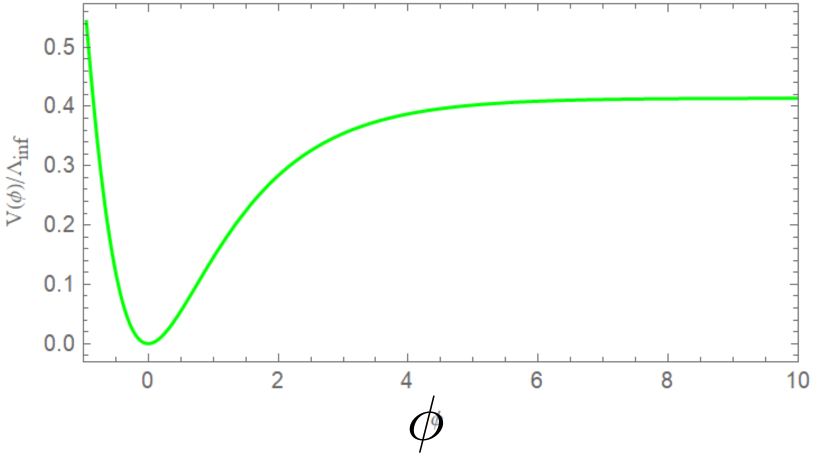
$$ds^2 = c^2dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)$$

- Energy density

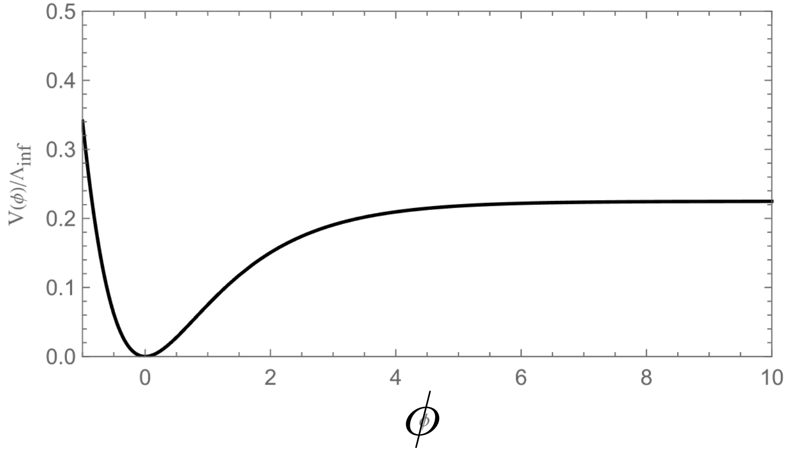
Radiation	$a(t) \propto t^{1/2}$
Matter	$a(t) \propto t^{2/3}$
Potential energy	$a(t) \propto \exp(\alpha t)$
Cosmological const.	

