Maintaining lasing topological zero-mode in distorted photonic lattice by nonlinearity

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Motivation

We are inspired by question how **nonlinearity deals with properties of photonic topological insulators.**

Pre-steps:

- ability to scan the topological properties of photonic bands by modulation instability [1]
- excitation of topological transition by nonlinearity [1],
- topological properties of nonlinear Floquet lattices [2]

Breaking point:
Menssen [3]=> FIRST EXPERIMENTAL REALIZATION OF A STATE BOUND TO VORTEX REPRESENTED BY A POINT DEFECT IN 2D LINEAR PHOTONIC GRAPHENE
→
How nonlinearity shapes the properties of this protected state?

Talk overview:

I **Introduction**

II **Model equations**

Graphene like hexagonally shaped lattice
Vortex distortion –> topological phase transition

III **Topological zero mode**

Genesis of zero-mode

Tuning the zero-mode lasing by driving effect (saturable nonlinear gain & linear loss)

Lasing zero-mode in the presence of nonlinear lattice response

IV **Summary**

I INTRODUCTION

Topological photonics is a rapidly emerging field of research in which geometrical and topological ideas are exploited to design and control the behavior of light [4,5].

Topological insulators and topologically protected modes

We considered photonic equivalent of graphene - ‘Dirac material’ – hexagonal (honeycomb) bipartite lattice consisting of two sublattices A and B, with sites from one sublattice directly interacting only with sites from the other sublattice. Energy spectrum is symmetric around zero energy.

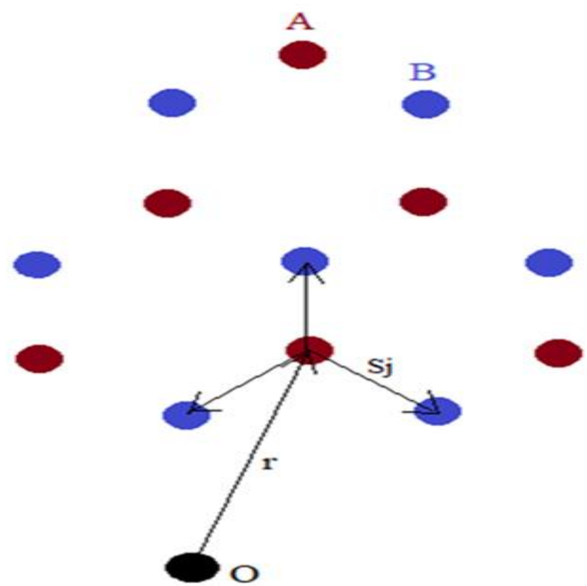


Fig. 1 Hexagonal lattice with bipartite symmetry

In the absence of distortion – topologically trivial – with two strongly coupled bands (Dirac points at the 1st BZ boundaries)

Vortex distortion opens the gap between bands which hosts zero-mode – topologically nontrivial state

We investigated ability to manage light propagation (guiding, coupling, lasing) via zero-modes utilizing nonlinear response and driving [6].

II Model equations

Hamiltonian of hexagonally shaped bipartite lattice with vortex distortion:

$$\hat{H} = \hat{H}_{latt} + \hat{H}_{Kerr} + \hat{H}_{gain} \tag{1}$$

Linear lattice Hamiltonian (tight-binding approximation)

$$\hat{H}_{latt} = -\sum_{\vec{r},\vec{r}'}\left(t + \delta t_{\vec{r},\vec{r}'}\right) \hat{a}_{\vec{r}}{}^{\dagger}\hat{b}_{\vec{r}} + +h.c. \tag{2}$$

