

Nonlinear analysis of the emission of close binary supermassive black hole candidates

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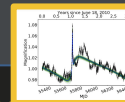
University of Belgrade-Faculty of Mathematics

Time domain surveys

ID	RA	Dec	z	Mass	Period	$\log(L_{\text{bol}})$	σ	Year	Size
	deg	deg		M_{\odot}	yr		%		arcsec
19M201	08 03 00.7	-02 23 07.6	1.996	37.26	1806	4.157	80.53	1.2 × 10 ⁴	3.0
19M224	08 23 02.2	-05 15 33.9	0.729	37.82	3184	4.118	8.107	8.900	1.8 × 10 ⁴
19M219	08 03 00.7	-02 23 07.6	1.996	37.26	1806	4.157	8.107	8.900	1.8 × 10 ⁴

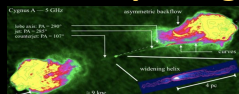
Graham+15,
Liu+19,
Guo+19

Periodic emission,
lensing



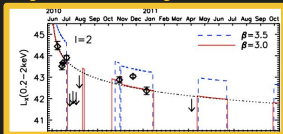
Graham+15,
Hu+19

Jet morphology



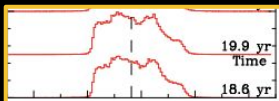
Gower +82,
Krause +18

tidal disruptions by a binary

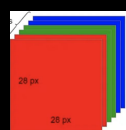
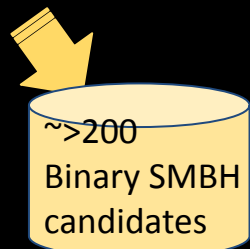


Liu, Li, Komossa+14
Huang+21

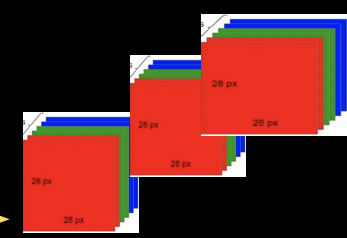
Emission-line dynamics



Bogdanović +09,
Shen & Loeb +10;
Popovic+21



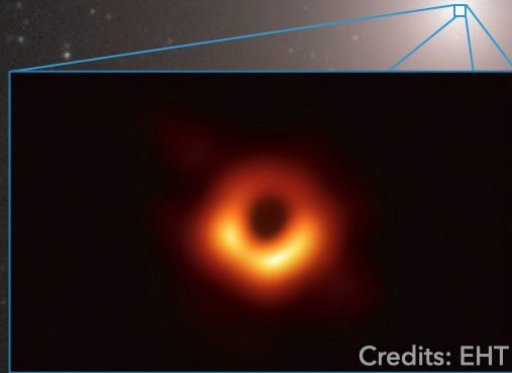
2D /3D tensors



4D tensors

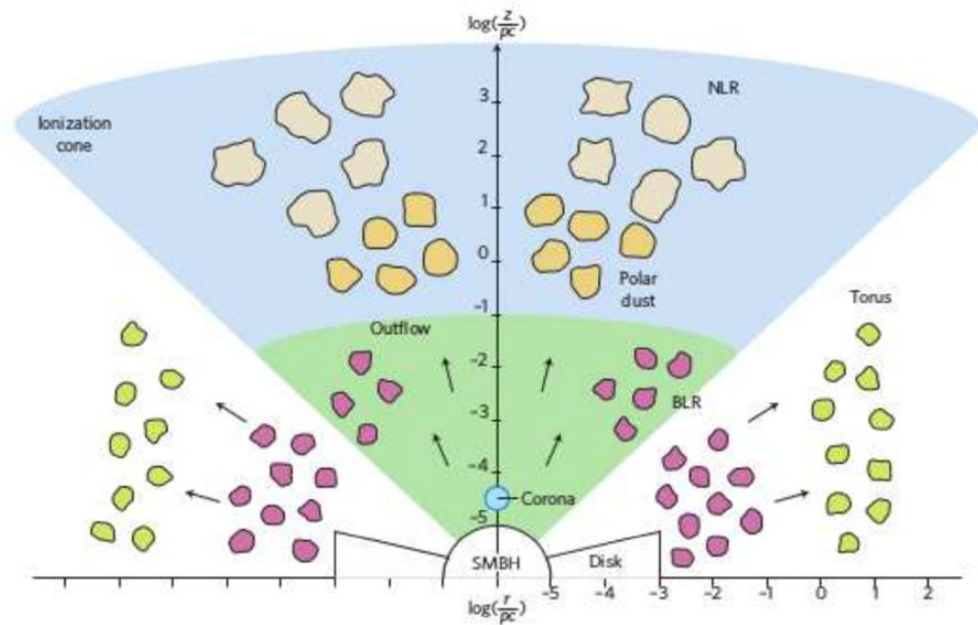
Active Galactic Nuclei (AGN)

AGNs are powered by the release of gravitational energy related with the accretion of material onto a supermassive black hole (SMBH), with masses larger than $10^6 M_{\odot}$



Active Galactic Nuclei (AGN) - *ID*

card



Ramos Almeida & Ricci (2017)

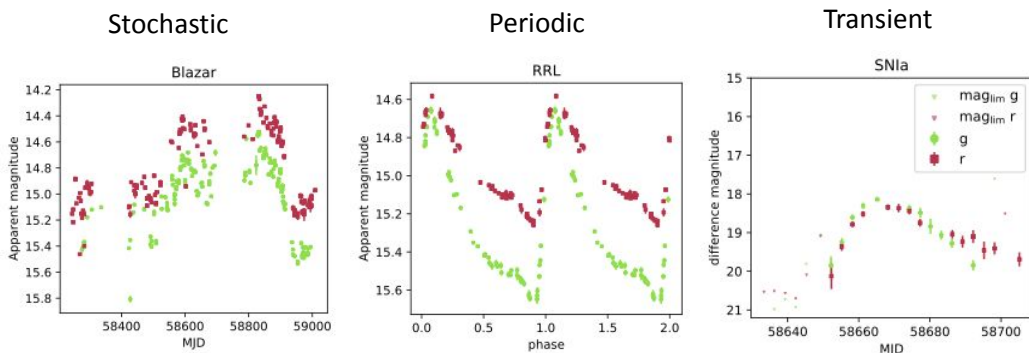
- **Black Hole Mass** $10^6 \lesssim M_{\text{BH}} \lesssim 10^9 (M_{\odot})$
- **Luminosity** $10^{12} \lesssim L_{\text{AGN}} \lesssim 10^{15} (L_{\odot})$
- **Spin** $-1 \lesssim a_* \lesssim 1$
- **Accretion rate** $0.01 \lesssim \dot{M} \lesssim 10 (M_{\odot}/\text{yr})$
- **Eddington ratio** $0.01 \lesssim \frac{L}{L_{\text{Edd}}} \lesssim 1$

$$\frac{\dot{L}}{L_{\text{Edd}}} \propto \frac{L_{\text{AGN}}}{M_{\text{BH}}}$$

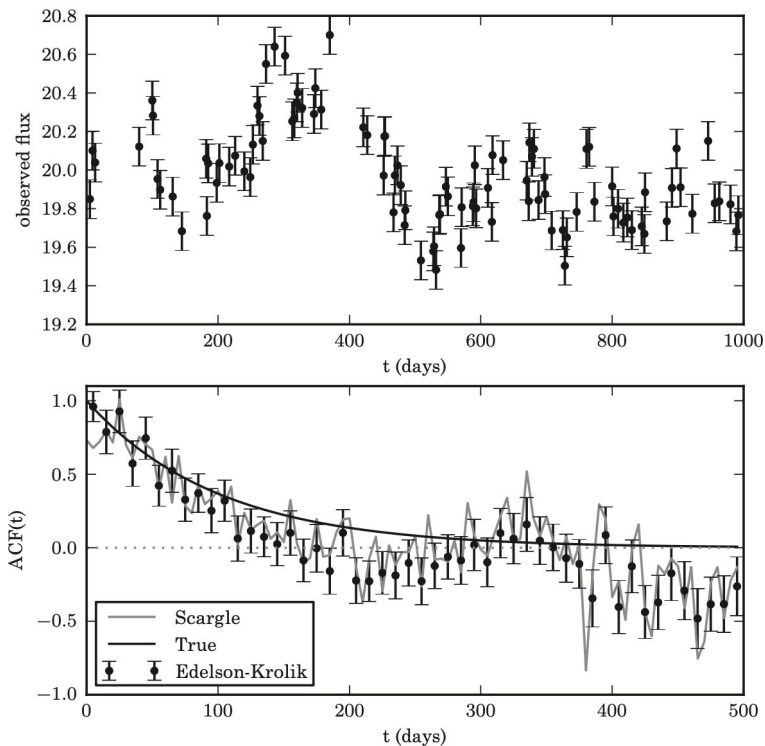
How can we reveal the inner structure?

AGN variability: what do we know?

