Estimation of graviton mass from Schwarzschild precession in S2 star orbit around Sgr A*

Predrag Jovanović

Astronomical Observatory Belgrade

Outline of the talk

- Motivation: recent detection of Schwarzschild precession (and its possible small deviation from GR prediction) in the orbit of S2 star around Sgr A* by GRAVITY Collaboration in 2020
- Outline:
  - GR vs theories of massive gravity
  - $f(R)$ gravity and Yukawa-like nonlinear correction to the gravitational potential
  - Observed stellar orbits around Sgr A* at Galactic Center
  - Our old results: constraints on the graviton mass from analysis of the observed stellar orbits in Yukawa gravity
  - Detection of Schwarzschild precession in S2 star orbit
  - Our new results: improved constraints on Yukawa gravity parameters and mass of graviton from the observed Schwarzschild precession in S2 star orbit
  - Conclusions
- The presented results are obtained in collaboration with:
  - Dr. Vesna Borka Jovanović and Dr. Duško Borka (Serbia)
  - Prof. Alexander F. Zakharov (Russia)
  - Prof. Salvatore Capozziello (Italy)

GR vs theories of massive gravity

- General Relativity (GR): graviton is massless and travels along null geodesics (like photon), i.e. at the speed of light c
- Theories of massive gravity (introduced by Fierz & Pauli, 1939, RSPSA, 173, 211): gravitation is propagated by a massive field (i.e. by graviton with small, nonzero mass m_g)
- Important predictions of massive gravity theories (Will, 1998, PRD, 57, 206):
  1) The effective Newtonian potential has a Yukawa form: $\propto r^{-1} \exp(-r/\lambda_g)$,
     $\lambda_g = h/(m_g\, c)$ being the Compton wavelength of graviton
  2) Massive graviton propagates at an energy (or frequency) dependent speed v_g
- Modified dispersion relation: $E^2 = p^2c^2 + m_g^2c^4 \quad \wedge \quad v_g^2/c^2 \equiv c^2p^2/E^2 \quad \Rightarrow$
  $v_g^2/c^2 = 1 - m_g^2c^4/E^2 = 1 - h^2c^2/(\lambda_g^2E^2) = 1 - c^2/(f\lambda_g)^2$
- LIGO bound on m_g from GW150914 using a potential with Yukawa type correction (Abbott et al., LIGO Scientific and Virgo Collaborations, 2016, PRL, 116, 221101):
  $\varphi(r) = \frac{GM}{r}\left(1 - e^{-r/\lambda_g}\right) \Rightarrow \lambda_g > 1.6 \times 10^{13}\ \text{km} \Rightarrow \boxed{m_g \leq 1.2 \times 10^{-22}\ \text{eV}/c^2}$
- Constraints on speed of gravity from the time difference Δt between GW and γ-rays from a binary neutron star merger in the galaxy NGC 4993 at $z \approx 0.01$ and $D = 26$ Mpc (Abbott et al. 2017, ApJL, 848, L13): $-3 \times 10^{-15} \leq \frac{v_g}{c} - 1 \leq +7 \times 10^{-16}$

$f(R)$ gravity and Yukawa-like nonlinear correction

- Gravitational potential with a Yukawa correction can be obtained in the Newtonian limit of any analytic $f(R)$ gravity model (Capozziello et al. 2014, PRD, 90, 044052)
- Action for $f(R)$ gravity: $\mathcal{S} = \int d^4x\sqrt{-g}\,[f(R) + \mathcal{X}\mathcal{L}_m]\,, \quad \mathcal{X} = \frac{16\pi G}{c^4}$
- 4th-order field equations: $f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - f'(R)_{;\mu\nu} + g_{\mu\nu}\Box f'(R) = \frac{\mathcal{X}}{2}T_{\mu\nu}$
- Trace: $3\Box f'(R) + f'(R)R - 2f(R) = \frac{\mathcal{X}}{2}T$
- Analytic Taylor expandable function $f(R)$:

$$f(R) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} R^n = f_0 + f_1 R + \frac{f_2}{2}R^2 + \ldots \quad \Rightarrow$$

- Metric: $ds^2 = \left[1 + \frac{2\Phi(r)}{c^2}\right] c^2dt^2 - \left[1 - \frac{2\Psi(r)}{c^2}\right] dr^2 - r^2 d\Omega^2$
- **Yukawa-like nonlinear correction** to the grav. potential in the weak field limit:

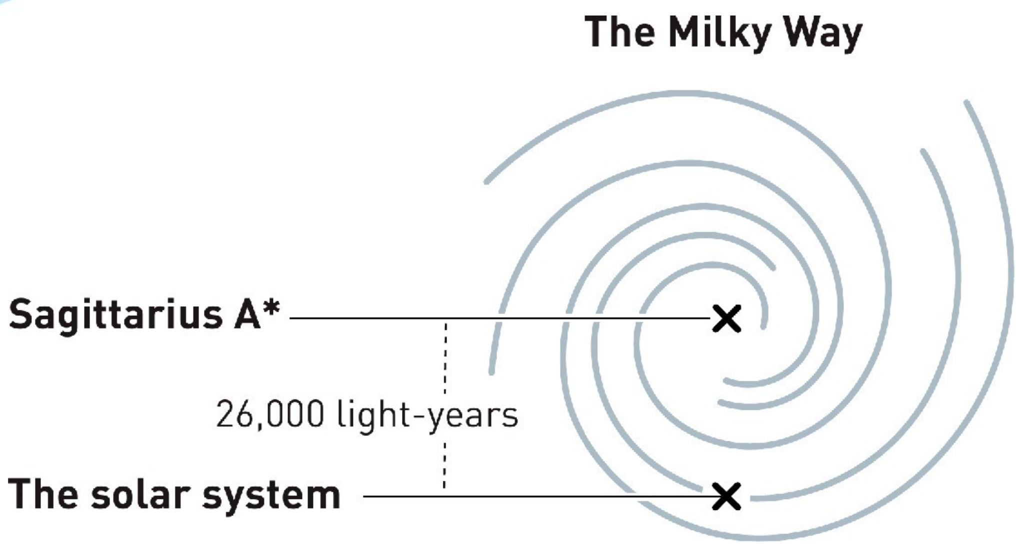
$$\boxed{\Phi(r) = -\frac{GM}{(1+\delta)r}\left(1 + \delta e^{-\frac{r}{\Lambda}}\right)}$$

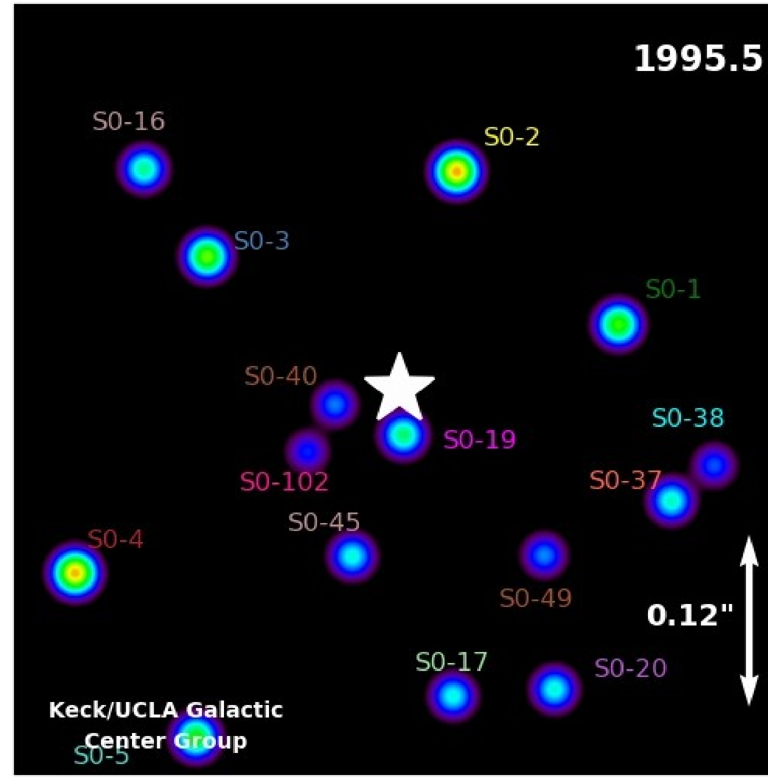
$$\Psi(r) = \frac{GM}{(1+\delta)r}\left[\left(1 + \frac{r}{\Lambda}\right)\delta e^{-\frac{r}{\Lambda}} - 1\right]$$