t – coupling strength

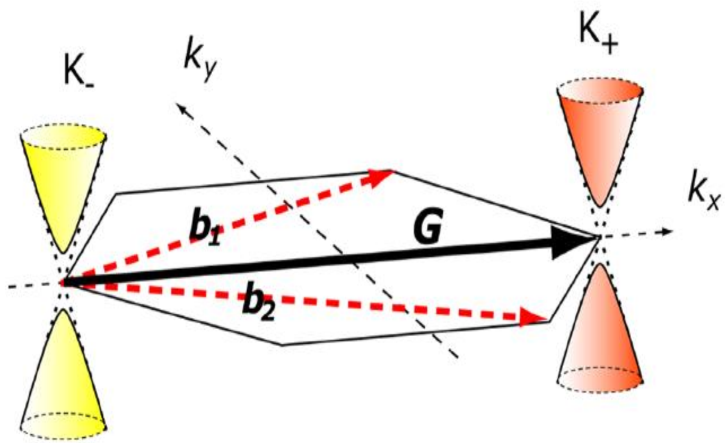
$\delta t_{\vec{r},\vec{r}'}$ -a Kekule vortex-like distortion [3,6,7]
(experimentally induced by small shifts in the WGs positions [3])

$$\delta t_{\vec{r},\vec{r}'} = \frac{\Delta(\vec{r})\exp(i\vec{K}_+\vec{s}_j)}{3} + \text{c.c.}$$

$$\Delta(\vec{r}) = \Delta_0 \tanh\left(\frac{r}{l_0}\right) e^{i(\alpha+\chi\Theta)} \quad (3)$$

α – vortex phase

- Vortex distortion → Coupling of two Dirac valleys by distortion → opening of the gap in energy spectrum → creation of topological zero-mode ('Majorana' mode)

(a) 

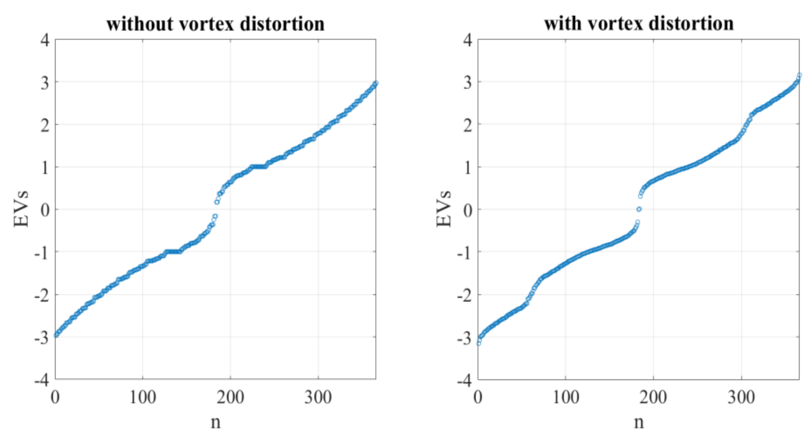
(b) 

Fig. 2 (a) Dirac points at the 1st BZ boundaries are connected by reciprocal vector [7]. (b)The eigenvalues of lattice without and with vortex distortion.

Nonlinear Hamiltonian

$$\hat{H}_{Kerr} = \frac{G}{2}\sum_{\vec{r}}\left[\frac{1}{1+\left|a_{\vec{r}}\right|^2}\hat{a}_{\vec{r}}{}^{\dagger}\hat{a}_{\vec{r}} + \frac{1}{1+\left|b_{\vec{r}}\right|^2}\hat{b}_{\vec{r}}{}^{\dagger}\hat{b}_{\vec{r}}\right] \tag{4}$$

Hamiltonian of ‘driving’

$$\hat{H}_{gain} = i\sum_{\vec{r}}\left[\frac{\Gamma_A}{1+\left|a_{\vec{r}}\right|^2}\hat{a}_{\vec{r}}^{\dagger}\hat{a}_{\vec{r}} + \frac{\Gamma_B}{1+\left|b_{\vec{r}}\right|^2}\hat{b}_{\vec{r}}^{\dagger}\hat{b}_{\vec{r}} - \gamma_A\hat{a}_{\vec{r}}^{\dagger}\hat{a}_{\vec{r}} - \gamma_B\hat{b}_{\vec{r}}^{\dagger}\hat{b}_{\vec{r}}\right] \tag{5}$$