Scalaron potential in Cartan F(R) gravity

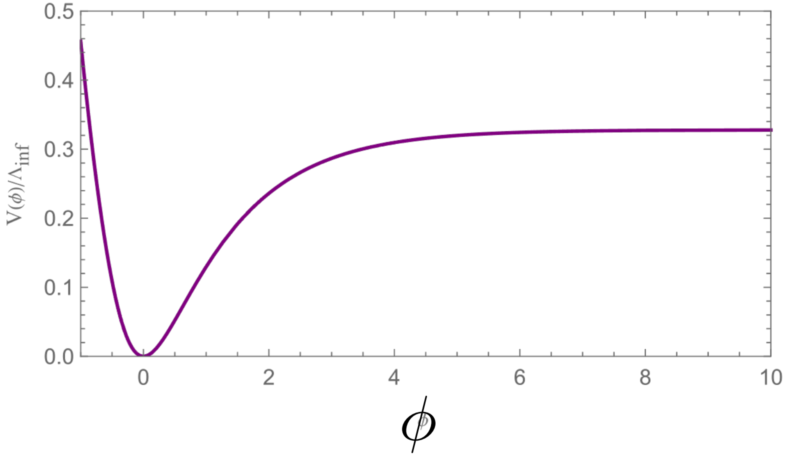
$$f(R) = 1 - \cosh R$$



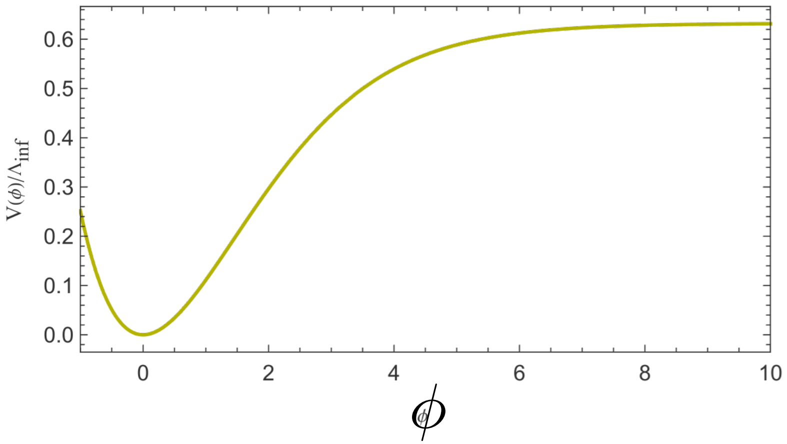
$$f(R) = -R \sinh R$$



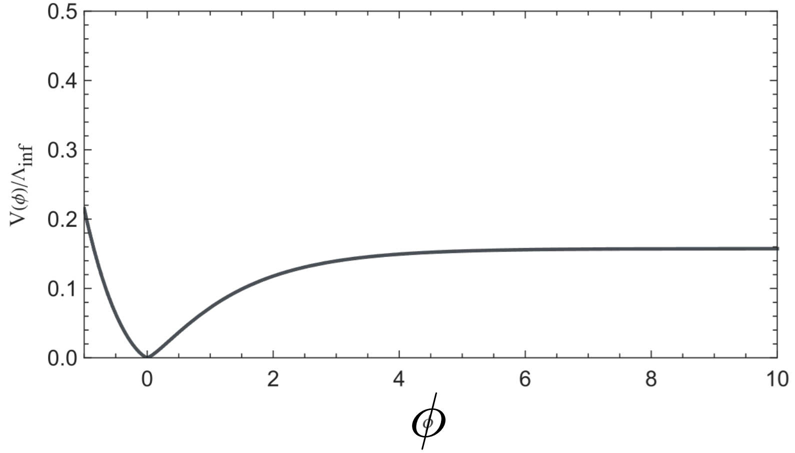
$$f(R) = 1 - e^{R^2/2}$$



$$f(R) = Re^{-R} - R$$



$$f(R) = -R^4$$



$$V(\phi) = -\frac{M_P^2}{2}(F(R) - R)$$

$$= -\frac{M_P^2}{2} f(R)$$

$$\phi \equiv -\sqrt{\frac{3}{2}} M_{\rm Pl} \ln F'(R)$$

Quasi de-Sitter expansion