$$\Lambda^2 = -f_1/f_2 \quad \wedge \quad \delta = f_1 - 1$$

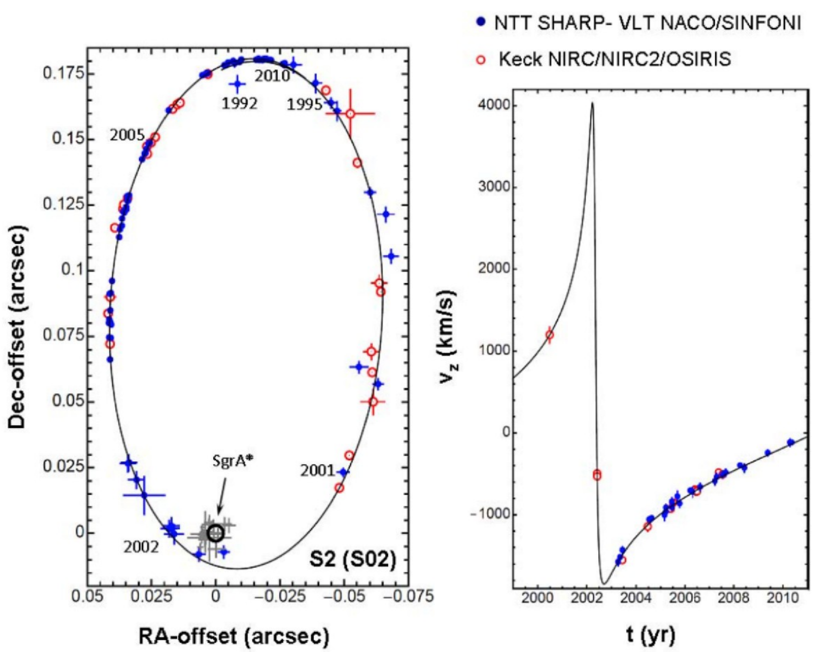
- **Λ - range of Yukawa interaction**
- **δ - universal constant**

Observed stellar orbits around Sgr A*

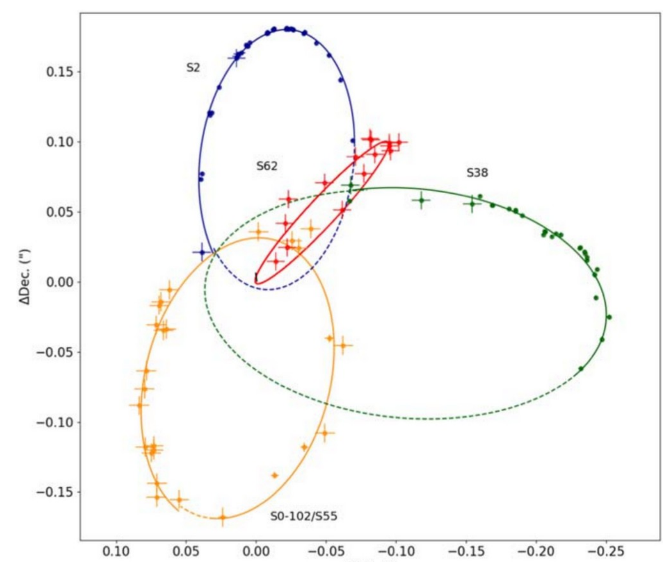




- Observations of S2 star by NTT/VLT and Keck:
  - Gillessen et al. 2009, ApJ, 692, 1075
  - Ghez et al. 2008, ApJ, 689, 1044

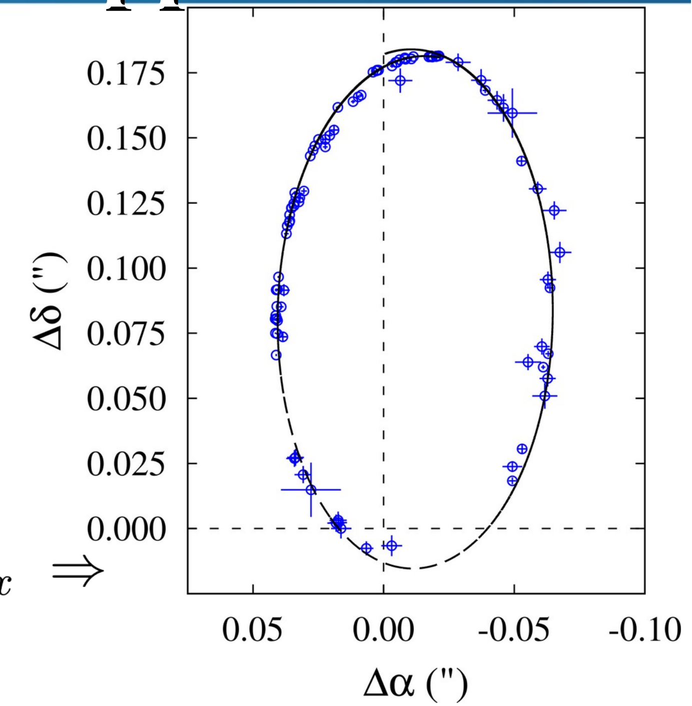


- New observations:
  - Gillessen et al. 2017, ApJ, 837, 30
  - GRAVITY, 2018, A&A, 615, L15
  - Peißker et al. 2020, ApJ, 889, 61 (right)

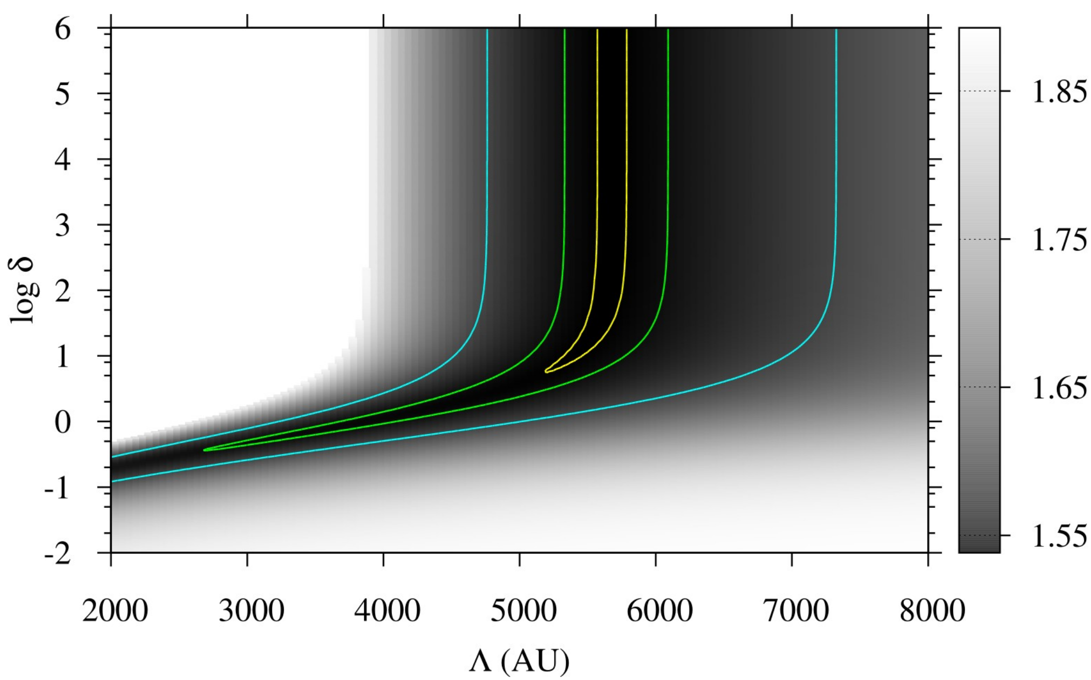


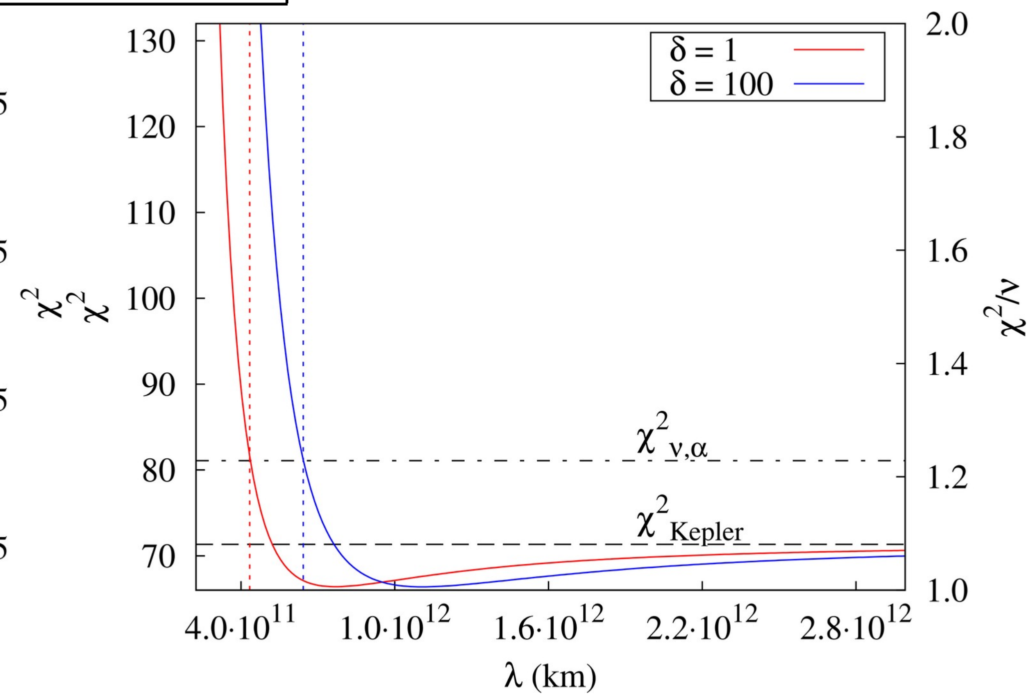
Our estimates for graviton mass upper bound