AGN variability seems to be well described as a **stochastic process**: behavior that is not predictable forever as in the periodic case, but unlike temporally localized events (transient), variability is always there

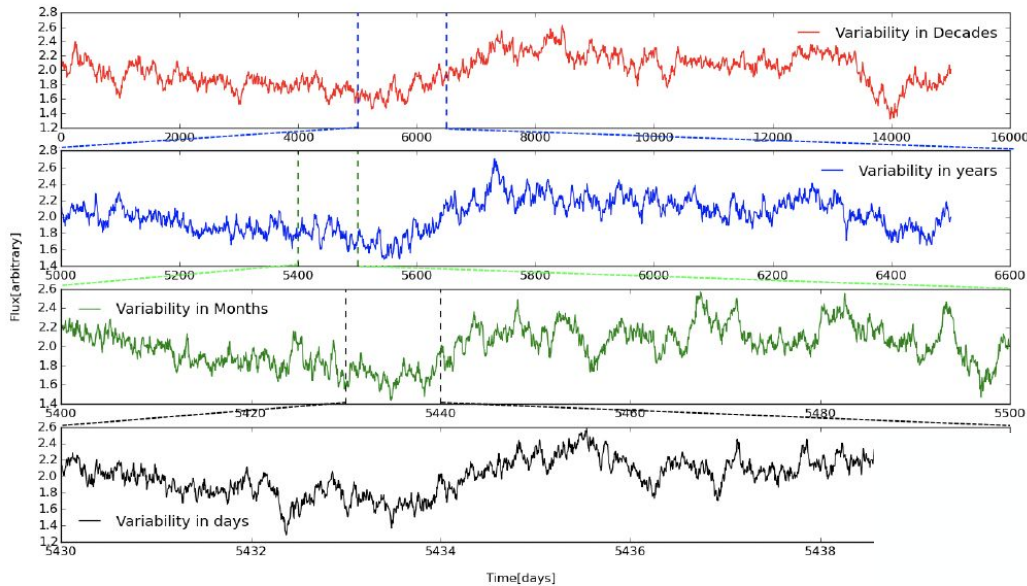


credits: Sánchez-Sáez et al. 2021a.



for more details see [Kozłowski et al. 2016](#)

fractals in time



**Astronomical
red noise**

1/f~fractals in time

Table 4.1 Nomenclature of noise spectra.

Power spectrum	Power index	Spectrum nomenclature
$P(v) \propto v^0$	$p = 0$	white noise
$P(v) \propto v^{-1}$	$p = 1$	pink noise, flicker noise, 1/f noise
$P(v) \propto v^{-2}$	$p = 2$	red noise, Brown(ian) noise
$P(v) \propto v^{-3}$	$p = 3$	black noise



Astronomical tomography

Czerny & Hryniewicz 2011
Dusty Disk Wind Model

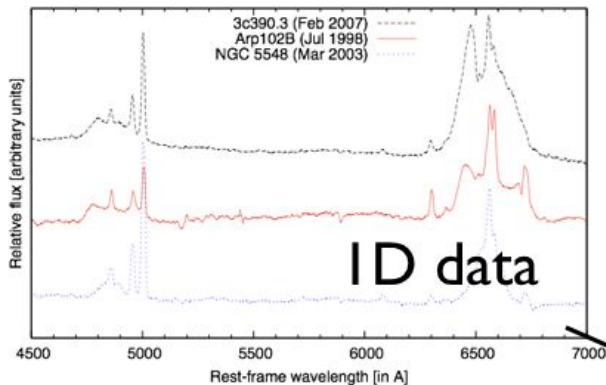
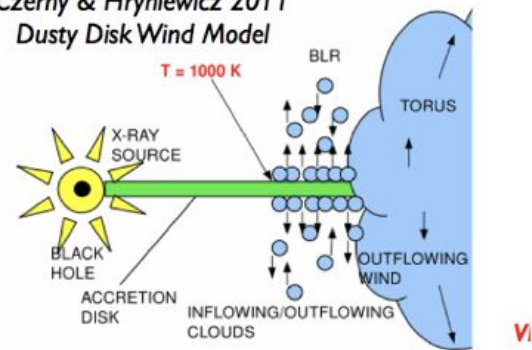


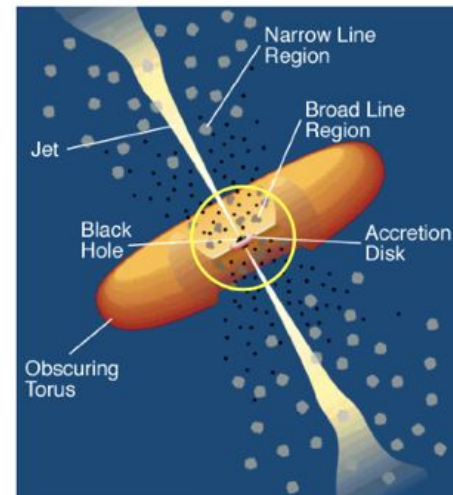
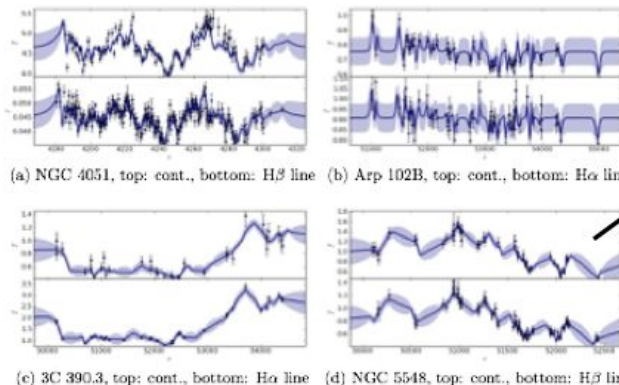
Figure 1: Examples of spectra of objects from our monitoring campaign: 3C 390.3, Arp 102B, and NGC 5548. The dates of observations are given in brackets.

3D
model

BH hrs Disk days BLR weeks Torus months

- **time-delay** between continuum and line flux
- time-delay = size of the broad line region (BLR)

ID data

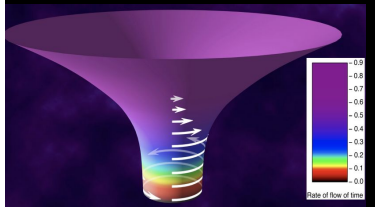
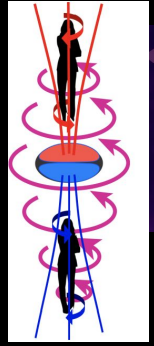


Credit: C.M. Urry and P. Padovani

Tornados of whirling space

fast-spinning BH = 2 vortices

Vortex 1

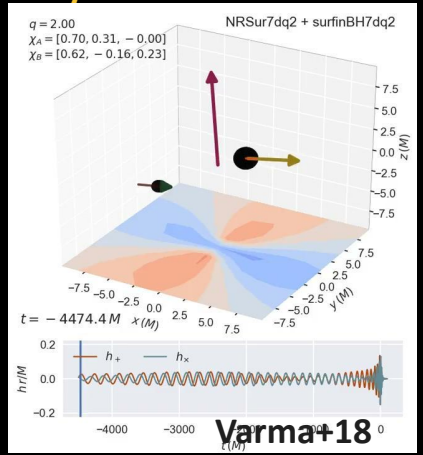


Horizon

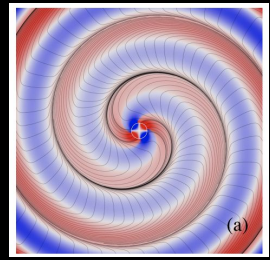
Vortex 2

Thorne+20

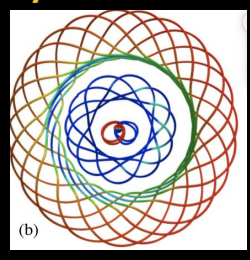
binary BH = 4 vortices



GW from binary SMBH



inspiral



head on collision

Perfect storm: Fujiwhara effect

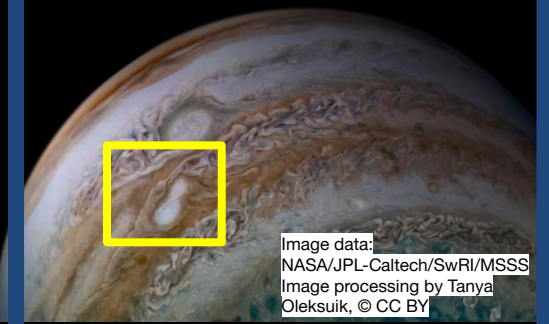


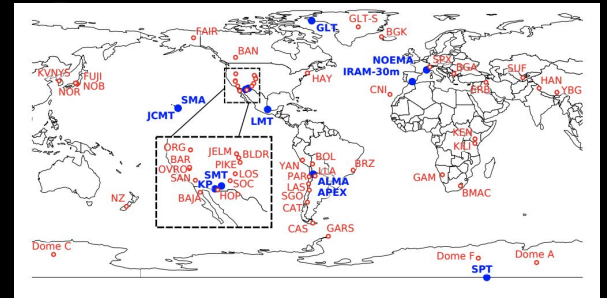
Image data:
NASA/JPL-Caltech/SwRI/MSSS
Image processing by Tanya
Oleksuik, © CC BY