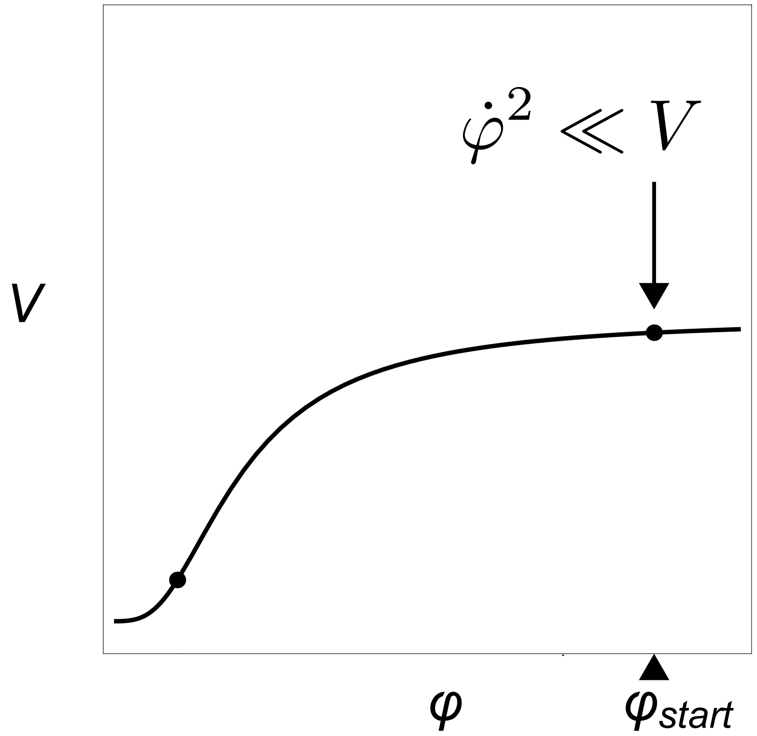
- Friedmann eq.

$$3\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{2}\dot{\varphi}^2 + V$$

- Assumption $\dot{\varphi}^2 \ll V$

⇒

$$a(t+\Delta t) \sim a(t) e^{\sqrt{\frac{V_E}{3}}\Delta t}$$



A. D. Linde, Contemp. Concepts Phys. 5, 1 (1990).

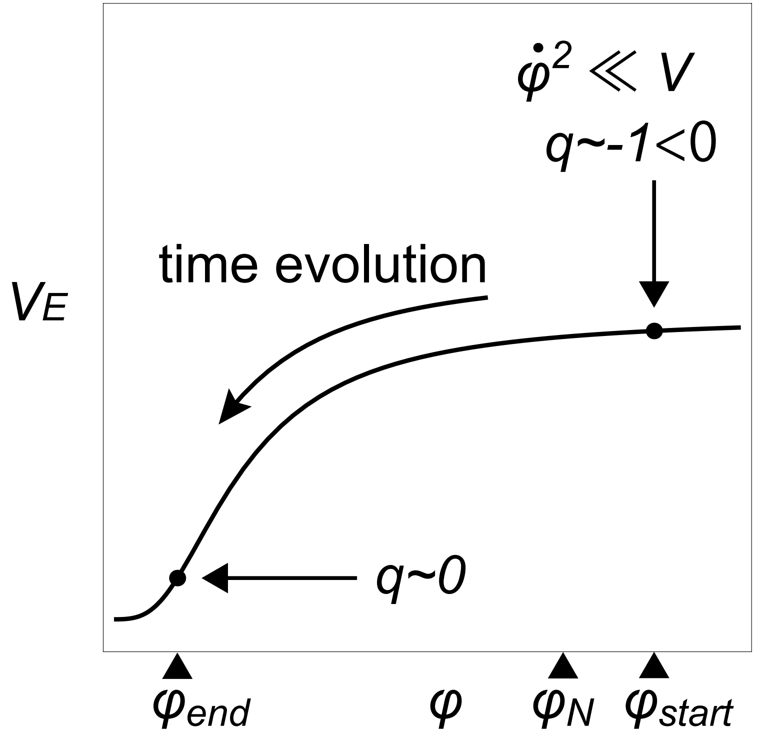
Exit from inflation

- Equation of motion

$$\ddot{\varphi} + 3\frac{\dot{a}}{a}\dot{\varphi} = -\frac{\partial V}{\partial \varphi}$$