- Simulated orbits in Yukawa gravitational potential were fitted to the astrometric observations of S2 star
- χ^2 test of goodness of the fits for significance level $\alpha = 0.1$: regions $\lambda < \lambda_x$ where $\chi^2 > \chi^2_{\nu,\alpha}$ can be excluded with 90% probability
- Upper bounds for graviton mass for $\delta = 1$ and $\delta = 100$, respectively (Zakharov, Jovanović, Borka, Borka Jovanović, 2016, JCAP, 2016, No. 05, 045): $m_g = h\,c/\lambda_x \Rightarrow$



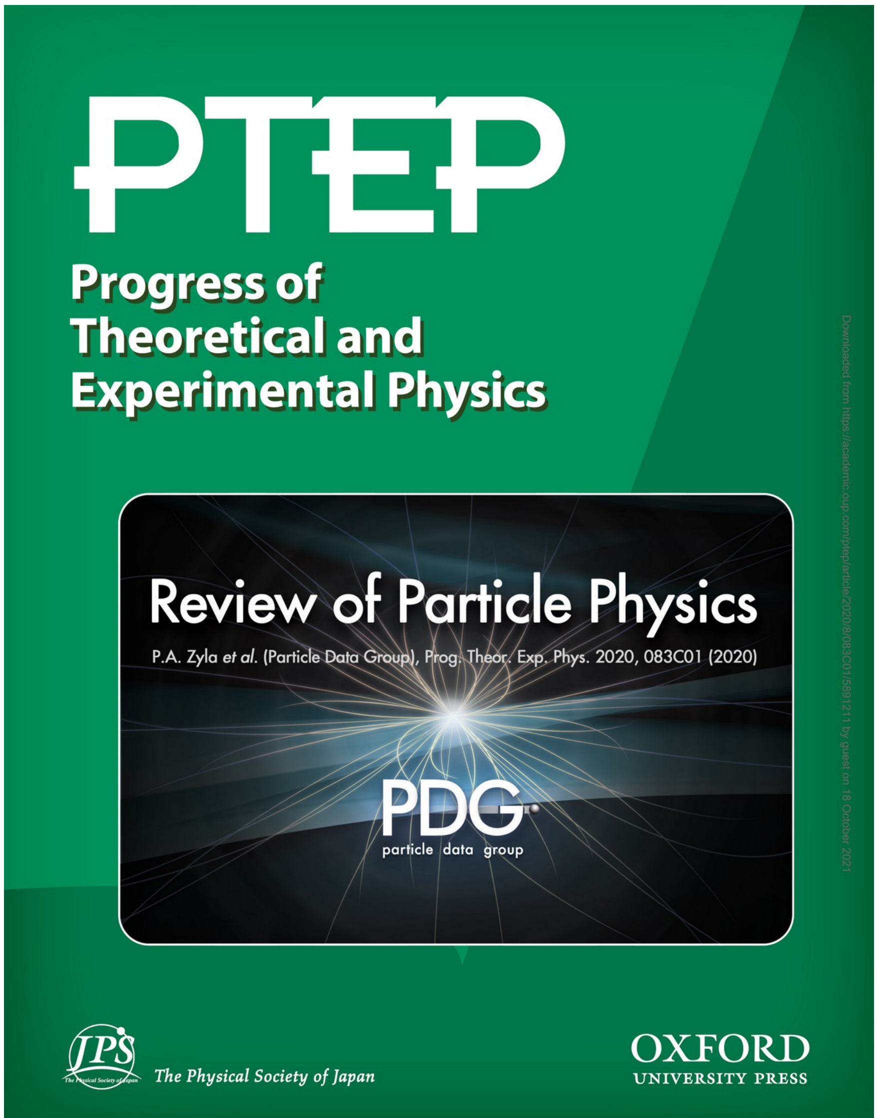
$$m_g = 2.9 \times 10^{-21}\ \text{eV} \quad \wedge \quad m_g = 1.9 \times 10^{-21}\ \text{eV}$$





Our estimates for graviton mass accepted by PDG

- From 2019, our estimate is in *Gauge and Higgs Boson Particle Listings* by PDG (Zyla et al., PDG, 2020, PTEP, 083C01)



Gauge & Higgs Boson Particle Listings

γ, g, graviton, W

graviton

$J = 2$

graviton MASS

VALUE (eV)	DOCUMENT ID		TECN	COMMENT
<6 × 10^{-32}	[1] CHOUDHURY	04	YUKA	Weak gravitational lensing
• • • We do not use the following data for averages, fits, limits, etc. • • •				
<6.8 × 10^{-23}	BERNUS	19	YUKA	Planetary ephemeris INPOP17b
<1.4 × 10^{-29}	[2] DESAI	18	YUKA	Gal cluster Abell 1689
<5 × 10^{-30}	[3] GUPTA	18	YUKA	SPT-SZ
<3 × 10^{-30}	[3] GUPTA	18	YUKA	Planck all-sky SZ
<1.3 × 10^{-29}	[3] GUPTA	18	YUKA	redMaPPer SDSS-DR8
<6 × 10^{-30}	[4] RANA	18	YUKA	Weak lensing in massive clusters
<8 × 10^{-30}	[5] RANA	18	YUKA	SZ effect in massive clusters
<7 × 10^{-23}	[6] ABBOTT	17	DISP	Combined dispersion limit from three BH mergers
<1.2 × 10^{-22}	[6] ABBOTT	16	DISP	Combined dispersion limit from two BH mergers
<2.9 × 10^{-21}	[7] ZAKHAROV	16	YUKA	S2 star orbit
<5 × 10^{-23}	[8] BRITO	13		Spinning black holes bounds
<4 × 10^{-25}	[9] BASKARAN	08		Graviton phase velocity fluctuations
<6 × 10^{-32}	[10] GRUZINOV	05	YUKA	Solar System observations
<9.0 × 10^{-34}	[11] GERSHTEIN	04		From Ω_{tot} value assuming RTG
>6 × 10^{-34}	[12] DVALI	03		Horizon scales
<8 × 10^{-20}	[13,14] FINN	02	DISP	Binary pulsar orbital period decrease
	[14,15] DAMOUR	91		Binary pulsar PSR 1913+16
<7 × 10^{-23}	TALMADGE	88	YUKA	Solar system planetary astrometric data
< 2 × 10^{-29} h_0^{-1}	GOLDHABER	74		Rich clusters
<7 × 10^{-28}	HARE	73		Galaxy
<8 × 10^{4}	HARE	73		2γ decay

graviton REFERENCES

ABBOTT	16	PRL 116 061102	B.P. Abbott *et al.*	(LIGO and Virgo Collabs.)
ZAKHAROV	16	JCAP 1605 045	A.F. Zakharov *et al.*	
BRITO	13	PR D88 023514	R. Brito, V. Cardoso, P. Pani	(LISB, MISS, HSCA+)

PPN equations of motion

- The parameterized post-Newtonian (PPN) formalism (Will & Nordtvedt 1972, ApJ, 177, 757) completely characterizes the weak-field behavior of a gravity theory by a set of ten PPN parameters in which this theory differs from Newtonian gravity
- The standard PPN formalism is not viable for massive gravity theories because Newtonian order terms are modified by the presence of massive fields so that the Newtonian potential acquires a Yukawa-like correction (Clifton, 2008, PRD, 77, 024041; Alsing et al. 2012, PRD, 85, 064041)
- *f*(*R*) gravity in the low energy limit gives the Yukawa potential, but it also includes the first post-Newtonian approximation that in the limit $f(R) \rightarrow R$ should coincide with GR $\Rightarrow$ **extended PPN formalism** which includes both Yukawa correction and the post-Newtonian correction (Jovanović et al. 2023, JCAP, 056)
- Equations of motion in Yukawa gravity: $\ddot{\vec{r}}_Y = \ddot{\vec{r}}_N + \ddot{\vec{r}}_{cor,PPN} + \ddot{\vec{r}}_{cor,Y}$