ASTRONOMY MOVIE OF XXI CENTURY: ngEHT mid-2020 production

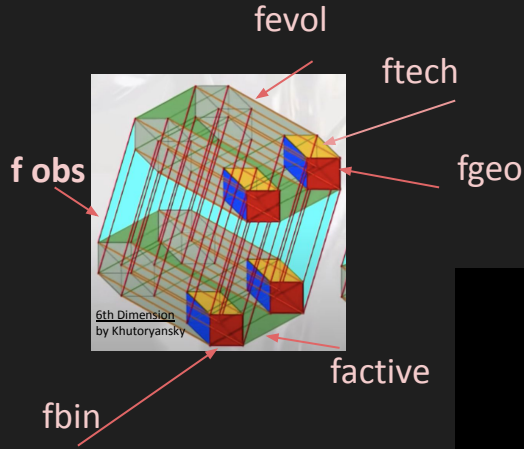


John Rowe Animation/CSIRO Astronomy and Space Science



ngEHT potential sites, Raymond+21

“WHERE ARE THEY ?” NAVIGATING PARAMETER HYPERSPACE OF BINARY SMBH



$$P_{\text{obs}} = f_{\text{bin}} \times f_{\text{active}} \times f_{\text{geo}} \times f_{\text{tech}} \times f_{\text{evo}}$$

Shen&Loeb+10
D’Orazio&Loeb+18



$$P_{\text{obs}}(M_{\text{BH}}, q, \Delta t, \dot{m}_{\text{Edd}}) = \int_{R_{\text{BLR}}}^{0.1 \text{ pc}} P_{\text{obs}}(r, M_{\text{BH}}, q, \Delta t) \frac{t_{\text{res}}(r, M_{\text{BH}}, \dot{m}_{\text{Edd}})}{r} dr \quad \text{Jun+13}$$

$$R_{\text{BLR}} \approx 2.2 \times 10^{-2} \left(\frac{\dot{m}_{\text{Edd}}}{0.1} \right)^{1/2} \left(\frac{M_{\text{BH}}}{10^8 M_{\odot}} \right)^{1/2} \text{ pc.}$$

$$N_{\text{LISA}} = [t_{\text{m}} / t_{\text{Q}}] f_{\text{Q,b}} \mathcal{N} \quad \text{Xin \& Haiman +21}$$

$$\mathcal{N} = \int_{m_{\text{min}}}^{m_{\text{max}}} dm \int_{z_{\text{min}}}^{z_{\text{max}}} dz f(m, z) \phi(M_{1450}(m), z) \frac{dV}{dz},$$

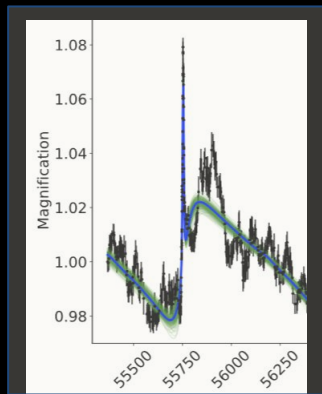
QLF Richards+06
Hopkins+07
Shen+20

Ordered Information: Non-Face-on Binaries

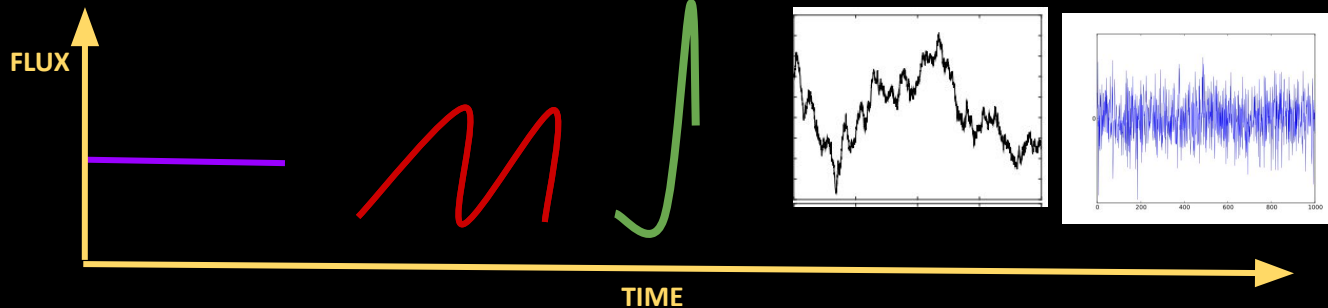
Observed Flux

~

Steady mean flux \times Doppler boost \times Gravitational microlensing \times Red noise + Photometric noise



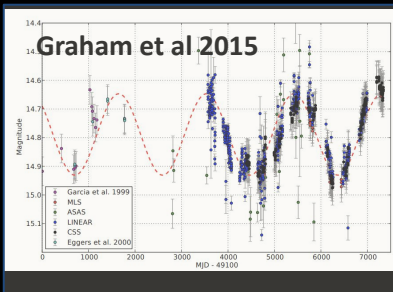
Optical Hu+20



Observed Flux

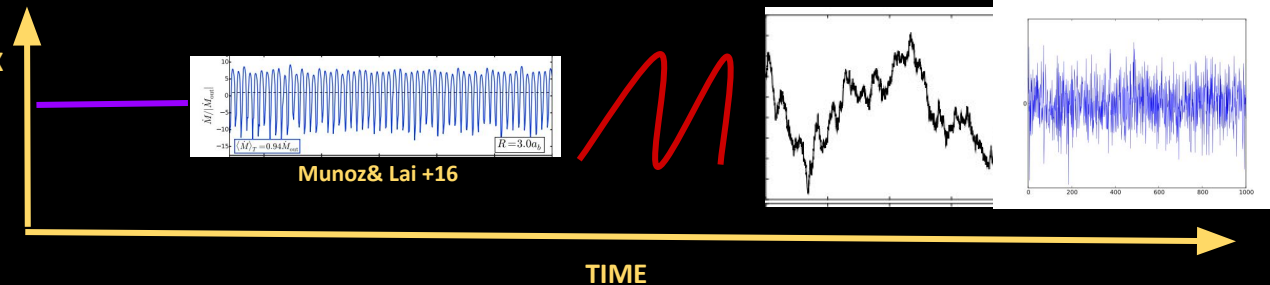
~

Steady mean flux \times (periodic accretion) \times Doppler boost \times Red noise + Photometric noise



FLUX

~



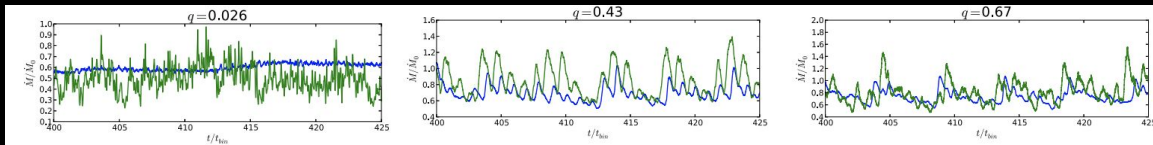
Munoz& Lai +16

Red Noise: Mimicking ordered information

Burst model
(Farris+14)

Periodicity signal immersed in red noise

Red Noise mimics periodicity (Vaughan+16)



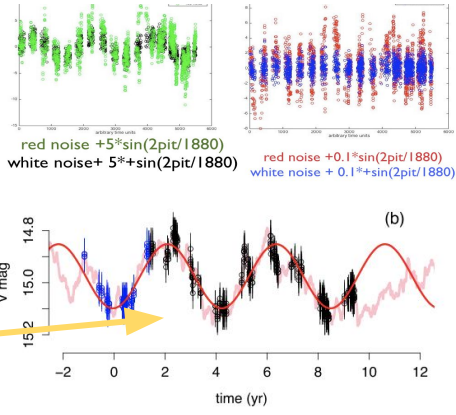
The power spectral density (PSD) for the DRW is (Kelly+09)

$$dX(t) = -\frac{1}{\tau}X(t)dt + \sigma\sqrt{dt}\epsilon(t) + b dt, \quad \tau, \sigma, t > 0. \quad (1)$$

$$\text{PSD}(f) = \frac{\tau^2 SF_\infty^2}{1 + (2\pi f\tau)^2}$$

$\text{PSD} \propto f^{-2}$, for $f > 1/(2\pi\tau)$ red noise

$\text{PSD} \propto \text{const}$ (i.e. $\text{const} \times SF_\infty^2$), for $f < 1/(2\pi\tau)$ white noise



light crossing
time scale

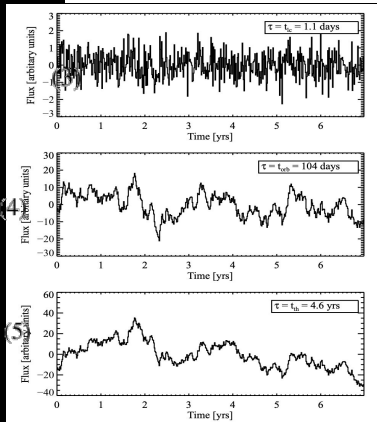
$$t_{lc} = 1.1 \times \left(\frac{M_{BH}}{10^8 M_\odot} \right) \left(\frac{R}{100 R_S} \right) \text{ days,}$$

the gas orbital
timescale

$$t_{orb} = 104 \times \left(\frac{M_{BH}}{10^8 M_\odot} \right) \left(\frac{R}{100 R_S} \right)^{3/2} \text{ days,}$$

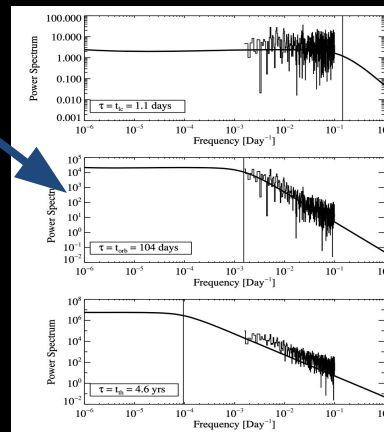
accretion disk
thermal timescale

$$t_{th} = 4.6 \times \left(\frac{\alpha}{0.01} \right)^{-1} \left(\frac{M_{BH}}{10^8 M_\odot} \right) \left(\frac{R}{100 R_S} \right)^{3/2} \text{ yr,}$$