- Deceleration parameter

$$q \equiv -\frac{a\ddot{a}}{\dot{a}^2} \to 0$$



A. D. Linde, Contemp. Concepts Phys. 5, 1 (1990).

Slow-roll Inflation

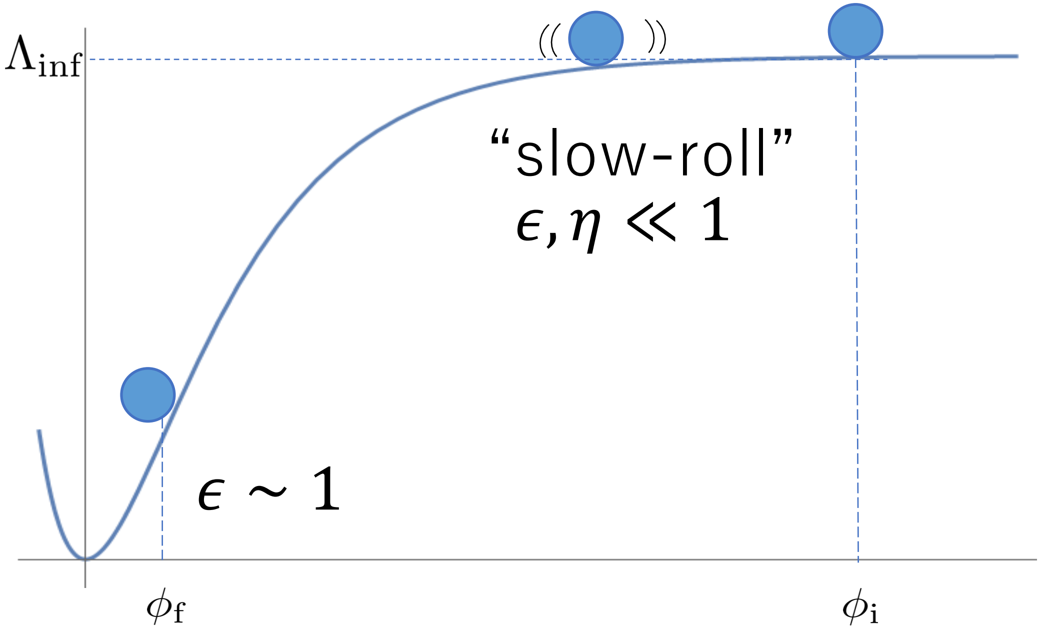
- Slow-roll parameters

$$3\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi),\ \dot{\phi}^2 \ll V(\phi)$$

$V(\Phi)$ behaves as constant at $\Lambda_{\mathbf{inf}}$

$$\epsilon = \frac{{M_{\mathrm{Pl}}}^2}{2}\left(\frac{V_\phi}{V}\right)^2,\ \eta = \frac{{M_{\mathrm{Pl}}}^2 V_{\phi\phi}}{V}$$

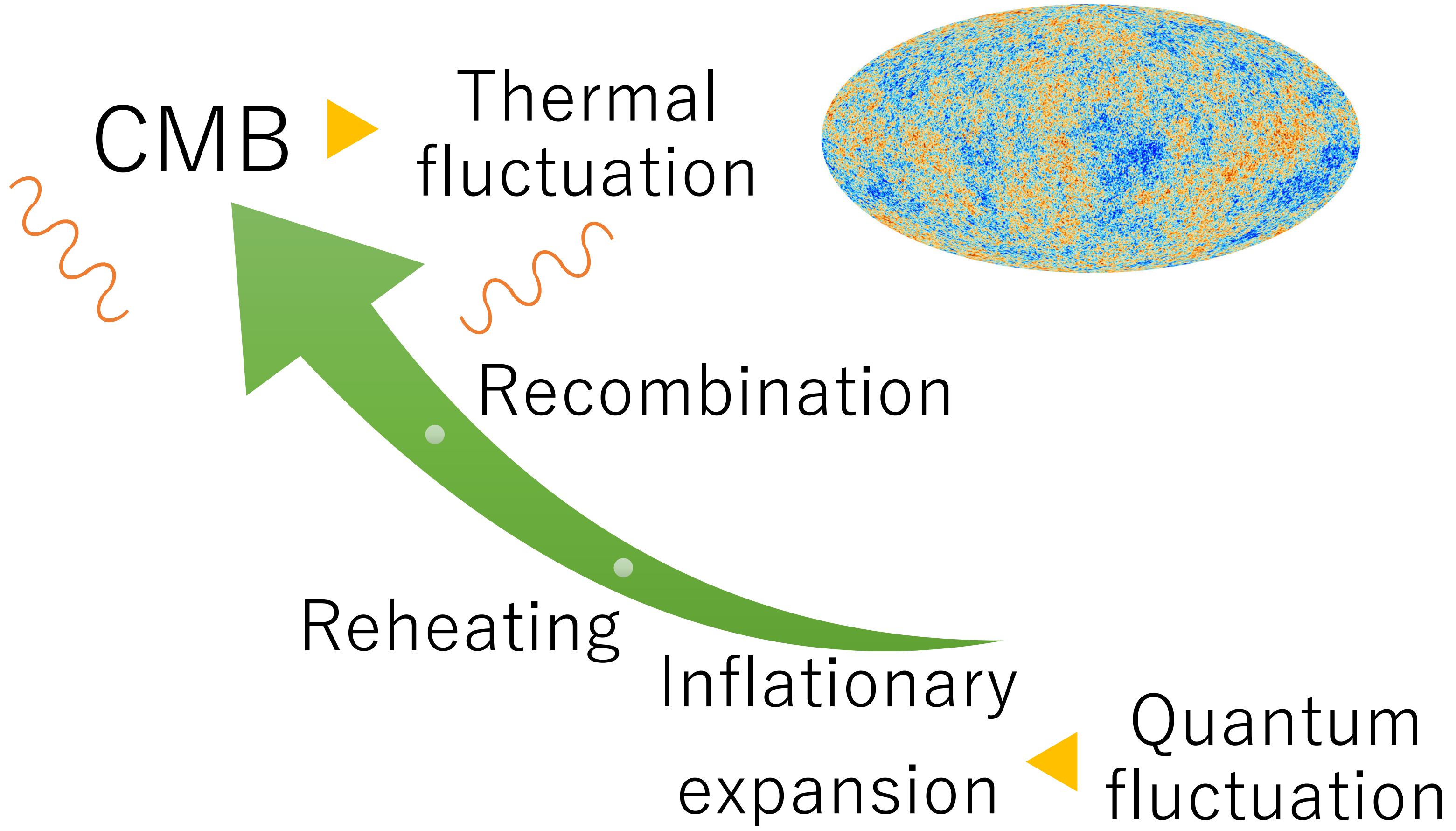
$\begin{cases} \text{During inflation } \epsilon,\eta \ll 1 \\ \text{Exit from inflation } \epsilon,\eta \sim 1 \end{cases}$