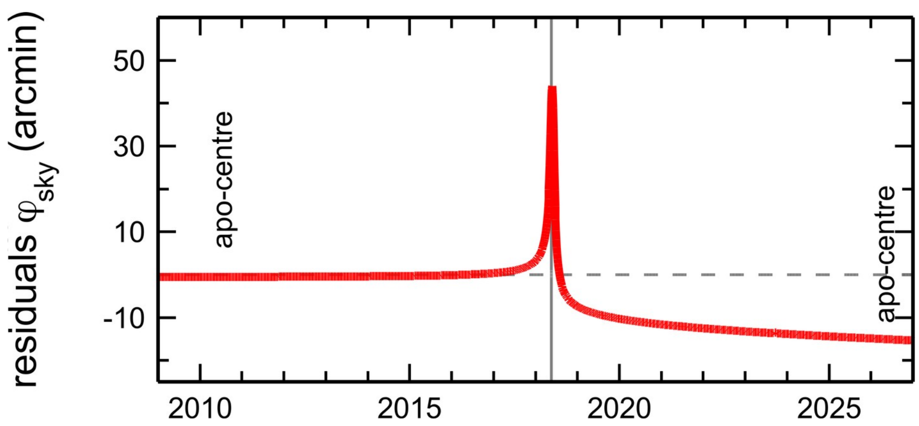
$$\ddot{\vec{r}}_N = -GM\frac{\vec{r}}{r^3} \qquad \ddot{\vec{r}}_{cor,Y} = \frac{\delta \cdot GM}{1+\delta}\left[1-\left(1-\frac{r}{\Lambda}\right)e^{-\frac{r}{\Lambda}}\right]\frac{\vec{r}}{r^3}$$

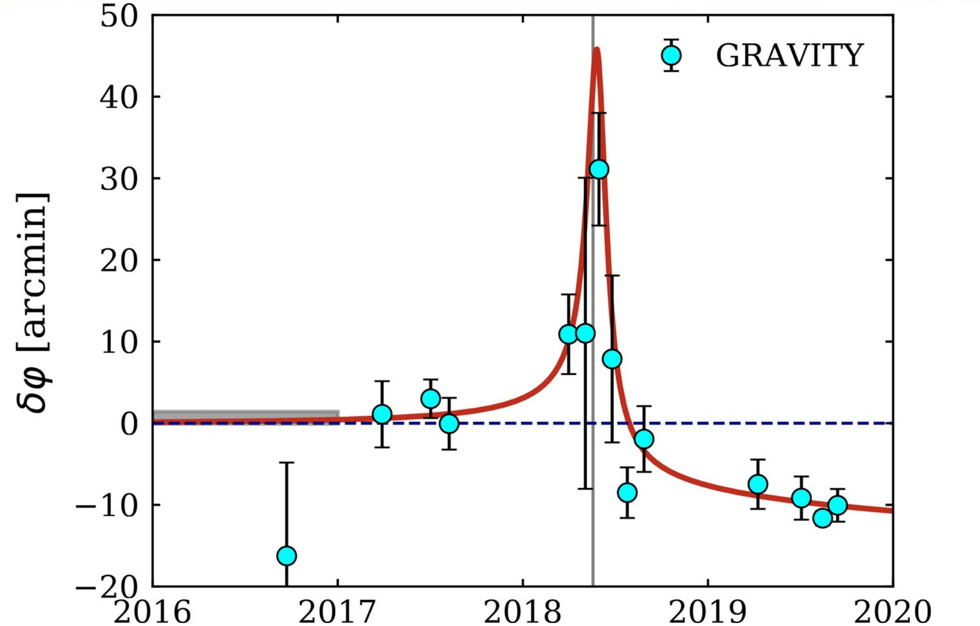
$$\ddot{\vec{r}}_{cor,PPN} = \frac{GM}{c^2 r^3}\left\{\left[2\left(\beta+\gamma\right)\frac{GM}{r} - \gamma\left(\dot{\vec{r}}\cdot\dot{\vec{r}}\right)\right]\vec{r} + 2\left(1+\gamma\right)\left(\vec{r}\cdot\dot{\vec{r}}\right)\dot{\vec{r}}\right\}$$

- Standard PPN equations of motion in the GR: $\ddot{\vec{r}}_{GR} = \ddot{\vec{r}}_N + \ddot{\vec{r}}_{cor,PPN}, \quad \beta = \gamma = 1$

Detection of Schwarzschild precession in S2 star orbit

- GRAVITY Collaboration, Abuter et al. 2020, A&A, 636, L5:





- Theoretical expectations (left) and observed residuals of $\delta\varphi$ (right) between GR and Keplerian orbits of the S2 star around 2018 pericentre
- Schwarzschild precession detected as a change in $\delta\varphi$ by ≈12' between two apocentres
- Modified PPN equation of motion was used to parametrize the effect of the Schwarzschild metric: $$\ddot{\vec{r}} = -GM\frac{\vec{r}}{r^3} + f_{SP}\frac{GM}{c^2r^3}\left[\left(4\frac{GM}{r} - \dot{\vec{r}}\cdot\dot{\vec{r}}\right)\vec{r} + 4\left(\vec{r}\cdot\dot{\vec{r}}\right)\dot{\vec{r}}\right]$$
- An ad hoc factor f_{SP} in front of the first post-Newtonian correction of GR shows to which extent some gravitational model is relativistic: $f_{SP} = (2 + 2\gamma - \beta)/3$
- GR and Keplerian orbits are obtained for $f_{SP} = 1$ and $f_{SP} = 0$, respectively
- The best-fit of the observed S2 orbit around Sgr A* resulted with $f_{SP} = 1.10 \pm 0.19$

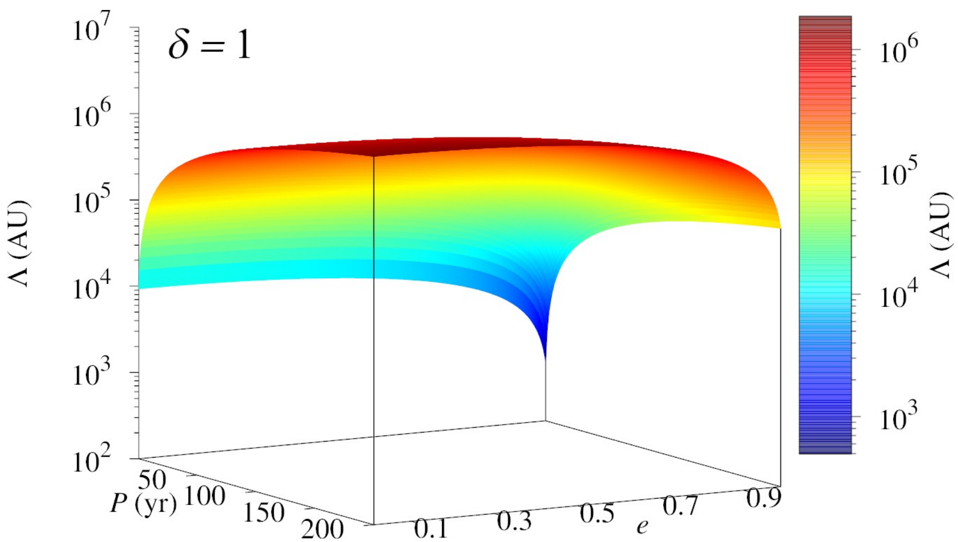
Constraints on Yukawa gravity parameters I