Fourier transform

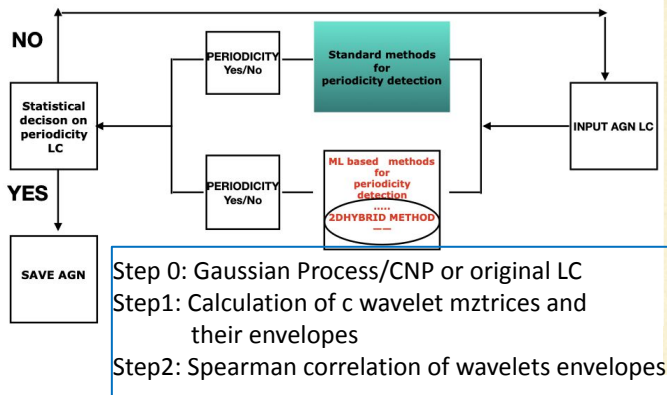
Kelly+09



Under the hood period detection unit: 2D Hybrid method

Under the hood periodicity detection-Sci case (data) supplied by CPG Inkind Team

- preprocessing with DGP, ANN+DGP
- period detection with WWZmatrix coefficients (inhomogenous cadence) + error and significance estimate
- period detection with SUPERLET matrix coefficients (homogenous cadenced LC) + error and significance estimate



HYBRID METHOD: 3 STEPS

OUGP=Ornstein-Uhlenbeck Gaussian Process
 OUGP(raw LC1)=OUGP1
 OUGP(raw LC2)=OUGP2

0 STEP: raw data preprocessing

$$CWT(a, b) = \frac{1}{\sqrt{2\pi a}} \int_{-\infty}^{+\infty} x(t) e^{i(t/b)^2/a/2} e^{i\omega(t-b)/a} dt$$

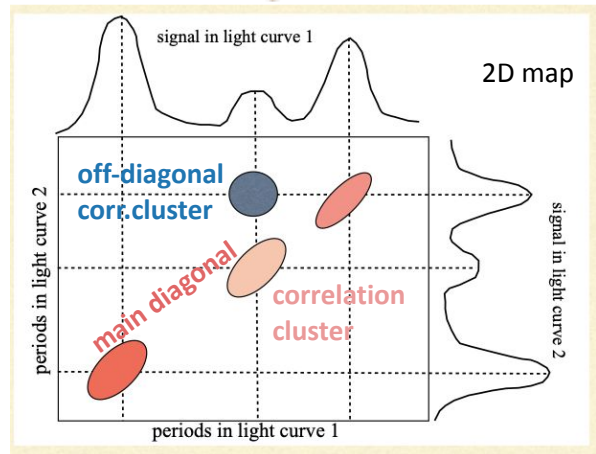
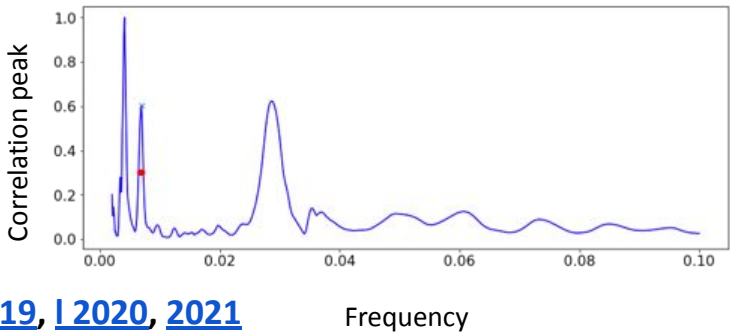
$$env(a, b) = \sqrt{\text{Re}\{[CWT(a, b)]^2\} + \text{Im}\{[CWT(a, b)]^2\}}$$

1 STEP

SpearmanCorrCoeff(env(OUGP1), env(OUGP2))

2 STEP

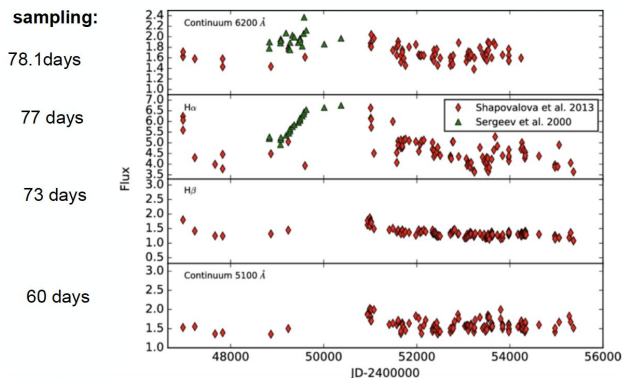
map integration along any period axis



EXAMPLES OF DATA

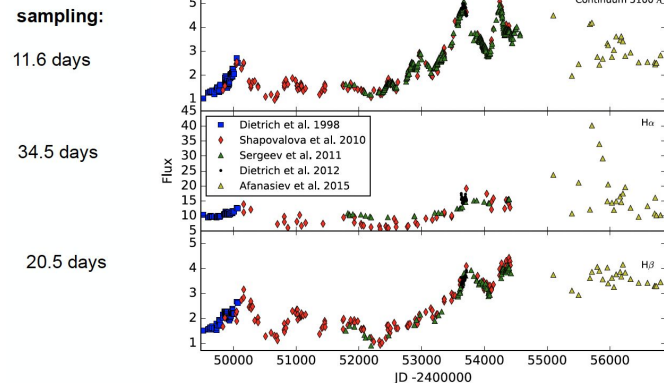
ARP 102 B

observed:
1987–2010



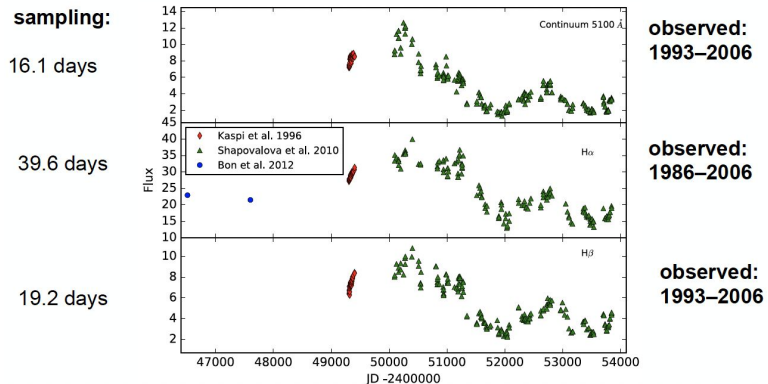
3C 390.3

observed:
1994–2014



Object name	Type	z	Period	CLC	Sampling (days)	EV	Reference ^a	
3C 390.3	BLRG	0.056	1994-2014	Continuum 5100 Å	11.6	0.1623	1, 2, 3, 4, 5	
				H α	34.5	0.1055		
				H β	20.5	0.1099		
				Continuum 1370 Å	64.4	0.1737		6, 7
				L $\gamma\alpha$	64.4	0.2539		
				CIV	64.4	0.2167		
Arp 102b	LINER	0.024	1987-2010	Continuum 6200 Å	78.1	0.0080	8, 9	
				H α	77.0	0.0245		
				Continuum 5100 Å	73.0	0.0073		8
NGC 4151	Seyfert I	0.003	1993-2006	H β	60.0	0.0090	8	
				Continuum 5100 Å	16.1	0.2847	10, 11, 12	
				H α	39.6	0.0740		
				H β	19.2	0.1367		
NGC 5548	Seyfert I	0.017	1972-2015	Continuum 5100 Å	6.9	0.0648	13	
				H β	11.2	0.0917		
E1821+643	Quasar	0.297	1990-2014	Continuum 5100 Å	68.4	0.0357	14	
				H β	68.4	0.0049		
				Continuum 4200 Å	114.9	0.0359		
				H γ	114.9	0.0356		

NGC 4151



^a (1) Dietrich et al. (1998), (2) Shapovalova et al. (2010a), (3) Dietrich et al. (2012), (4) Sergeev et al. (2011), (5) Afanasiev et al. (2015), (6) Wamstecker et al. (1997), (7) O' Brien et al. (1998), (8) Shapovalova et al. (2013), (9) Sergeev et al. (2000), (10) Kaspi et al. (1996), (11) Shapovalova et al. (2010b), (12) Bon et al. (2012), (13) Bon et al. (2016), (14) Shapovalova et al. (2016).