Fluctuations in CMB (cosmic microwave background)

T. I., H. Sakamoto, M. Taniguchi, arXiv:2304.14769 [gr-qc] to be appear in JCAP.

Evidence of inflation



Quantum fluctuations

$$\varphi + \delta\varphi \rightarrow \mathcal{P}_s(k)$$

Scalar type fluctuation

Origin: quantum fluctuation of scalar field

Tensor type fluctuation

Origin: quantum fluctuation of space-time

$$g^{\mu\nu} + \delta h^{\mu\nu} \rightarrow \mathcal{P}_t(k)$$

Observed CMB fluctuations

- Rescaled scalar type fluctuation

$$\mathcal{P}_s(k) \equiv A_s \left(\frac{k}{k_0}\right)^{n_s - 1}$$

- Rescaled tensor type fluctuation

$$\mathcal{P}_t(k) \equiv A_t \left(\frac{k}{k_0}\right)^{n_t}$$

- Tensor to scalar ratio

$$r \equiv \frac{\mathcal{P}_t(k)}{\mathcal{P}_s(k)}$$

$$n_s - 1 = -6\epsilon + 2\eta$$

$$r = 16\epsilon$$

expressed by ϵ, η

Power law model

E. Elizalde, S. Nojiri, S. D. Odintsov, L. Sebastiani and S. Zerbini (2011)

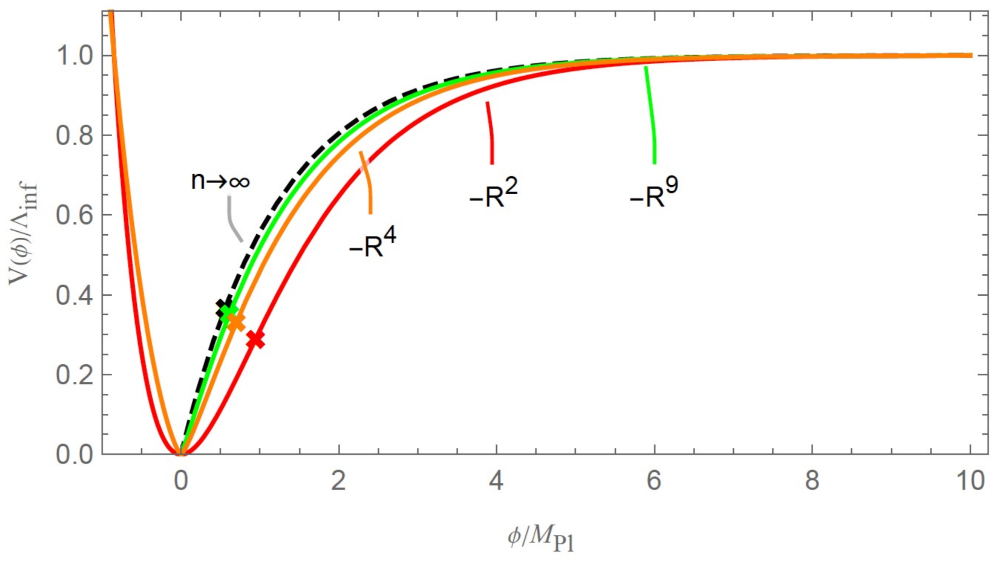
- Modified theory of gravity

$$F(R) = R - \frac{R^n}{(2nM^2)^{n-1}n}$$

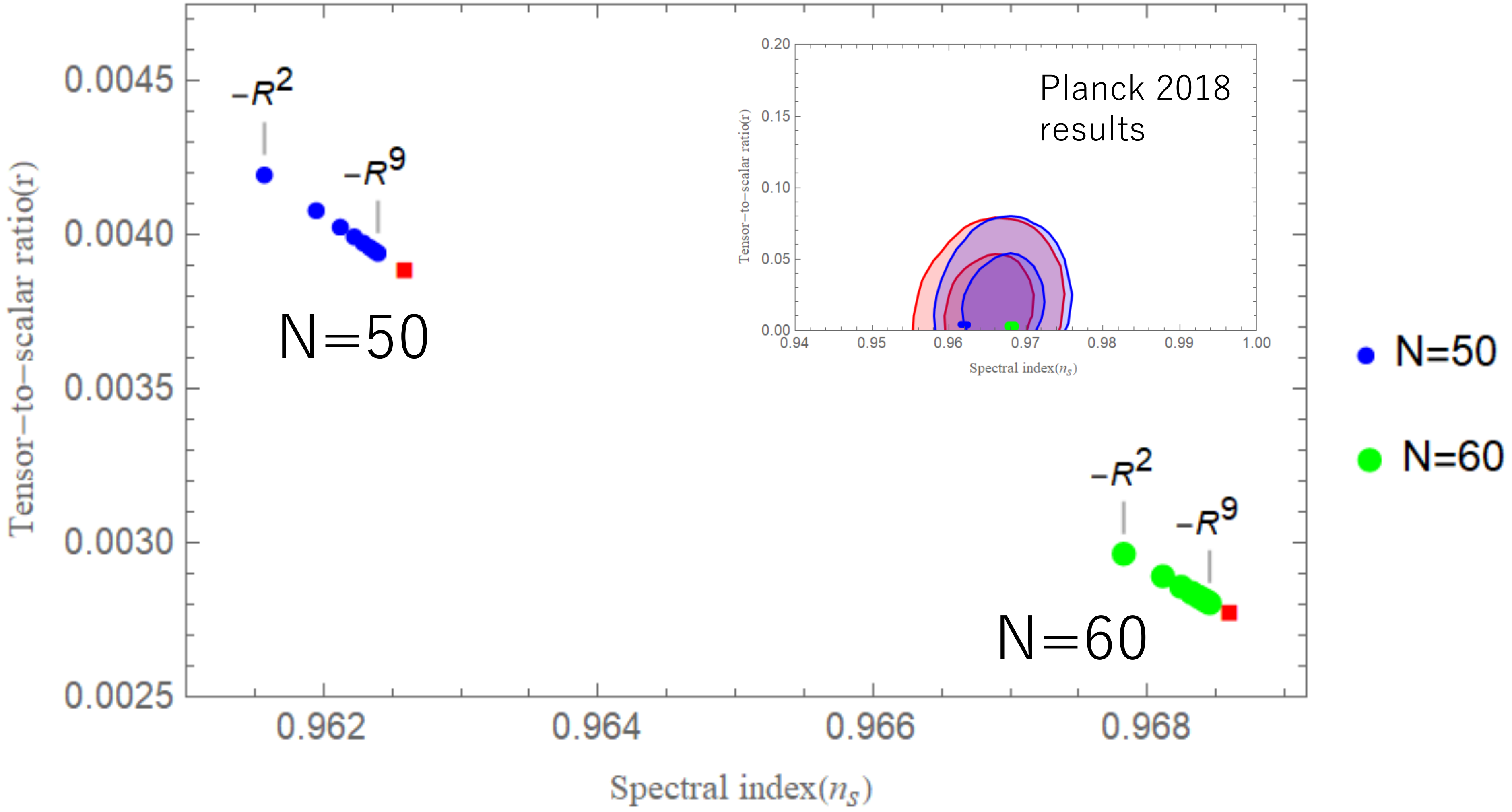
- Scalaron potential

$$V(\phi) = -\frac{{M_{\rm Pl}}^2}{2}\, f(R)\big|_{R=R(\phi)}$$

$$= {M_{\rm Pl}}^2 M^2 \left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm Pl}}}\right)^{1+\frac{1}{n-1}}$$



CMB fluctuations (n_s and r)