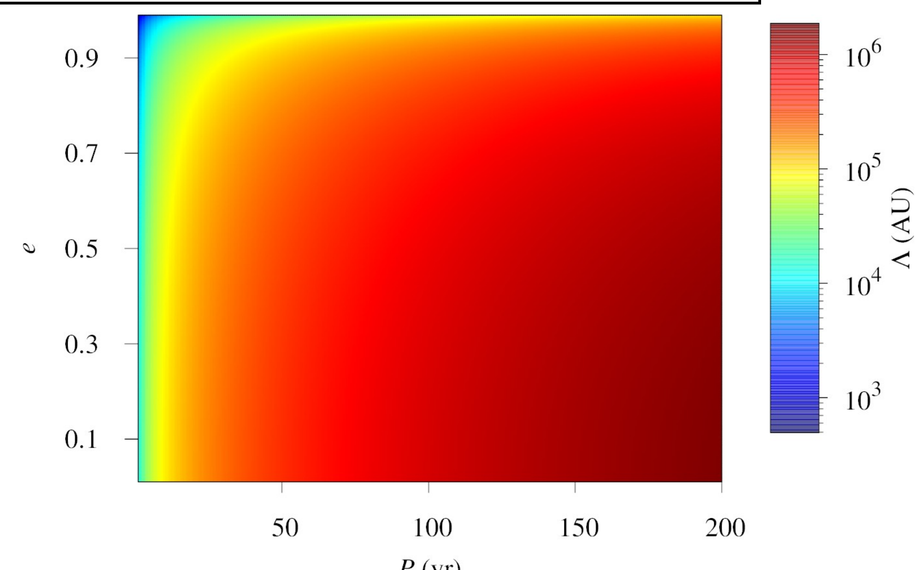
- Schwarzschild precession: $\Delta\varphi_{GR}^{rad} \approx \dfrac{6\pi GM}{c^2 a(1-e^2)}$
- Additional contribution of Yukawa gravity to the Schwarzschild precession (Zakharov, Jovanović, Borka & Borka Jovanović, 2016, JCAP, 045):

$$\Delta\varphi_{Y}^{rad} \approx \frac{\pi\delta\sqrt{1-e^2}}{1+\delta}\frac{a^2}{\Lambda^2}, \qquad a \ll \Lambda$$

- Observed precession detected by GRAVITY: $\Delta\varphi_{obs} \approx \dfrac{2\pi GM}{c^2 a(1-e^2)}(3f_{sp})$
- Constraints on the range of Yukawa interaction Λ from observed S-star orbits:

$$\Delta\varphi_Y + \Delta\varphi_{GR} \approx \Delta\varphi_{obs} \quad \Rightarrow \quad \boxed{\Lambda(P,e;\delta) \approx \frac{cP}{2\pi}\sqrt{\frac{\delta(\sqrt{1-e^2})^3}{2(3f_{sp}-3)(1+\delta)}}}$$





Stellar orbits around Sgr A* simulated using PPN EoM

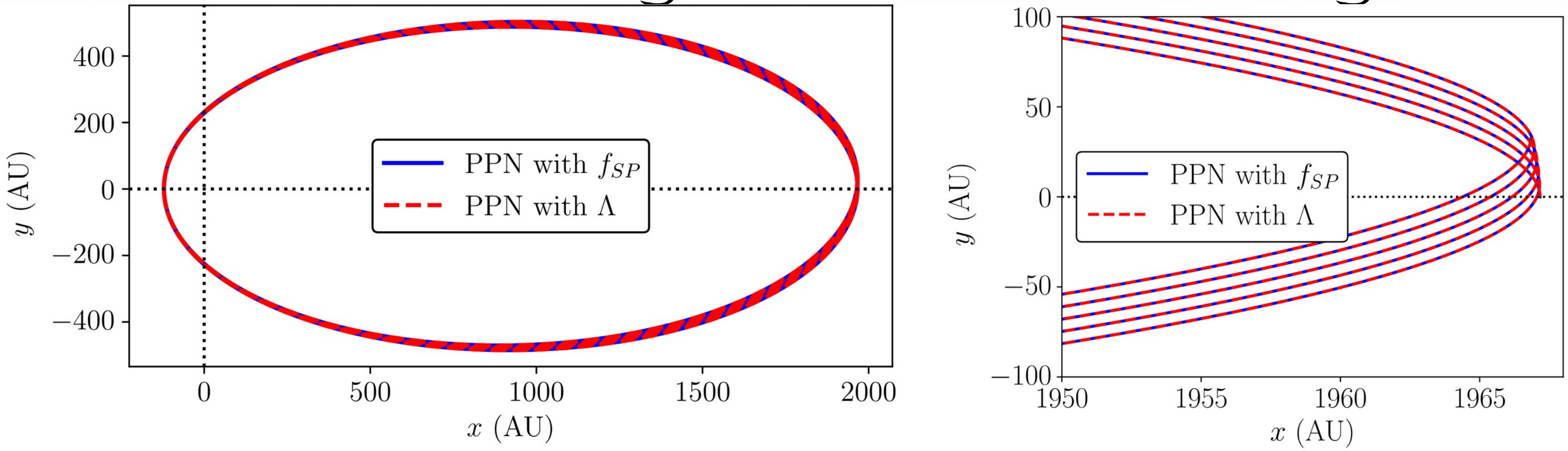


FIG. 1. Comparisons between the simulated orbits of S2 star, obtained by numerical integration of equation of motions in PPN formalism [2] for $f_{SP} = 1.10$ (blue solid line) and in PPN formalism [3] for $\Lambda = 46924.6$ AU (red dashed line). The orbits are calculated during five orbital periods, and their zoomed parts around the apocenter, where the largest discrepancy could occur, are presented in the right panel for better insight.

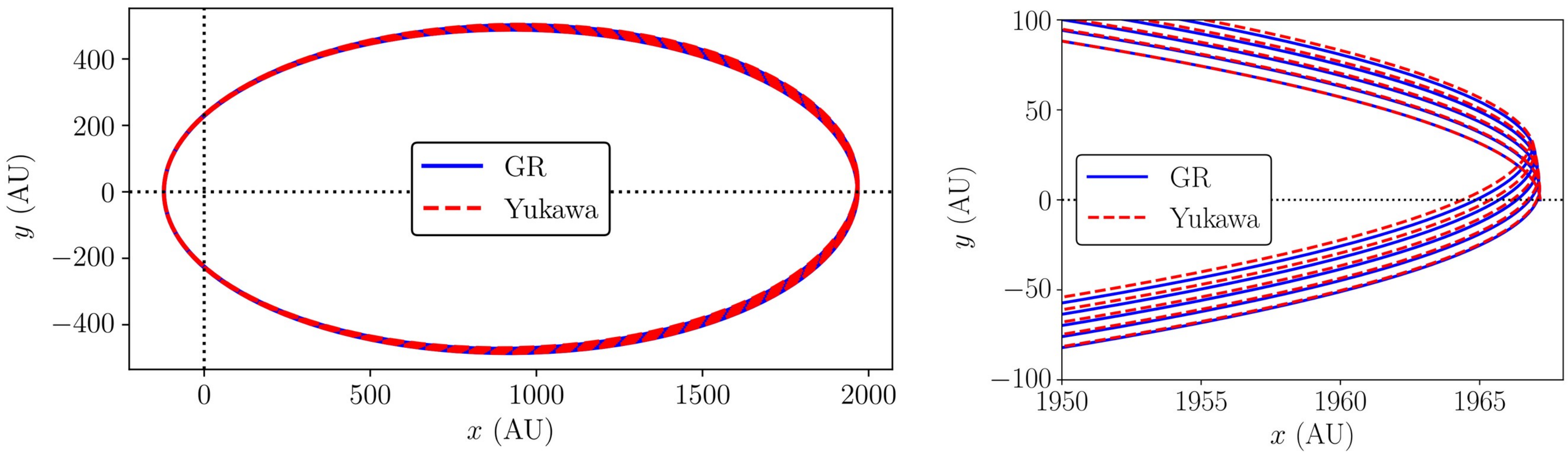


FIG. 2. Comparisons between the simulated orbits of S2 star, obtained by numerical integration of PPN equation in GR (blue solid line) and in Yukawa gravity for $\Lambda = 46924.6$ AU (red dashed line, which corresponds to $f_{SP} = 1.10$) using PPN formalism [3]. The orbits are calculated during five orbital periods, and their zoomed parts around the apocenter, where the largest discrepancy could occur, are presented in the right panel for better insight.