CHECKING RESULTS ON 3 LEVELS

- **1L**>non linear least square fitting of multisinusoidal models to the observed light curves

$$y = \sum_{i=1}^n c_i \sin\left(\frac{2\pi t}{p_i} + \phi_i\right) + B.$$

- **2L**>comparison of dynamics of observed light curves and time-series models

MODEL1 (linearly coupled oscillators of 2 units)

MODEL 2 (linearly coupled oscillators of 3 units)

$$U_a(t) = A(t) \sin(2\pi f_a t + \phi) + cp_{b \rightarrow a} \quad (3)$$

$$\times B(t) \sin(2\pi f_b t + 2\pi f_b \tau) + W(t) \quad (4)$$

$$U_b(t) = B(t) \sin(2\pi f_b t) + cp_{a \rightarrow b} \quad (5)$$

$$\times A(t) \sin(2\pi f_a t + 2\pi f_a \tau + \phi) + W(t). \quad (6)$$

$$U_a(t) = A(t) \sin(2\pi f_a t + \phi) + cp_{b \rightarrow a} \quad (7)$$

$$\times B(t) \sin(2\pi f_b t + 2\pi f_b \tau) + cp_{c \rightarrow a} \quad (8)$$

$$\times C(t) \sin(2\pi f_c t + 2\pi f_c \tau_1) + W(t) \quad (9)$$

$$U_c(t) = B(t) \sin(2\pi f_b t) + C(t) \sin(2\pi f_c t) + cp_{a \rightarrow b} \quad (10)$$

$$\times A(t) \sin(2\pi f_a t + 2\pi f_a \tau + \phi) + cp_{a \rightarrow c} \quad (11)$$

$$\times A(t) \sin(2\pi f_a t + 2\pi f_a \tau_1 + \phi_1) + W(t). \quad (12)$$

- **MODEL3 (nonlinearly coupled oscillators of 3 units)**

$$U_a(t) = A(t) \sin(2\pi f_a t + \phi) + cp_{b \rightarrow a} \quad (16)$$

$$\times B(t) \sin(2\pi f_b t + 2\pi f_b \tau) + W(t) \quad (17)$$

$$U_b(t) = B(t) \sin(2\pi f_b t) + cp_{a \rightarrow b} \quad (18)$$

$$\times U_c(t)^2 + W(t) \quad (19)$$

3L of checking: HILBERT TRANSFORM

$$H[x(t)] = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau$$

$$z(t) = a(t)e^{j\theta(t)}$$

$$a(t) = \sqrt{x^2(t) + \{H[x(t)]\}^2}, \text{ and } \theta(t) = \arctan\left(\frac{H[x(t)]}{x(t)}\right)$$

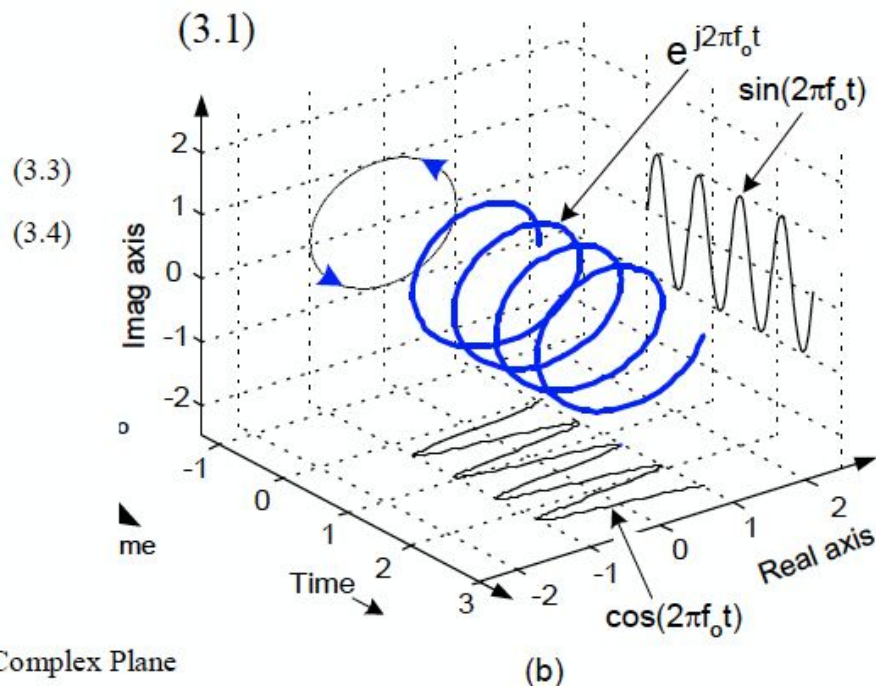
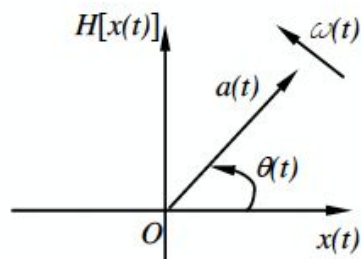
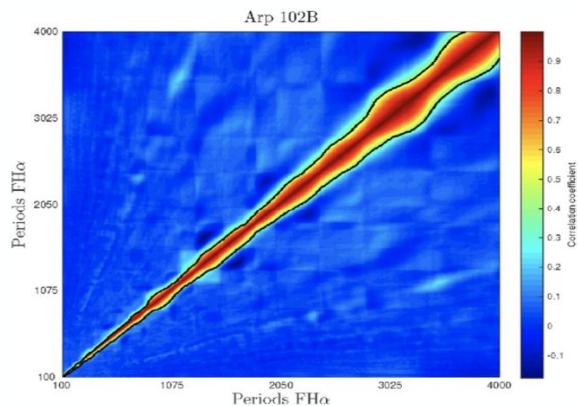


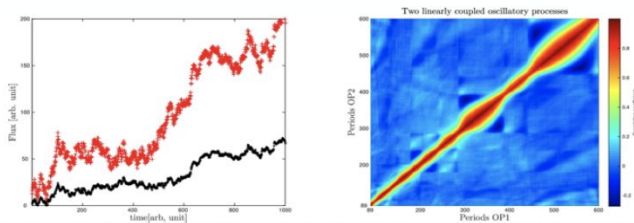
Figure 3.1. Instantaneous Amplitude, Phase Angle and Frequency in Complex Plane

ARP 102B



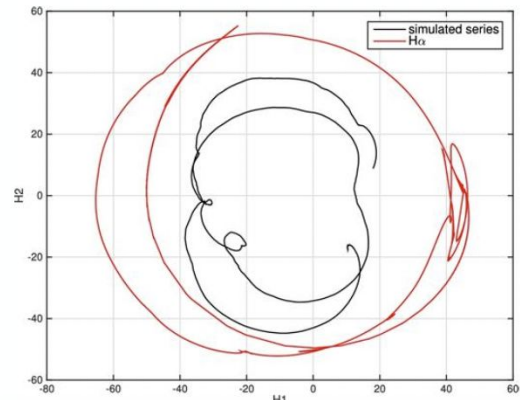
Note the prominent stationarity of the diagonal correlation line and the absence of correlation clusters.

Object name	CLC1	CLC2	$P \pm \Delta P$ (yr)	r	95% CI	p
Arp 102B	Continuum 6200 Å	H α	—	—	—	—
	Continuum 5100 Å	H β	—	—	—	—



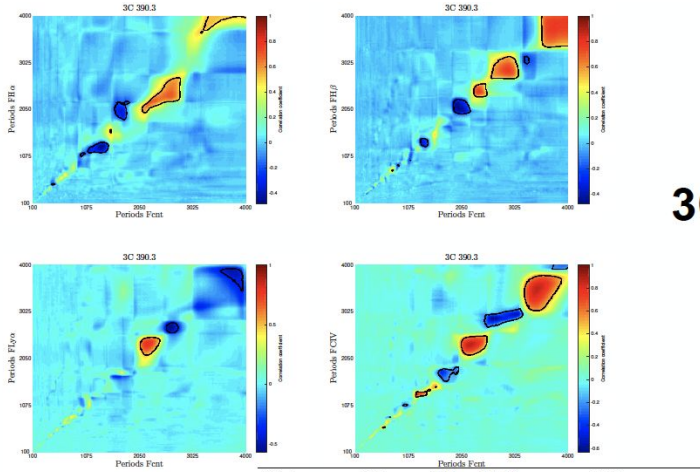
ARP 102B

Simulation of two bidirectional coupled oscillators for the case of Arp 102B. Left: random realization of equation from two time series (black is $U_a = OP1$ and red is $U_b = OP2$) of amplitudes $A = 5.29$, $B = 1.99$, phase $\phi = 0.4174$ rad, coupling strengths $cp_{a \rightarrow b} = 0.4$, $cp_{b \rightarrow a} = 0.2$, time delay is 100 and periods are 500 and 300 arbitrarily chosen time units. Right: corresponding 2D correlation map.



Both curves are similar and non-closed, indicating either weak coupling or the absence of periodicity. They appear to intersect themselves due to projection on to 2D phase space.