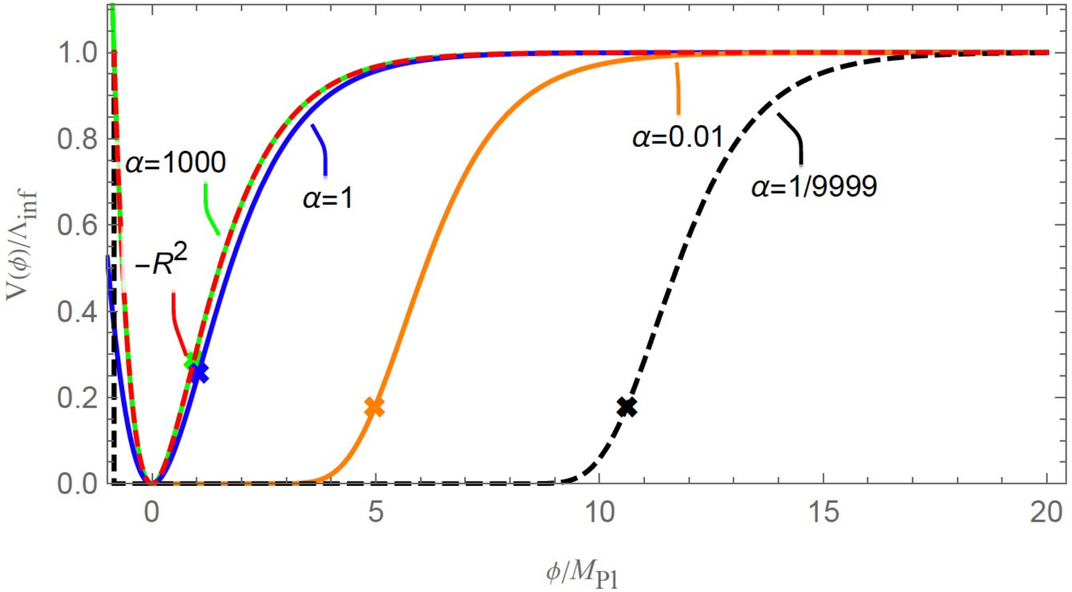
Logarithmic model

S. Nojiri, S. D. Odintsov (2014)

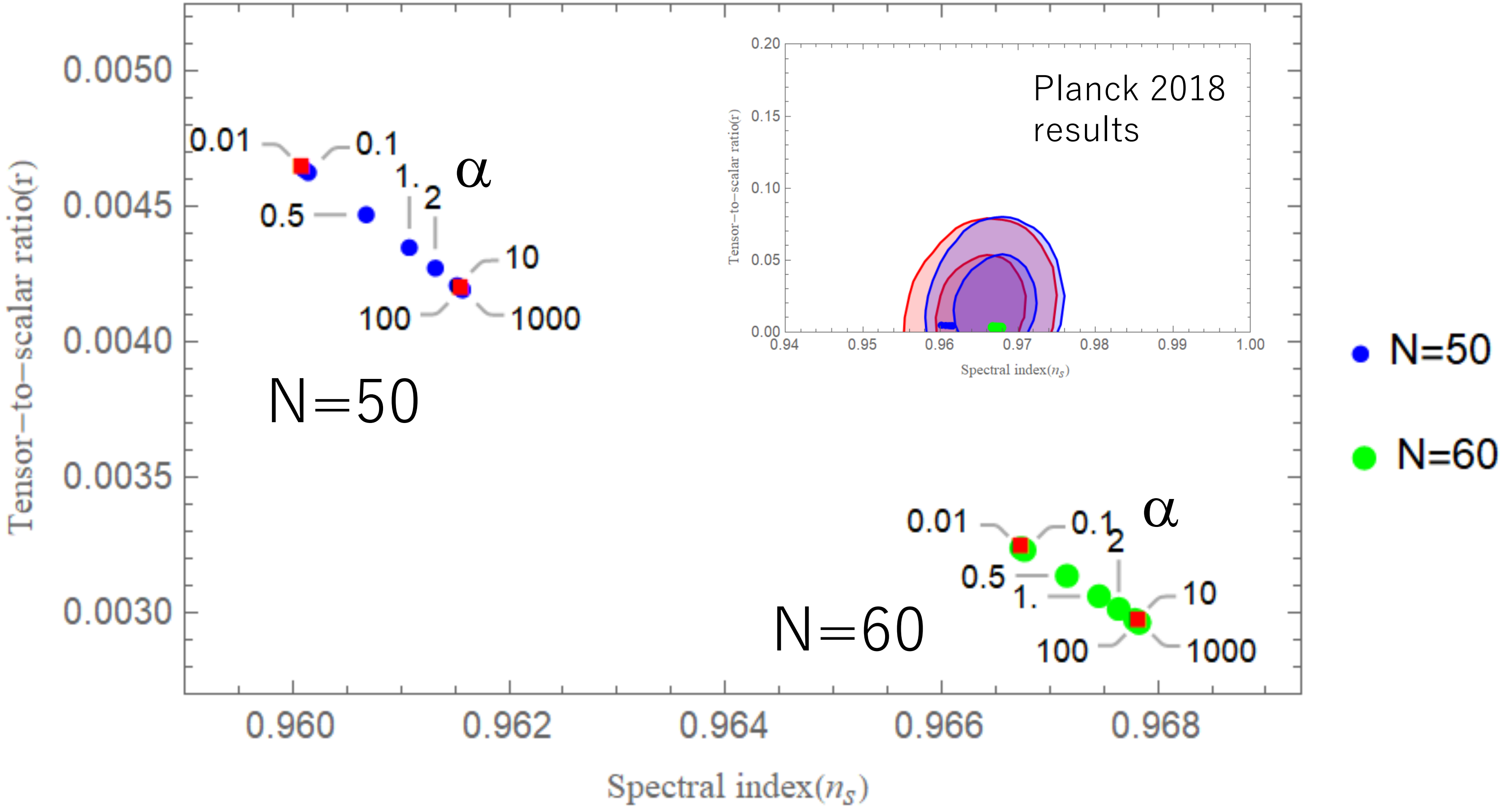
- Modified theory of gravity

$$F(R) = R - \alpha R \ln\left(\frac{R}{R_0} + 1\right)$$