Constraints on Yukawa gravity parameters II

- For the given strength of Yukawa interaction δ and parameter f_{SP}, the following condition holds: $\Lambda \propto c\,P^*, \quad P^* = P(1-e^2)^{3/4}$
- P^* can be used as a criterion for classification of the gravitational systems according to their scales, as well as to find the systems which could be described by similar values of the range of Yukawa interaction Λ

- Influence of strength of Yukawa interaction δ on its range Λ in the case of S-stars (Jovanović, Borka Jovanović, Borka & Zakharov, 2023, JCAP, 056)

Star	$\Delta\varphi$ (")	P^* (yr)	$\Lambda \pm \Delta\Lambda$ (AU) $\delta = 0.01$	$\delta = 0.1$	$\delta = 1$
S1	48.2	125.79	164626.9 ± 13537.8	498844.4 ± 41021.7	1169893.8 ± 96204.5
S2	722.1	5.12	6730.8 ± 149.9	20395.2 ± 454.3	47831.0 ± 1065.4
S4	65.5	68.01	89185.0 ± 1750.2	270244.0 ± 5303.3	633778.4 ± 12437.2
S6	102.4	76.74	100839.1 ± 267.5	305557.6 ± 810.6	716596.2 ± 1901.1
S8	138.0	42.73	56050.5 ± 1717.2	169841.6 ± 5203.5	398314.0 ± 12203.2
S9	124.3	34.33	45030.4 ± 2503.1	136448.8 ± 7584.9	320000.8 ± 17788.2
S12	314.6	18.33	24050.7 ± 475.7	72877.1 ± 1441.4	170912.0 ± 3380.4
S13	91.6	42.20	55328.1 ± 601.8	167652.4 ± 1823.5	393179.8 ± 4276.6
S14	1465.9	5.60	7346.3 ± 981.2	22260.4 ± 2973.2	52205.2 ± 6972.8
S17	66.1	67.35	88370.7 ± 4262.7	267776.5 ± 12916.7	627991.6 ± 30292.4
S18	107.1	34.71	45506.5 ± 926.2	137891.7 ± 2806.5	323384.7 ± 6581.8
S19	87.1	72.62	95481.5 ± 36447.7	289323.6 ± 110442.1	678523.9 ± 259009.7
S21	217.4	19.18	25142.3 ± 1261.7	76185.0 ± 3823.2	178669.6 ± 8966.2
S22	19.0	456.10	599449.8 ± 236689.3	1816423.7 ± 717204.5	4259891.2 ± 1681993.5
S23	114.1	34.54	45425.0 ± 11014.4	137644.5 ± 33375.3	322805.0 ± 78272.1
S24	107.5	97.28	127590.2 ± 14036.6	386617.6 ± 42533.2	906698.7 ± 99749.1
S29	98.5	57.33	75236.9 ± 14099.5	227979.3 ± 42723.7	534658.8 ± 100196.0
S31	63.3	82.46	108733.3 ± 3953.7	329478.4 ± 11980.3	772695.3 ± 28096.4
S33	47.9	135.83	178323.3 ± 27097.8	540346.5 ± 82110.5	1267224.9 ± 192566.2
S38	427.5	8.31	10917.4 ± 51.8	33081.5 ± 157.1	77582.9 ± 368.4
S39	364.5	19.25	25286.5 ± 2038.3	76622.1 ± 6176.3	179694.7 ± 14484.7
S42	30.8	250.45	327668.0 ± 127216.9	992883.5 ± 385486.4	2328518.3 ± 904045.8
S54	81.5	144.02	187868.3 ± 301214.1	569269.5 ± 912724.3	1335055.3 ± 2140528.3
S55	382.8	7.38	9666.0 ± 302.1	29289.4 ± 915.3	68689.7 ± 2146.6
S60	105.5	50.59	66370.5 ± 2549.6	201112.8 ± 7725.8	471651.3 ± 18118.6
S66	13.4	655.82	860607.2 ± 88872.3	2607770.2 ± 269296.7	6115763.2 ± 631556.7
S67	19.3	402.94	528751.9 ± 32803.8	1602198.3 ± 99400.4	3757488.1 ± 233114.5
S71	106.2	100.28	131669.4 ± 20154.0	398978.3 ± 61069.7	935687.0 ± 143221.2
S83	15.3	589.30	773374.9 ± 184565.2	2343443.1 ± 559260.5	5495861.3 ± 1311582.1
S85	11.0	1772.21	2311796.7 ± 3523752.4	7005094.4 ± 10677503.5	16428402.6 ± 25040965.4
S87	7.6	1577.89	2065566.0 ± 200654.2	6258978.1 ± 608012.7	14678604.7 ± 1425916.1
S89	31.0	273.90	358908.7 ± 49485.2	1087547.9 ± 149947.8	2550525.9 ± 351658.7
S91	11.4	891.25	1168818.2 ± 101284.2	3541696.1 ± 306906.4	8306013.7 ± 719759.3
S96	13.6	646.91	848925.2 ± 53447.7	2572372.1 ± 161954.7	6032747.3 ± 379817.5
S97	9.7	1151.43	1516520.2 ± 550839.2	4595286.1 ± 1669126.3	10776901.1 ± 3914448.1
S145	23.6	343.33	452172.2 ± 222048.9	1370150.3 ± 672841.7	3213287.3 ± 1577953.6
S175	1812.0	6.30	8263.6 ± 2000.9	25040.0 ± 6062.9	58723.9 ± 14218.7
R34	18.6	589.73	775088.0 ± 220322.6	2348634.1 ± 667611.0	5508035.2 ± 1565686.6
R44	5.5	2579.33	3444472.6 ± 2260986.2	10437273.8 ± 6851130.7	24477576.8 ± 16067325.7

- Constrains on Yukawa gravity parameters, obtained from the orbits of all studied S-stars using Markov chain Monte Carlo (MCMC) simulations (bottom right panel in the figure):