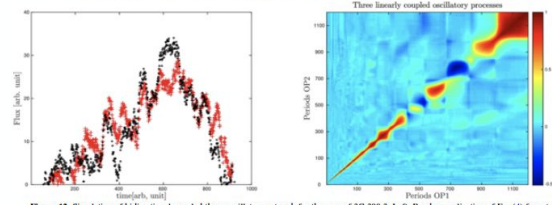
$$\begin{aligned}
 U_a(t) &= A(t) \cdot \sin(2\pi f_a t + \phi) + cp_{b \rightarrow a} \\
 B(t) &= \sin(2\pi f_b t + 2\pi f_b \tau) + W(t) \\
 U_b(t) &= B(t) \cdot \sin(2\pi f_b t) + cp_{a \rightarrow b} \\
 A(t) &= \sin(2\pi f_a t + 2\pi f_a \tau + \phi) + W(t)
 \end{aligned}$$



3C 390.3

Object name	CLC1	CLC2	$P \pm \Delta P$ [yr]	r	95%CI	p
3C 390.3	Continuum 5100 Å	$H\alpha$	9.5 ± 0.3	0.5	(0.49,0.51)	< 0.00001
			7.2 ± 1.2	0.69	(0.68,0.7)	< 0.00001
		6.3 ± 0.8	0.68	(0.67,0.69)	< 0.00001	
		4.0 ± 0.04	-0.47	(-0.48,-0.45)	< 0.00001	
	$H\beta$	5.44 ± 0.1	-0.35	(-0.37,-0.33)	< 0.00001	
		10.11 ± 0.1	0.77	(0.76,0.78)	< 0.00001	
		7.67 ± 0.02	0.71	(0.70,72)	< 0.00001	
		8.42 ± 1.6	0.75	(0.74,0.76)	< 0.00001	
Continuum 1370 Å	$Ly\alpha$	5.43 ± 0.8	-0.47	(-0.48,-0.45)	< 0.00001	
		3.6 ± 0.4	-0.33	(-0.35,-0.31)	< 0.00001	
		10.34 ± 0.1	-0.47	(-0.49,-0.45)	< 0.00001	
		7.1 ± 0.02	-0.53	(-0.54,-0.51)	< 0.00001	
	CIV	6.25 ± 1.42	0.77	(0.76,0.78)	< 0.00001	
		9.42 ± 0.02	0.85	(0.84,0.86)	< 0.00001	
		7.84 ± 0.02	-0.6	(-0.61,-0.59)	< 0.00001	
		6.4 ± 1.22	0.85	(0.84,0.86)	< 0.00001	
4.68 ± 0.7	-0.42	(-0.44,-0.40)	< 0.00001			
3.4 ± 0.4	0.75	(0.74,0.76)	< 0.00001			

2L of CHECKING: simulations of coupled oscillators

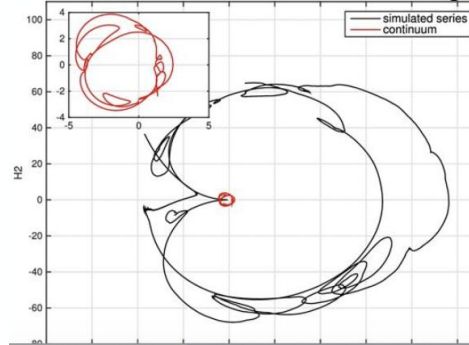


Simulation of bidirectional coupled three-oscillator network for the case of 3C 390.3.

$$\begin{aligned}
 U_a(t) &= A(t) \cdot \sin(2\pi f_a t + \phi) + c p_{b \rightarrow a} \\
 &B(t) \cdot \sin(2\pi f_b t + 2\pi f_b \tau) + c p_{b \rightarrow a} \\
 &C(t) \cdot \sin(2\pi f_c t + 2\pi f_c \tau_1) + W(t) \\
 U_b(t) &= B(t) \cdot \sin(2\pi f_b t) + C(t) \cdot \sin(2\pi f_c t) + c p_{a \rightarrow b} \\
 &A(t) \cdot \sin(2\pi f_a t + 2\pi f_a \tau + \phi) + c p_{a \rightarrow b} \\
 &A(t) \cdot \sin(2\pi f_a t + 2\pi f_a \tau_1 + \phi_1) + W(t)
 \end{aligned}$$

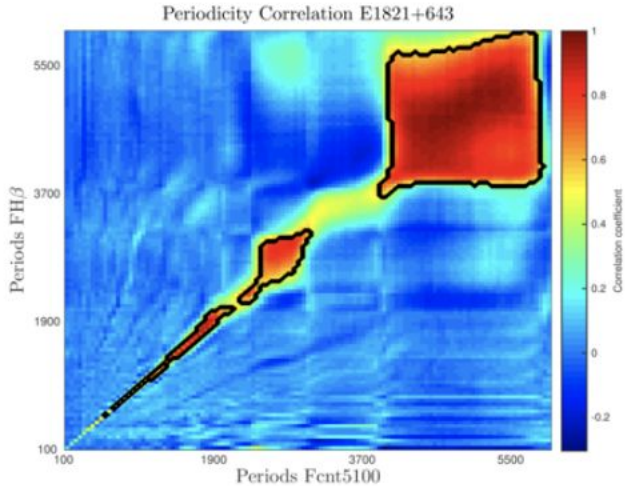
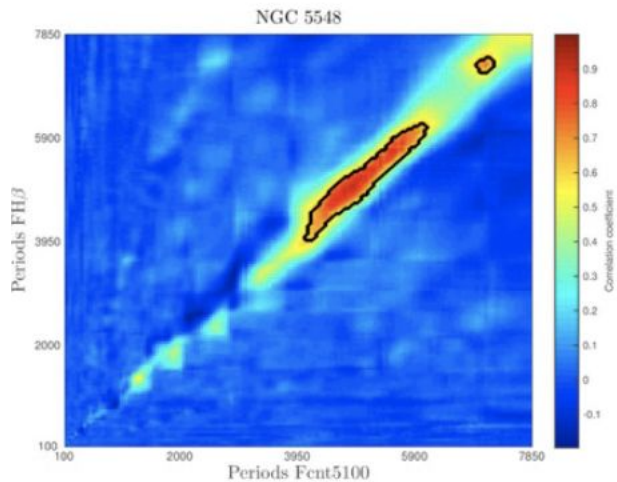
Figure 12. Simulation of bidirectional coupled three oscillators network for the case of 3C 390.3. Left: Random realization of Eq. (4) form two time series (black is $U_a = OP1$, and red is $U_b = OP2$) of amplitudes $A = 1.054, B = 1.729, C = 2.257$, phase $\phi = \phi_1 = 2.309$ rad coupling strengths $c p_{a \rightarrow b} = 0.7, c p_{b \rightarrow a} = 0.5, c p_{a \rightarrow c} = 0.2, c p_{b \rightarrow c} = 0.3$, frequencies $f_a = \frac{1}{1000}, f_b = \frac{1}{200}, f_c = \frac{1}{300}$, and time delay of 100 arbitrary chosen time unities. Right: corresponding 2D correlation map, which clearly shows three clusters related to fundamental periods as well as clusters of negative correlation.

3L of checking: Hilbert transform

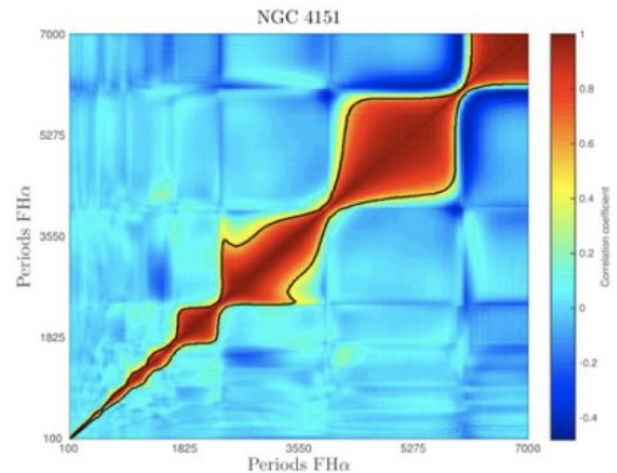


Comparison of the phase trajectories between the continuum of 3C 390.3 and simulated curve OP1 from the oscillatory network model

$$y^H(t) = \frac{1}{\pi} P V \int_{-\infty}^{\infty} \frac{y(t)}{t - \tau} d\tau$$

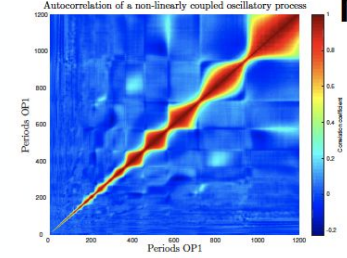
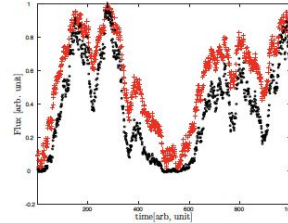
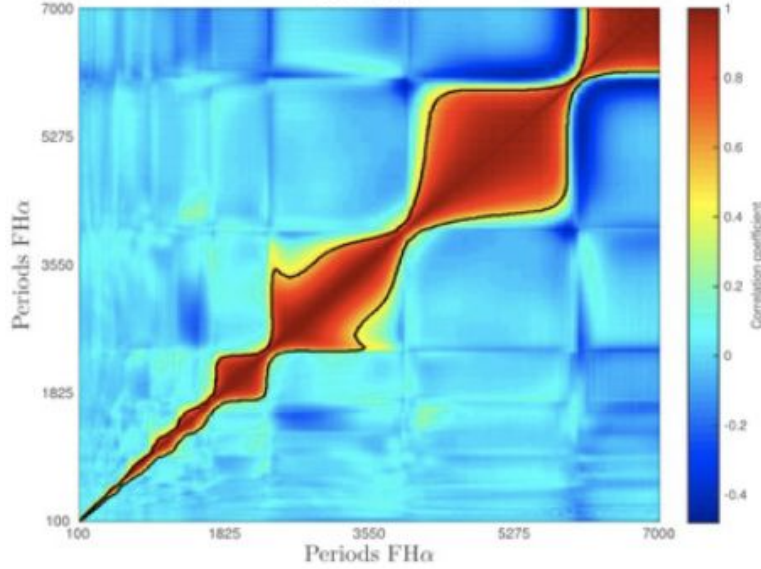