Equation of light propagation

$$i\partial_z\left|\psi(\vec{r},z)\right> = \hat{H}\ \left|\psi(\vec{r},z)\right> \tag{6}$$

- Dynamical considerations :

To solve Eq. (6) numerical Runge-Kutta method of 4th order

Total norm and norms in A and B

$$N(z) = N_A(z) + N_B(z), \qquad N_{A(B)} = \sum |a_r(b_r)|^2 \tag{7}$$

Participation numbers: total and in A/B

$$\mathrm{P}(\mathrm{z}) = P_A(z) + P_B(z), \qquad P_{A(B)} = (\sum |a_r(b_r)|^2)^2 / \sum(|a_r(b_r)|^4) \tag{8}$$

Fidelity

$$F(z) = \sum_{\vec{r}} \left| \langle \psi_{\vec{r}}(z) | \psi_{\vec{r}}{}^{ZM} \rangle \right| \tag{9}$$

III Topological zero-mode:

- Emerge after applaying distortion to the underlying lattice;
- Lie in the mid-gap at zero energy;
- Extremaly robust to the external perturbation – disorder.

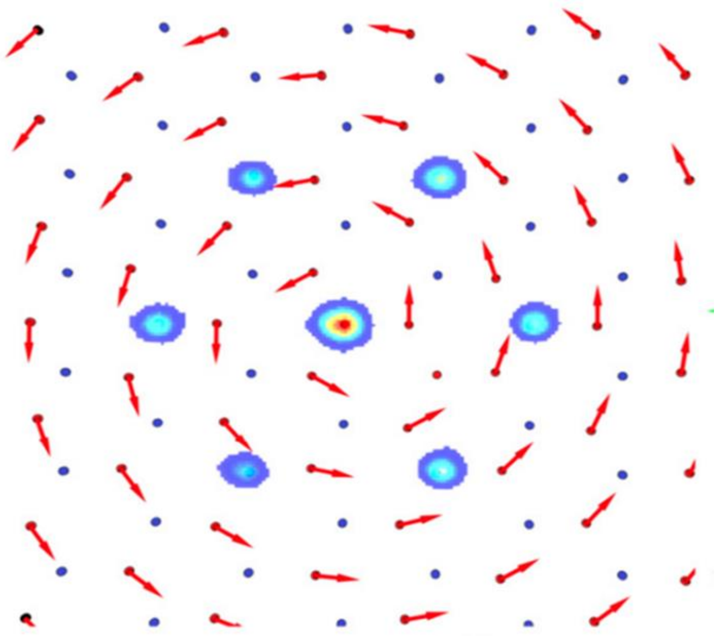


Fig. 3 Zero-mode indicated at the centre of the vortex presented in [3].

- Consists of components in each sublattices, which are either localized around the vortex center (vortex component) or along the edge (edge component)