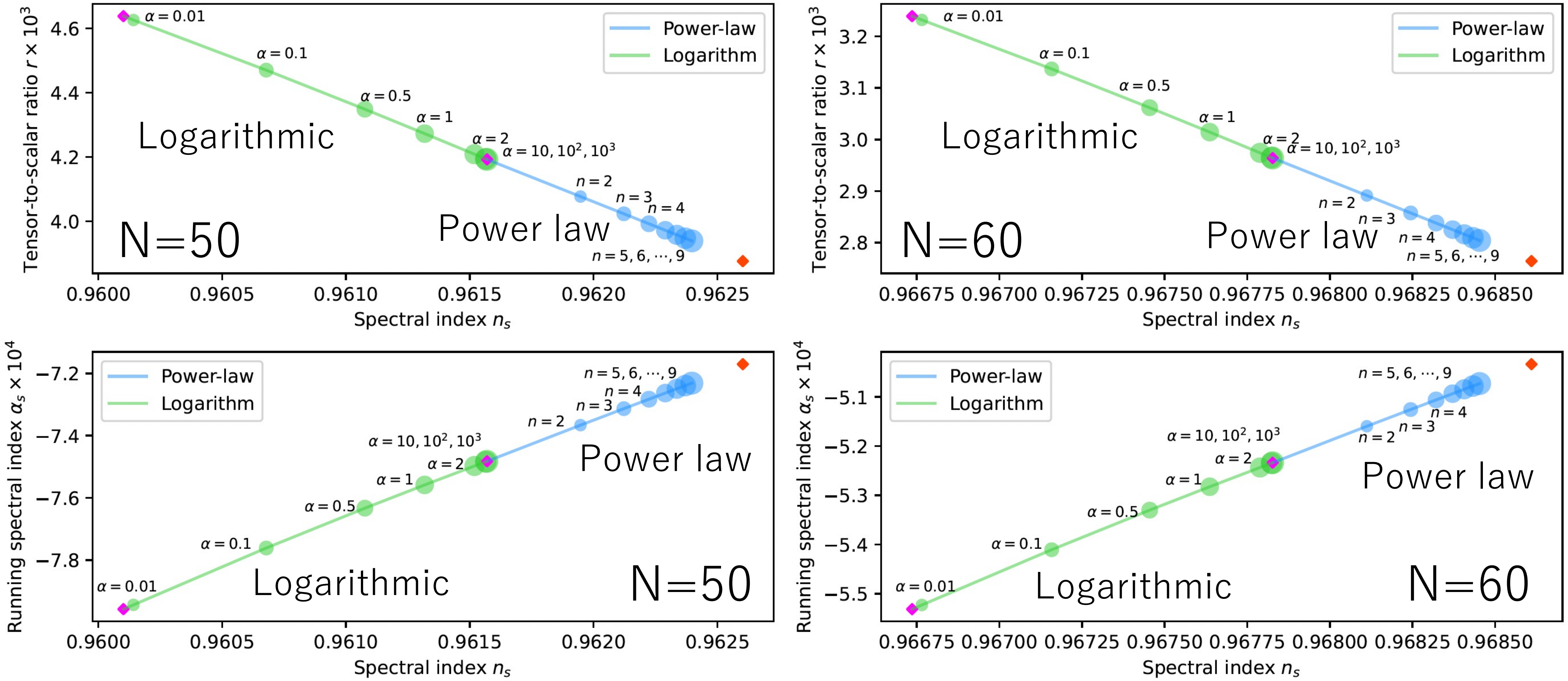
- Scalaron potential



CMB fluctuations (n_s and r)

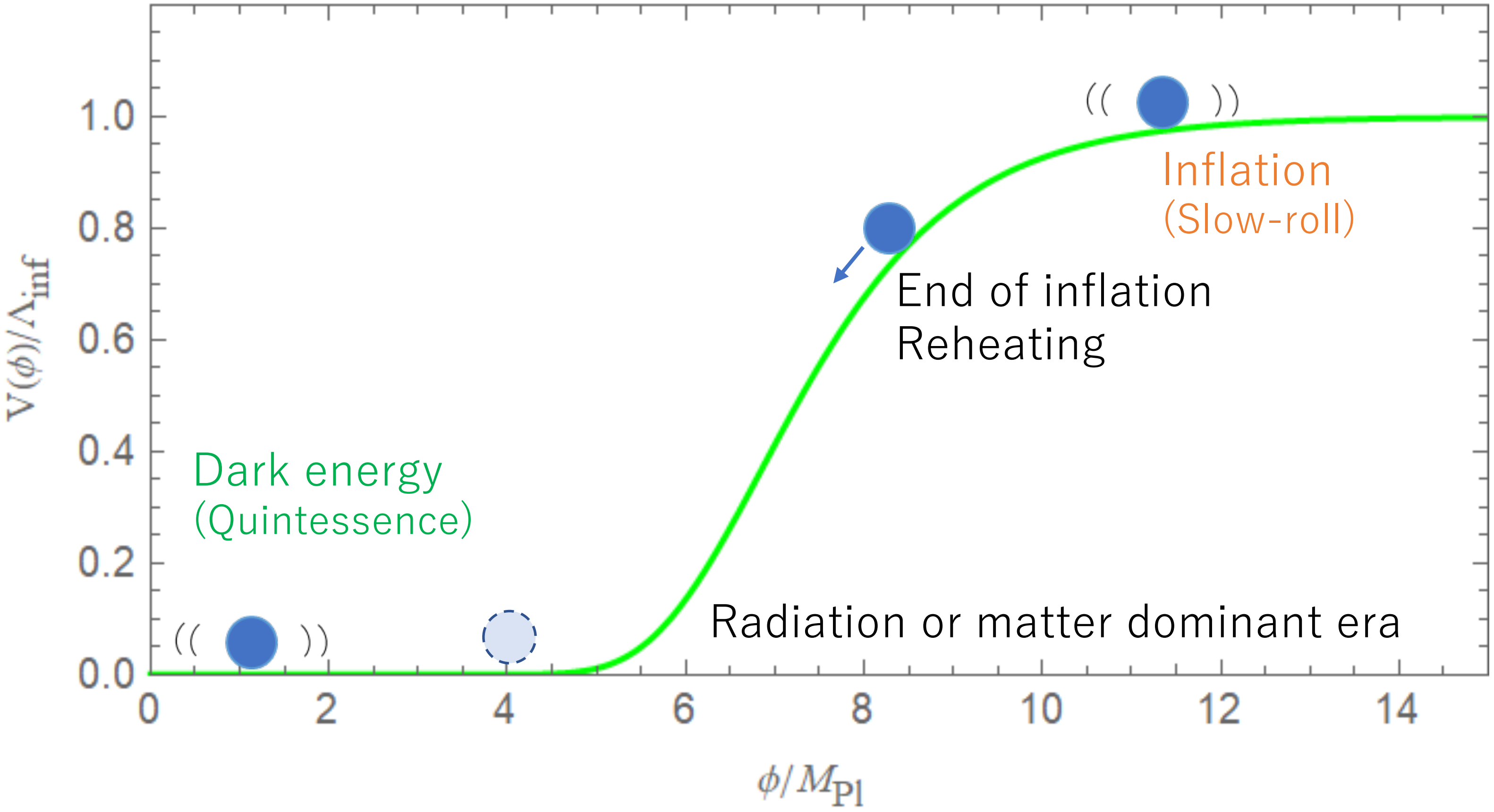


CMB fluctuations (n_s and r)