BINARY SMBH CANDIDATES



NGC 4151	Continuum 5100 Å	H α	13.76 ± 3.73	0.96	(0.956,0.962)	< 0.00001
			8.33 ± 2.33	0.97	(0.968,0.972)	< 0.00001
			5.44 ± 1.29	0.98	(0.978,0.981)	< 0.00001
NGC 5548	Continuum 5100 Å	H β	13.3 ± 2.26	0.87	(0.867,0.873)	< 0.00001
E1821+643	Continuum 5100 Å	H β	12.76 ± 5.6	0.98	(0.979,0.981)	< 0.00001
			6.93 ± 1.99	0.80	(0.792,0.808)	< 0.00001
			4.75 ± 0.79	0.80	(0.792,0.808)	< 0.00001
	Continuum 4200 Å	H γ	12.36 ± 6.1	0.99	(0.989,0.991)	< 0.00001
			6.52 ± 3.26	0.91	(0.906,0.914)	< 0.00001
			4.34 ± 0.74	0.94	(0.937,0.943)	< 0.00001

NGC 4151



NGC 4151

Figure 16. Simulation of two bidirectional coupled oscillators for the case of NGC 4151. Left: Random realization of Eq. (8) form two time series (black is $U_a = OP1$, and red is $U_b = OP2$) of amplitudes $A = 6.09, B = 1.04$, phase $\phi = 2.2 \text{ rad}$, coupling strengths $cp_{a \rightarrow b} = 0.7, cp_{b \rightarrow a} = 0.6$, periods are 500, 300 and time delay is 100 arbitrarily chosen time units. Note the similarity of sharpness of this signal 'bursts' with features in the observed light curves. Right: corresponding 2D correlation map.

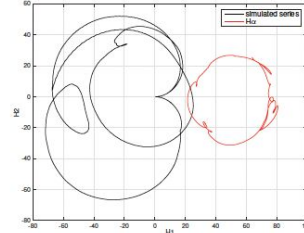
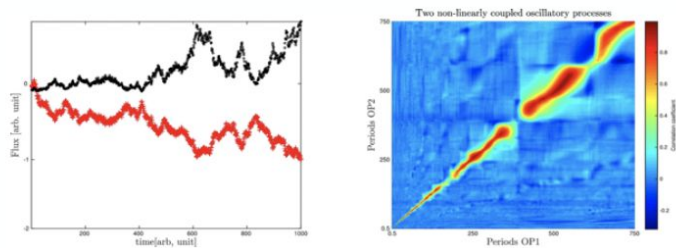
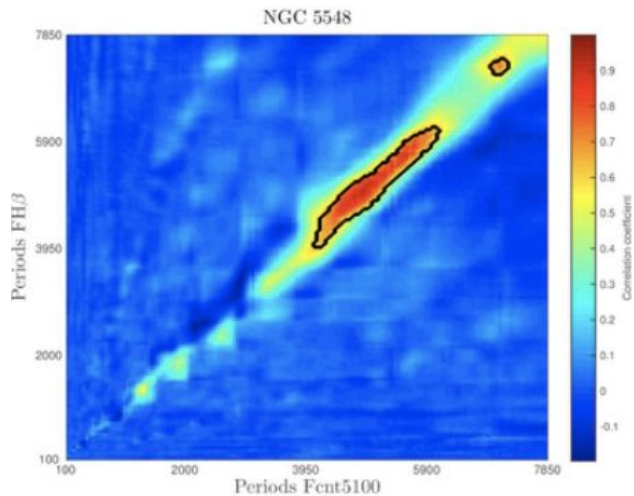


Figure 17. As in Fig. 13 but for the $H\alpha$ line of NGC 4151 and simulated $OP1$ curve described by Eq. (8) with parameter values as in (see Fig. 16). Note that phase curve of $H\alpha$ line is shifted by + 50 units on x axis for a better view.

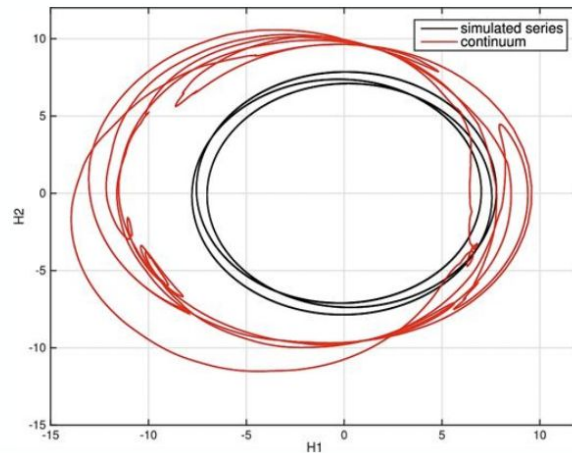
$$\begin{aligned}
 U_a(t) &= A(t) \cdot \sin(2\pi f_a t + \phi) + cp_{b \rightarrow a} \cdot \\
 &\quad B(t) \cdot \sin(2\pi f_b t + 2\pi f_b \tau) + W(t) \\
 U_b(t) &= B(t) \cdot \sin(2\pi f_b t) + cp_{a \rightarrow b} \cdot \\
 &\quad U_a(t)^2 + W(t)
 \end{aligned} \tag{8}$$

where the non-linear coupling is introduced by squared term $U_a(t)^2$. Simulated curves consists of sum and multiple of base sinus signals of periods of 500 and 300 arbitrary chosen time units. As a consequence, periods of $2 * 500, 2 * 300, 500, 300$ are accompanied with an interference patterns $500 + 300, 500 - 300$ (right plot in Fig. 16). Comparing this scenario with autocorrelation of periods in $H\alpha$ (see Fig. 7), the largest period of 13.76 yr can be interpreted as interference pattern (i.e. sum) of two smaller periods of 5.44 and 8.33 yr.



NGC 5548

Simulation of two bidirectional coupled oscillators for the case of NGC 5548. Left: random realization of equation (19) from two time series (black is $U_a = OP1$ and red is $U_b = OP2$) of amplitudes $A = 5.92$, $B = 1.27$, phases $\phi = 2.65$ rad, coupling strengths $cp_{a \rightarrow b} = 0.7$, $cp_{b \rightarrow a} = 0.2$, periods 500 and 300 and time delay is 100 arbitrarily chosen time units



Note the chaotic-like appearance of both curves.

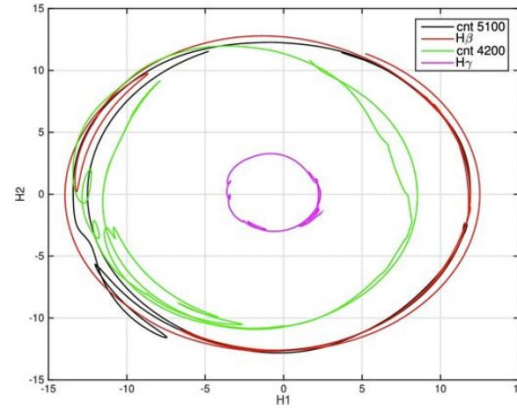
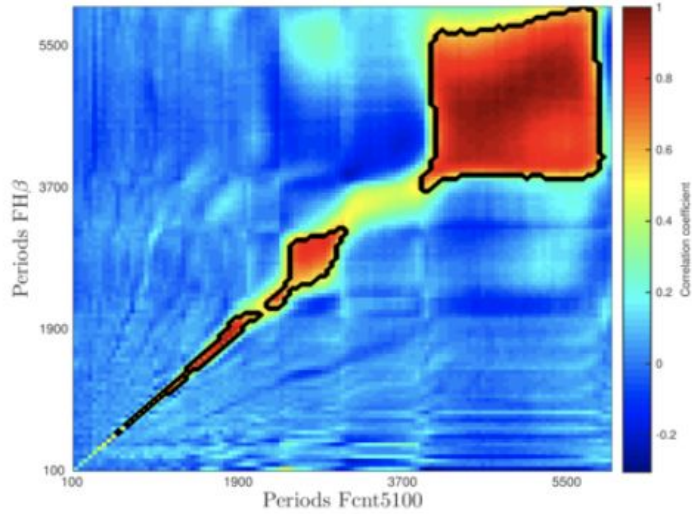
$$U_a(t) = A(t) \sin(2\pi f_a t + \phi) + cp_{b \rightarrow a} \quad (16)$$