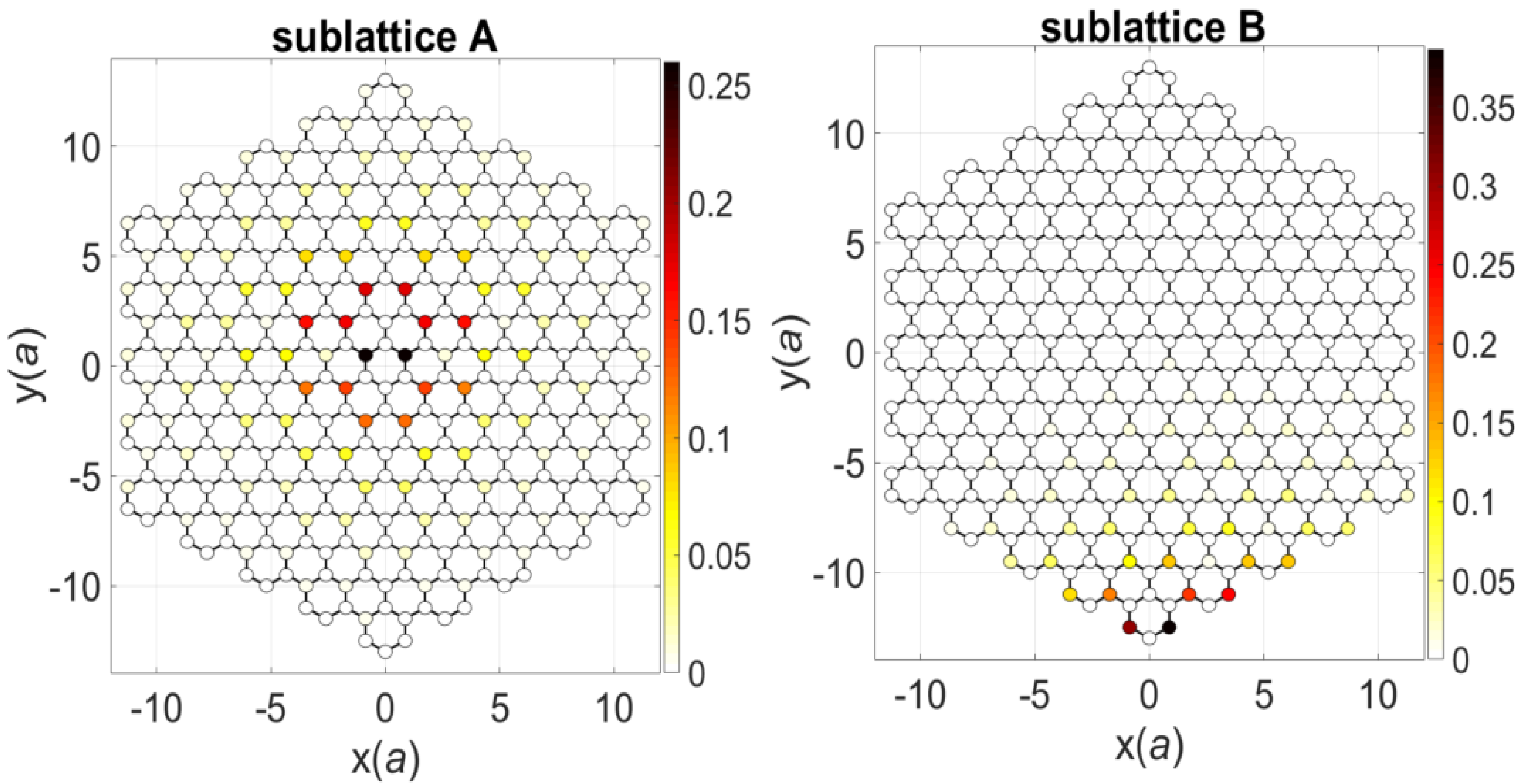


Fig. 4 The zero-mode components

- **Does the zero-mode robustness provide lasing light?**
- **Does local nonlinear intensity-dependent lattice response deal in favor of the zero-mode lasing?**

Nonlinear saturable gain + linear loss ***= generator of lasing zero-mode***
(zero or weak local nonlinearity)