$$\delta = 0.016^{+0.0065}_{-0.0049}$$

$$\Lambda = 11000^{+2000}_{-1800}\ \text{AU}$$

Jovanović, Borka Jovanović, Borka & Zakharov, 2023, JCAP, 056

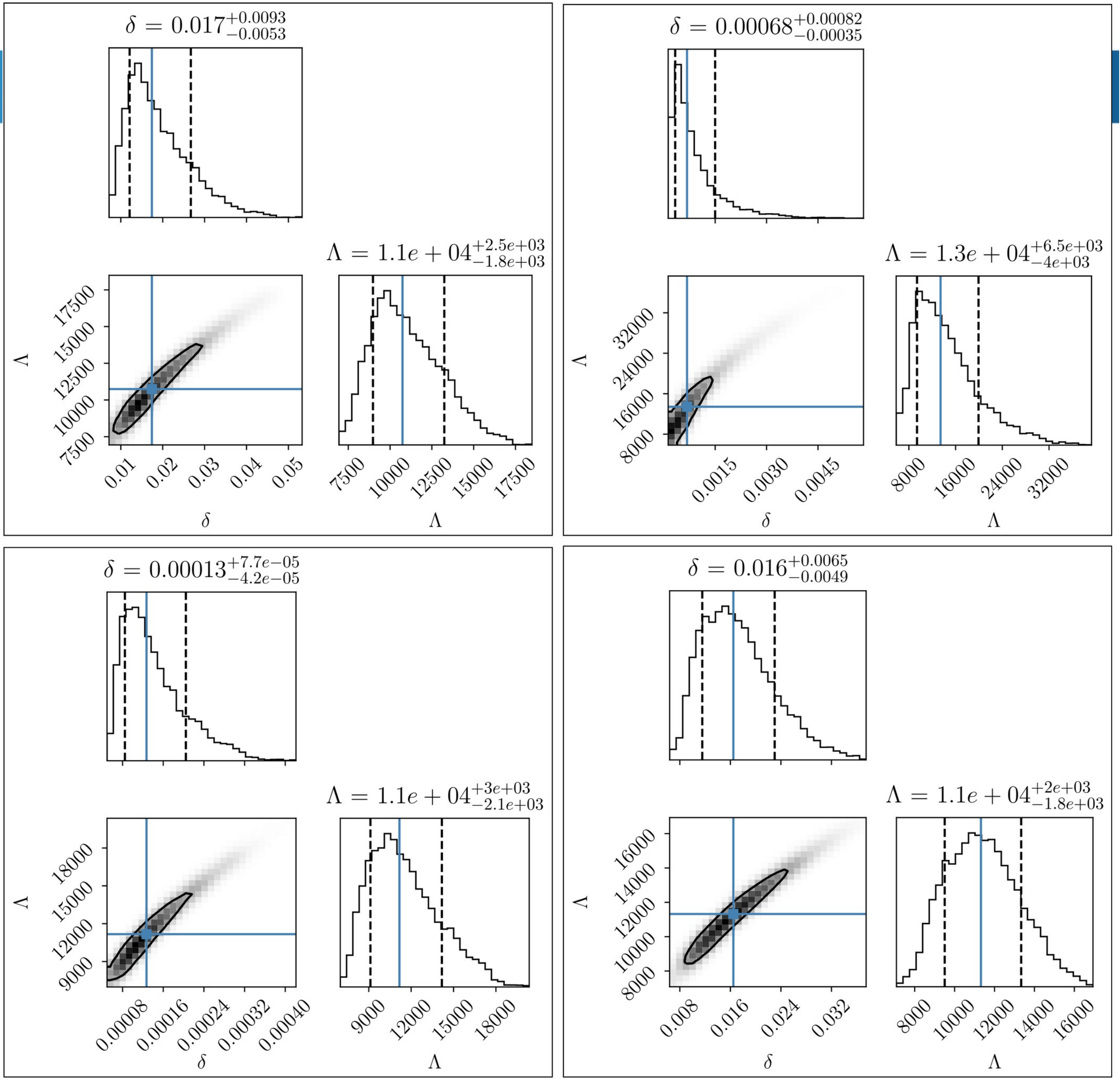


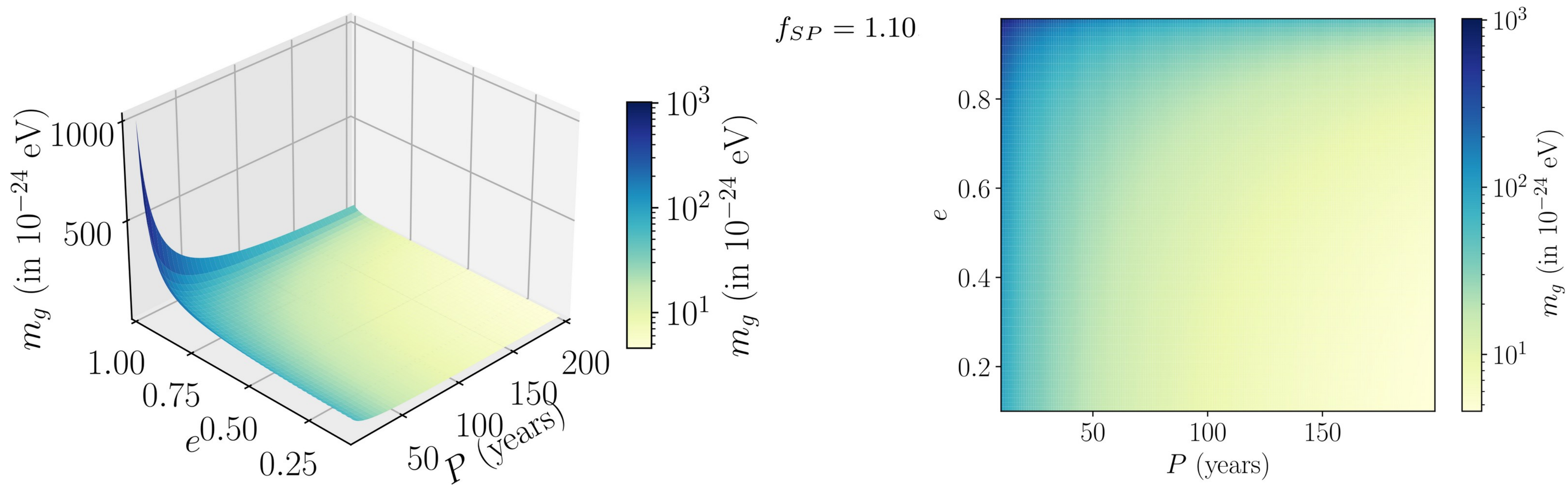
Figure 5. The posterior probability distributions and 68% confidence levels (closed contours) for the Yukawa gravity parameters in the case of S-stars, obtained by MCMC simulations. Top left panel corresponds to the following S-stars: S2, S12, S14, S21, S38, S39, S55 and S175, for which $5\,\text{yr} < P^* < 25\,\text{yr}$ (see table 1), top right panel corresponds to S8, S9, S13, S18 and S23 star for which $25\,\text{yr} < P^* < 50\,\text{yr}$, bottom left panel corresponds to S4, S6, S17, S19, S24, S29, S31 and S60 star for which $50\,\text{yr} < P^* < 100\,\text{yr}$, while bottom right panel correspond to all S-stars from table 1.

Bounds on graviton mass I

- A gravitational potential which does not include δ is usually used in the frame of a massive graviton theory and thus, in order to study the constraints on the graviton mass m_g, we assumed that $\delta = 1$ in Yukawa gravity potential
- We also assumed that the range of Yukawa interaction Λ corresponds to the graviton Compton wavelength λ_g:

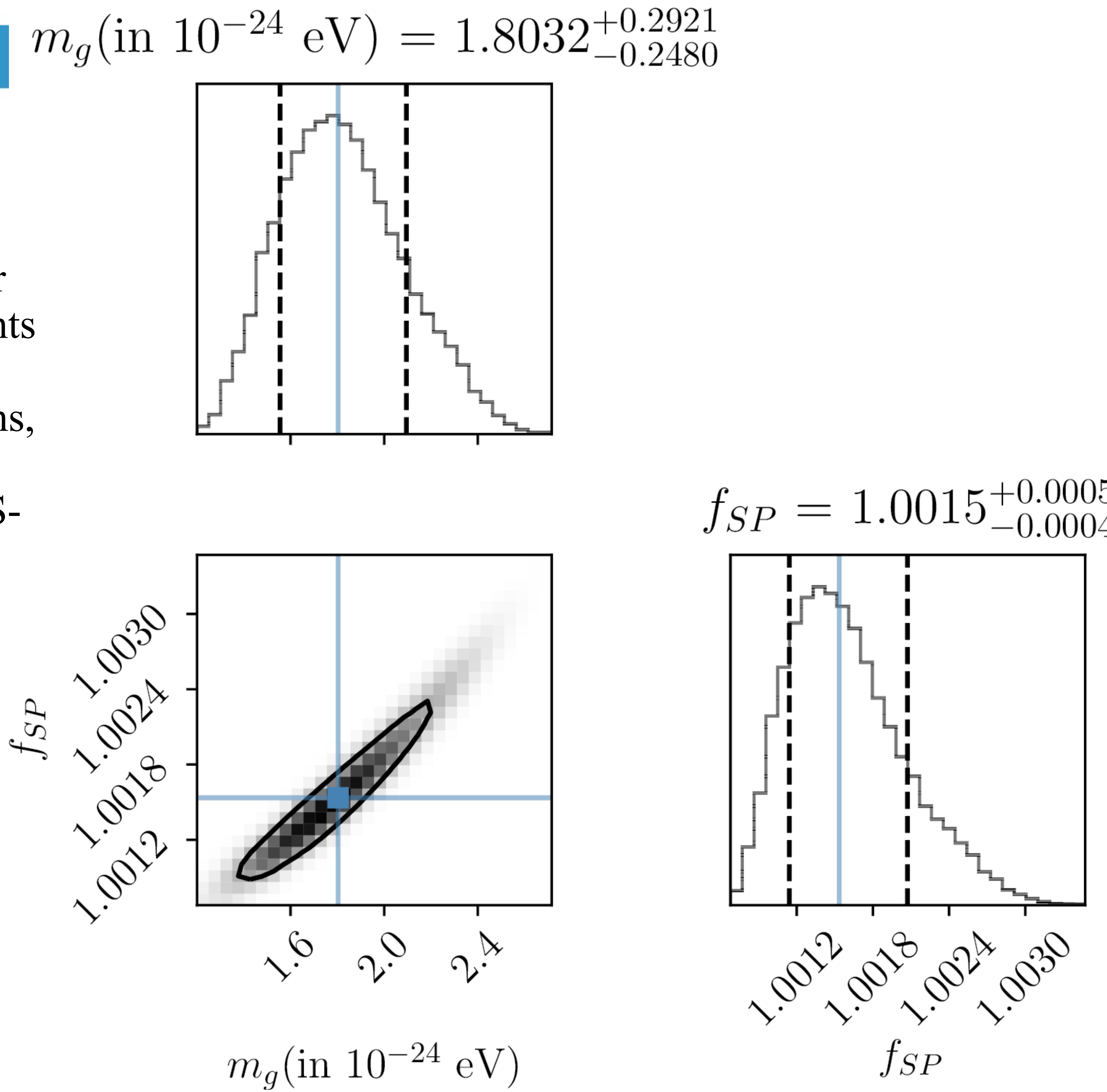
$$\lambda_g = \frac{hc}{m_g} \quad \Rightarrow \quad \boxed{m_g(P, e; f_{SP}) \approx \frac{4\pi h}{P}\sqrt{\frac{3(f_{SP}-1)}{(\sqrt{1-e^2})^3}}}$$

- Constraints on m_g from the observed S-star orbits:



Jovanović, Borka Jovanović, Borka & Zakharov, 2023, arXiv:2305.13448v3

- 68% confidence regions for graviton mass m_g and f_{SP}, as well as prediction for possible improvements by the future high precision observations, obtained from the orbits of all studied S-stars using MCMC simulations