$$\times B(t) \sin(2\pi f_b t + 2\pi f_b \tau) + W(t) \quad (17)$$

$$U_b(t) = B(t) \sin(2\pi f_b t) + cp_{a \rightarrow b} \quad (18)$$

$$\times U_a(t)^2 + W(t). \quad (19)$$

Periodicity Correlation E1821+643

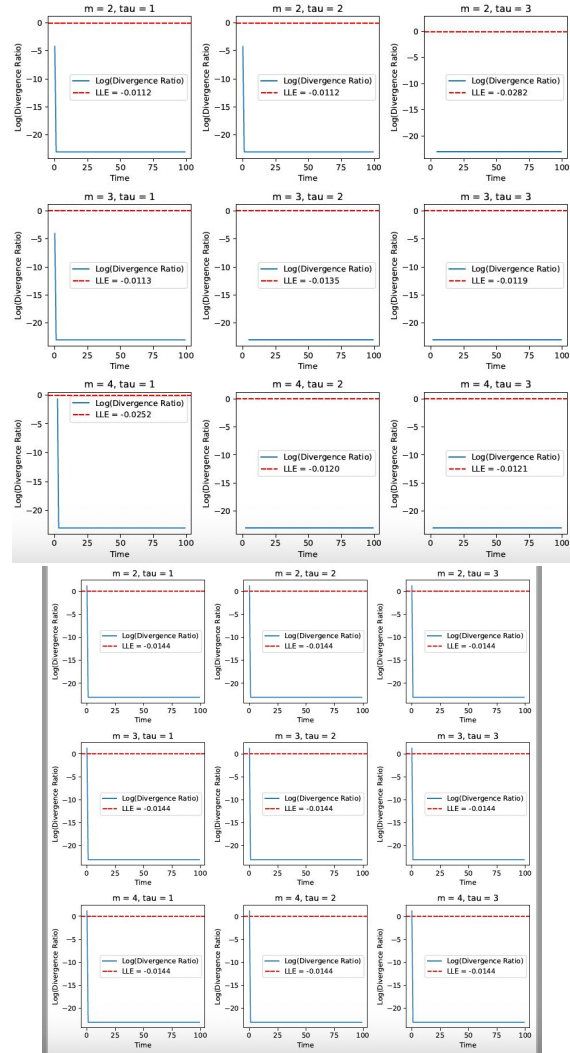
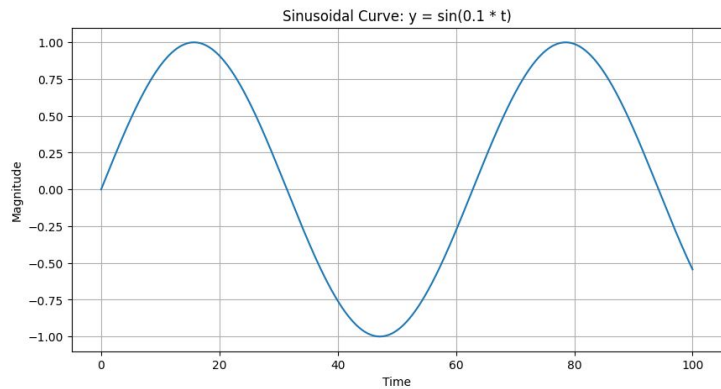
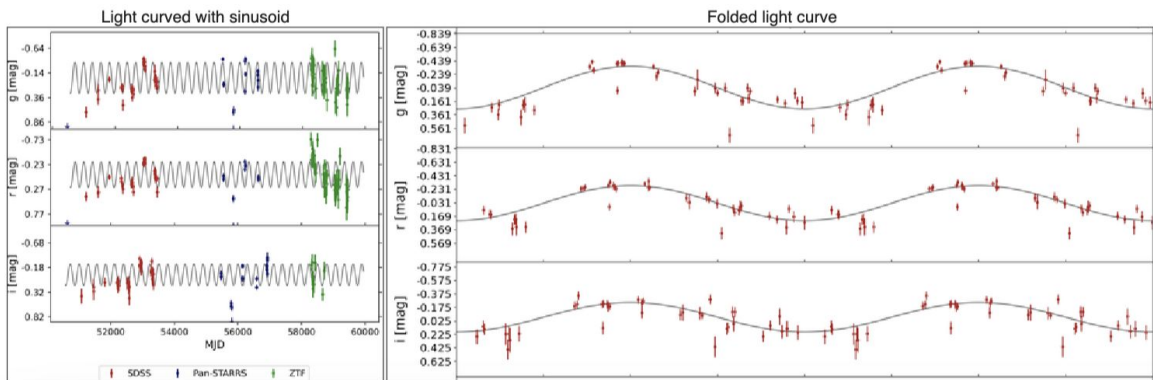


NGC E1821+643

Its 2D correlation maps are similar to the case of NGC 4151. Particularly, if we look at phase portraits of the light curves normal limit cycles are observed in the dynamics of E1821 + 643. They are similar to the phase portrait of regular sinusoids. We note the presence of two smaller elongated loops in all phase curves reflecting two smaller periods.

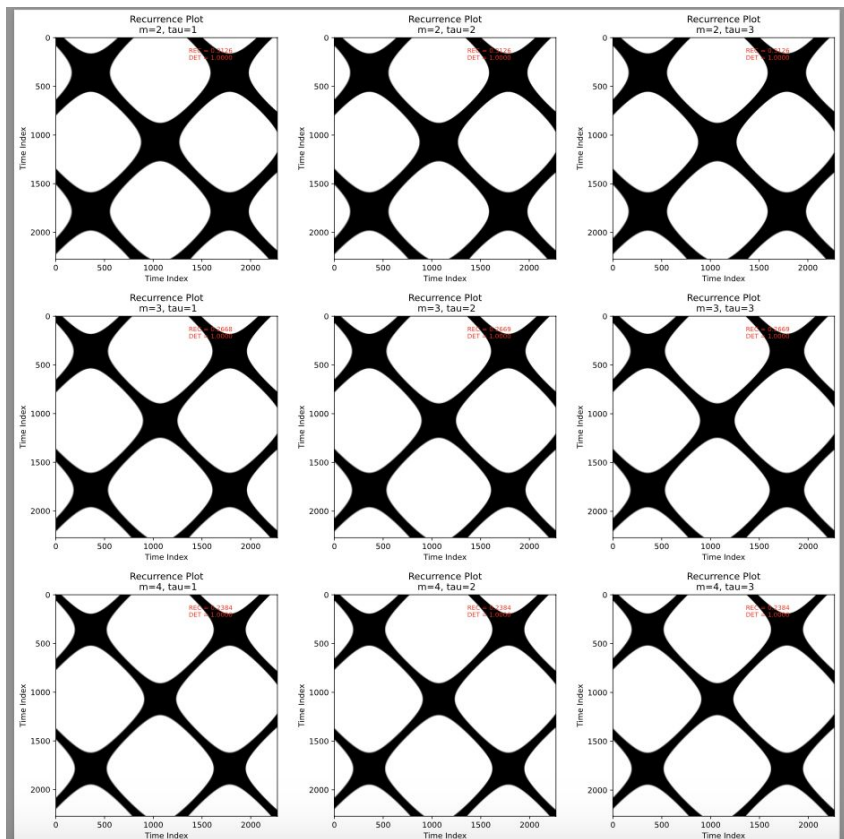
Lyapunov Exponents & Recurrence Analysis: Quasar vs. Sinusoidal Signal

Fatovic et al +23

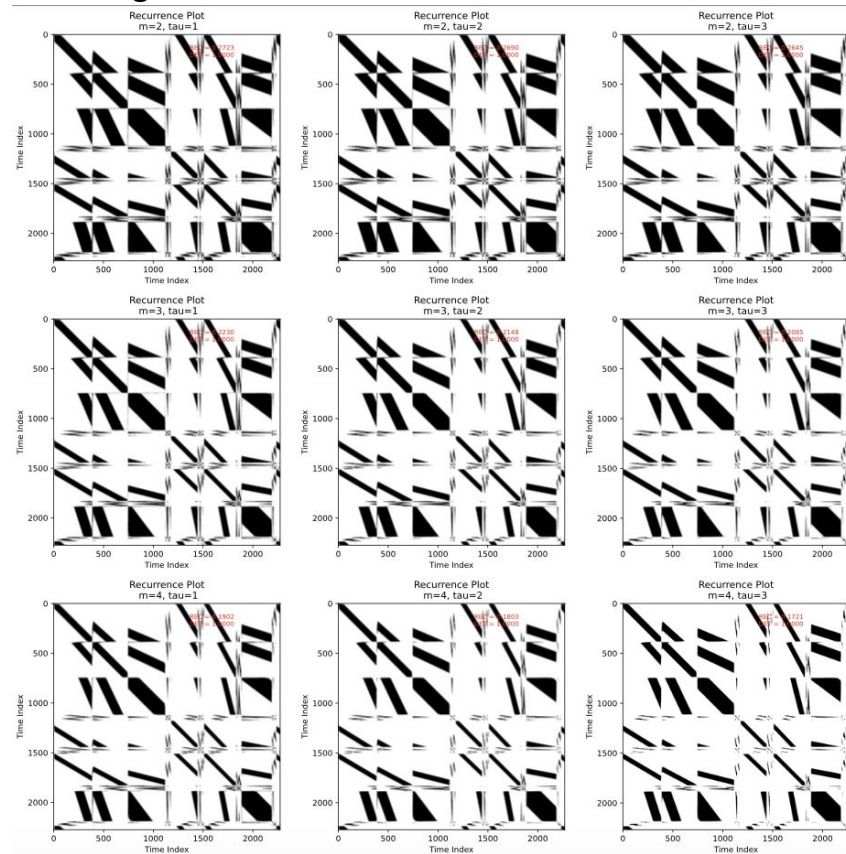


Complex Dynamics in Quasar Light Curves: A Recurrence Analysis Perspective

Pure Sinusoid



Quasar light curve



SUMMARY

- The integration of astronomical big data surveys with upcoming nHz GW observations is essential for revealing binary SMBH.
- The application of advanced methods on new astronomical big data can facilitate the detection of binary SMBH systems
- Given the intricacy of both binary and single AGN light curves, there's an emerging need for innovative observables (NOb).
- Novel methods, such as our 2D Hybrid approach, can detect periodicity of binary SMBH within a 2D correlation signal space, representing a potential NOb.
- Our recent pilot experiments using recurrence analysis of newly detected binary SMBH have indicated the stability and periodic signal characteristics of the observed object.