T. I., H. Sakamoto, M. Taniguchi, arXiv:2304.14769 [gr-qc] to be appear in JCAP.

After the inflation era (logarithmic model)

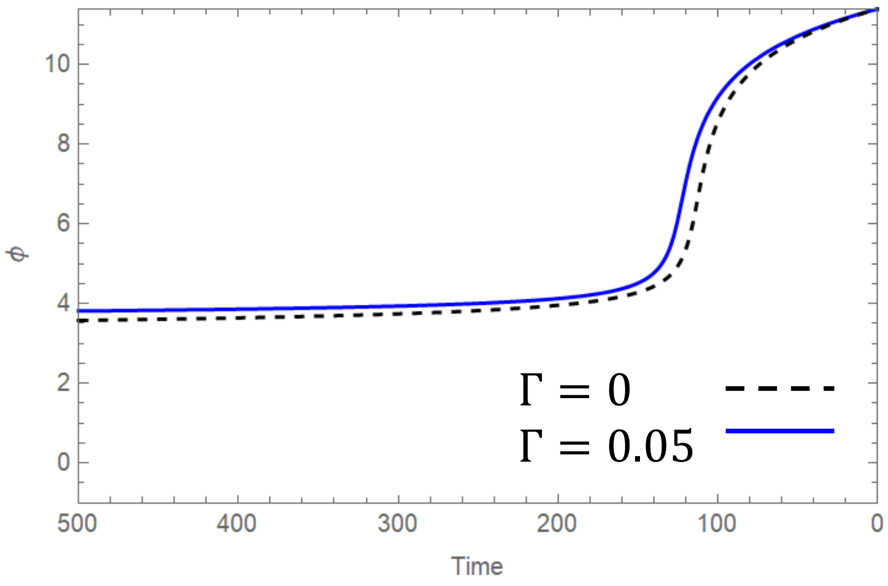


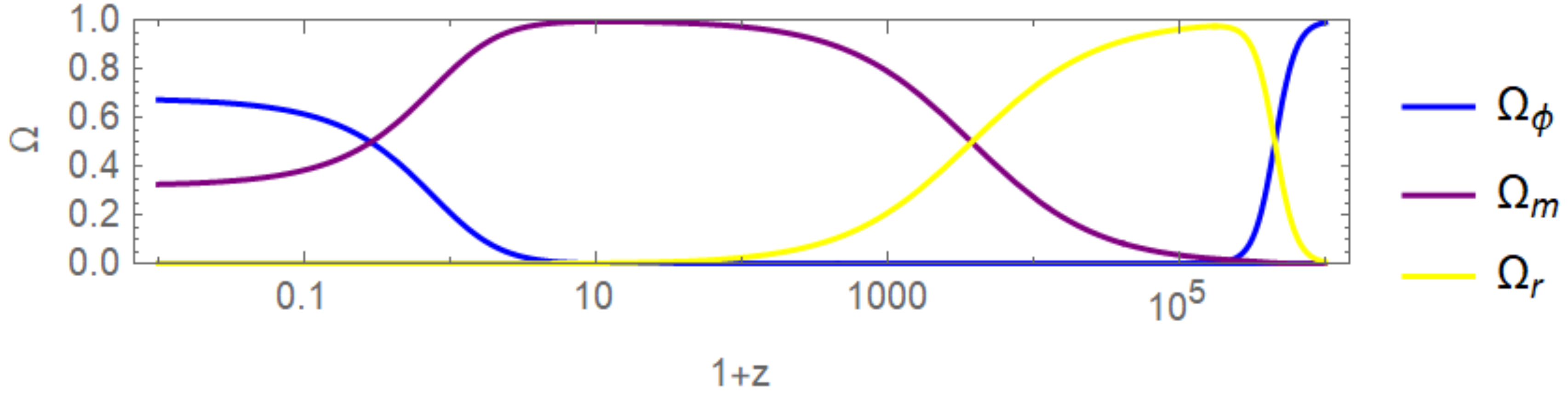
After the inflation era (logarithmic model)

- EoM for scalaron

$$3H(t)^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$\ddot{\phi} + [3H(t) + \Gamma]\dot{\phi} + V'(\phi) = 0$$





Conclusion

Summary

- We study a modified theory of gravity on Einstein-Cartan geometry.
- The Cartan F(R) can be rewritten as a scalar tensor theory without conformal transformation. T. I., M. Taniguchi, Symmetry 14, 1830 (2022).
- The inflaton potential energy can induce the inflationary expansion of the universe.
- We observed a robustness of the CMB fluctuations.

  T. I., H. Sakamoto, M. Taniguchi, arXiv:2304.14769 [gr-qc] to be appear in JCAP.
- There is a possibility to induce the current accelerated expansion.

Open questions

- Many models of Cartan F(R) gravity induce the inflationary expansion. How to distinguish the models?
- Matter fields may have an decisive role for the spacetime structure. What is a role of the matter fields?