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Bounds on graviton mass II

Star	$f_{SP} = 1.10 \pm 0.19$ $\Lambda \pm \Delta\Lambda$ (AU)	$m_g \pm \Delta m_g$ (10^{-24} eV)	R.E. (%)
S1	1.2e+06 ± 1.2e+06	7.2 ± 7.2	100.7
S2	4.7e+04 ± 4.5e+04	176.6 ± 170.0	96.3
S4	6.2e+05 ± 6.0e+05	13.3 ± 12.9	96.7
S6	7.0e+05 ± 6.7e+05	11.8 ± 11.2	95.2
S8	3.9e+05 ± 3.8e+05	21.2 ± 20.7	98.0
S9	3.1e+05 ± 3.1e+05	26.3 ± 26.2	99.7
S12	1.7e+05 ± 1.6e+05	49.3 ± 47.6	96.4
S13	3.9e+05 ± 3.7e+05	21.4 ± 20.4	95.5
S14	5.1e+04 ± 5.5e+04	161.3 ± 173.2	107.3
S17	6.2e+05 ± 6.0e+05	13.4 ± 13.0	97.1
S18	3.2e+05 ± 3.1e+05	26.0 ± 25.1	96.5
S19	6.7e+05 ± 7.8e+05	12.4 ± 14.5	116.4
S21	1.8e+05 ± 1.8e+05	47.1 ± 46.9	99.6
S22	4.2e+06 ± 4.8e+06	2.0 ± 2.3	114.1
S23	3.2e+05 ± 3.7e+05	26.2 ± 30.3	115.6
S24	8.9e+05 ± 9.2e+05	9.3 ± 9.6	103.2
S29	5.3e+05 ± 5.7e+05	15.8 ± 17.2	109.1
S31	7.6e+05 ± 7.3e+05	11.0 ± 10.6	96.4
S33	1.2e+06 ± 1.3e+06	6.7 ± 7.1	107.0
S38	7.6e+04 ± 7.3e+04	108.8 ± 103.7	95.4
S39	1.8e+05 ± 1.7e+05	47.0 ± 46.4	98.8
S42	2.3e+06 ± 2.8e+06	3.6 ± 4.4	122.7
S54	1.3e+06 ± 2.5e+06	6.3 ± 11.8	188.3
S55	6.8e+04 ± 6.6e+04	122.4 ± 119.4	97.6
S60	4.6e+05 ± 4.5e+05	17.9 ± 17.5	97.7
S66	6.0e+06 ± 6.1e+06	1.4 ± 1.4	101.4
S67	3.7e+06 ± 3.7e+06	2.2 ± 2.2	100.1
S71	9.2e+05 ± 9.9e+05	9.0 ± 9.7	107.3
S83	5.4e+06 ± 6.0e+06	1.5 ± 1.7	110.3
S85	1.6e+07 ± 3.4e+07	0.5 ± 1.1	211.0
S87	1.4e+07 ± 1.5e+07	0.6 ± 0.6	102.4
S89	2.5e+06 ± 2.7e+06	3.3 ± 3.6	107.8
S91	8.2e+06 ± 8.3e+06	1.0 ± 1.0	101.9
S96	5.9e+06 ± 5.9e+06	1.4 ± 1.4	100.0
S97	1.1e+07 ± 1.3e+07	0.8 ± 1.0	125.9
S145	3.1e+06 ± 4.3e+06	2.6 ± 3.6	136.7
S175	5.8e+04 ± 6.4e+04	143.4 ± 158.1	110.3
R34	5.4e+06 ± 6.5e+06	1.5 ± 1.8	120.5
R44	2.4e+07 ± 3.7e+07	0.4 ± 0.5	156.2

TABLE III. The same as in Table I but for $f_{SP} = 1.0015 \pm 0.0005$ (this value is obtained using MCMC analysis).

Star name	$\Lambda \pm \Delta\Lambda$ (AU)	$m_g \pm \Delta m_g$ (10^{-24} eV)	R.E. (%)
S1	9.4e+06 ± 2.1e+06	0.9 ± 0.2	22.3
S2	3.8e+05 ± 6.9e+04	21.6 ± 3.9	17.9
S4	5.1e+06 ± 9.4e+05	1.6 ± 0.3	18.4
S6	5.7e+06 ± 9.7e+05	1.4 ± 0.2	16.9
S8	3.2e+06 ± 6.3e+05	2.6 ± 0.5	19.7
S9	2.6e+06 ± 5.5e+05	3.2 ± 0.7	21.3
S12	1.4e+06 ± 2.5e+05	6.0 ± 1.1	18.1
S13	3.2e+06 ± 5.4e+05	2.6 ± 0.4	17.1
S14	4.2e+05 ± 1.2e+05	19.8 ± 5.7	29.0
S17	5.0e+06 ± 9.5e+05	1.6 ± 0.3	18.7
S18	2.6e+06 ± 4.7e+05	3.2 ± 0.6	18.2
S19	5.4e+06 ± 2.1e+06	1.5 ± 0.6	38.1
S21	1.4e+06 ± 3.1e+05	5.8 ± 1.2	21.3
S22	3.4e+07 ± 1.2e+07	0.2 ± 0.1	35.8
S23	2.6e+06 ± 9.6e+05	3.2 ± 1.2	37.3
S24	7.3e+06 ± 1.8e+06	1.1 ± 0.3	24.9
S29	4.3e+06 ± 1.3e+06	1.9 ± 0.6	30.7
S31	6.2e+06 ± 1.1e+06	1.3 ± 0.2	18.1
S33	1.0e+07 ± 2.9e+06	0.8 ± 0.2	28.6
S38	6.2e+05 ± 1.1e+05	13.3 ± 2.3	17.0
S39	1.4e+06 ± 3.0e+05	5.8 ± 1.2	20.5
S42	1.9e+07 ± 8.3e+06	0.4 ± 0.2	44.4
S54	1.1e+07 ± 1.2e+07	0.8 ± 0.8	110.0
S55	5.5e+05 ± 1.1e+05	15.0 ± 2.9	19.3
S60	3.8e+06 ± 7.4e+05	2.2 ± 0.4	19.4
S66	4.9e+07 ± 1.1e+07	0.2 ± 0.0	23.1
S67	3.0e+07 ± 6.6e+06	0.3 ± 0.1	21.7
S71	7.5e+06 ± 2.2e+06	1.1 ± 0.3	29.0
S83	4.4e+07 ± 1.4e+07	0.2 ± 0.1	31.9
S85	1.3e+08 ± 1.8e+08	0.1 ± 0.1	132.7
S87	1.2e+08 ± 2.8e+07	0.1 ± 0.0	24.0
S89	2.1e+07 ± 6.0e+06	0.4 ± 0.1	29.5
S91	6.7e+07 ± 1.6e+07	0.1 ± 0.0	23.6
S96	4.8e+07 ± 1.0e+07	0.2 ± 0.0	21.6
S97	8.6e+07 ± 4.1e+07	0.1 ± 0.0	47.6
S145	2.6e+07 ± 1.5e+07	0.3 ± 0.2	58.3
S175	4.7e+05 ± 1.5e+05	17.6 ± 5.6	31.9
R34	4.4e+07 ± 1.9e+07	0.2 ± 0.1	42.1
R44	1.9e+08 ± 1.5e+08	0.0 ± 0.0	77.9

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Conclusions

- We obtained new and improved constraints on Yukawa gravity parameters δ and Λ, as well as on mass of graviton m_g from the observed orbits of the S-stars around Sgr A*, assuming that orbital precession in Yukawa gravity has a small deviation from GR prediction, as recently indicated by GRAVITY Collaboration
- For that purpose we assumed modified and extended PPN formalisms and used the factor f_{SP}, which was recently introduced and measured by the GRAVITY Collaboration and which parametrizes the effect of the Schwarzschild metric
- We derived the relations describing the dependence of Λ and m_g on f_{SP} and used them to improve the corresponding constraints from the measured value of f_{SP}
- Our MCMC simulations resulted with the following best-fit values and uncertainties of Yukawa gravity parameters in the case of the studied observed sample of S-stars: $\delta = 0.016^{+0.0065}_{-0.0049}$ and $\Lambda = 11000^{+2000}_{-1800}$ AU
- According to the current GRAVITY estimate of $f_{SP} = 1.10 \pm 0.19$ our previous constraints on the upper bound of m_g can be improved for about 3 times, but with unrealistically high contribution of 95% to the relative error
- Our MCMC simulations also showed that if the future high precision observations will confirm the GR prediction for Schwarzschild precession with much higher accuracy, so that f_{SP} will be in the range 1.0011-1.0021, they could improve the previous constraints on the upper bound of m_g for about 25 times, reaching: $m_g < 1.8 \times 10^{-24}$ eV

Thank you for your attention!