-Zero-mode is excited from the noise background owing the gain-loss mechanism

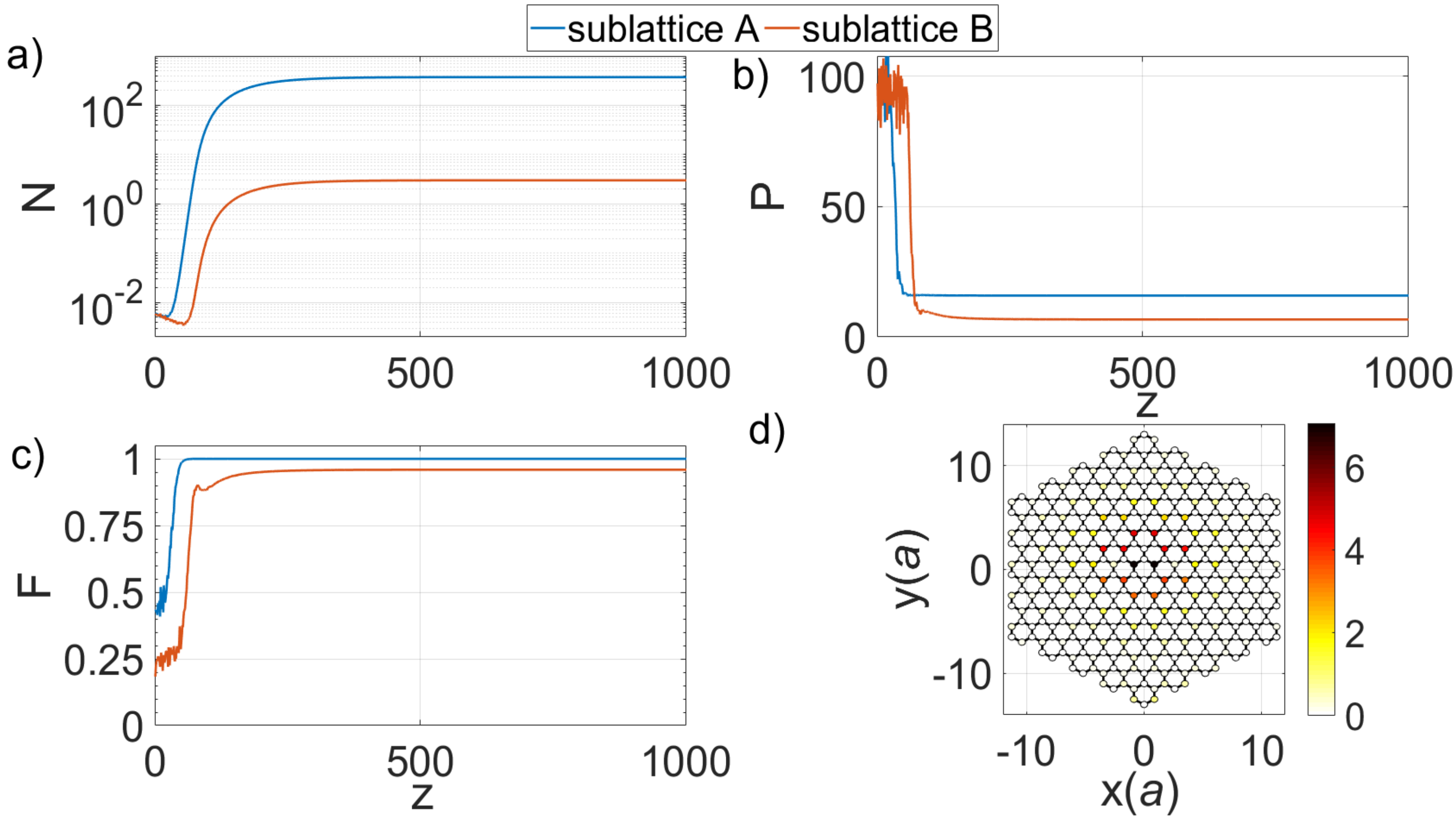


Fig. 5 Noise amplification and lasing via the zero-mode : Γ_A/γ_A = 10, Γ_B = 0, γ_A = 0.01, γ_B/γ_A = 10, G=0, $\alpha = -\pi/2$.
a) N vs. z in log scale, b) P, c) F, d) intensity profiles at z = 500 (steady state lasing regime)

Pumped vortex component (sublattice A)→ linear loss →

coupling between the sublattices →

redistribution of energy to edge component (sublattice B)→

feedback loop: the sublattice B loss stimulates the more efficient excitation of the sublattice A.

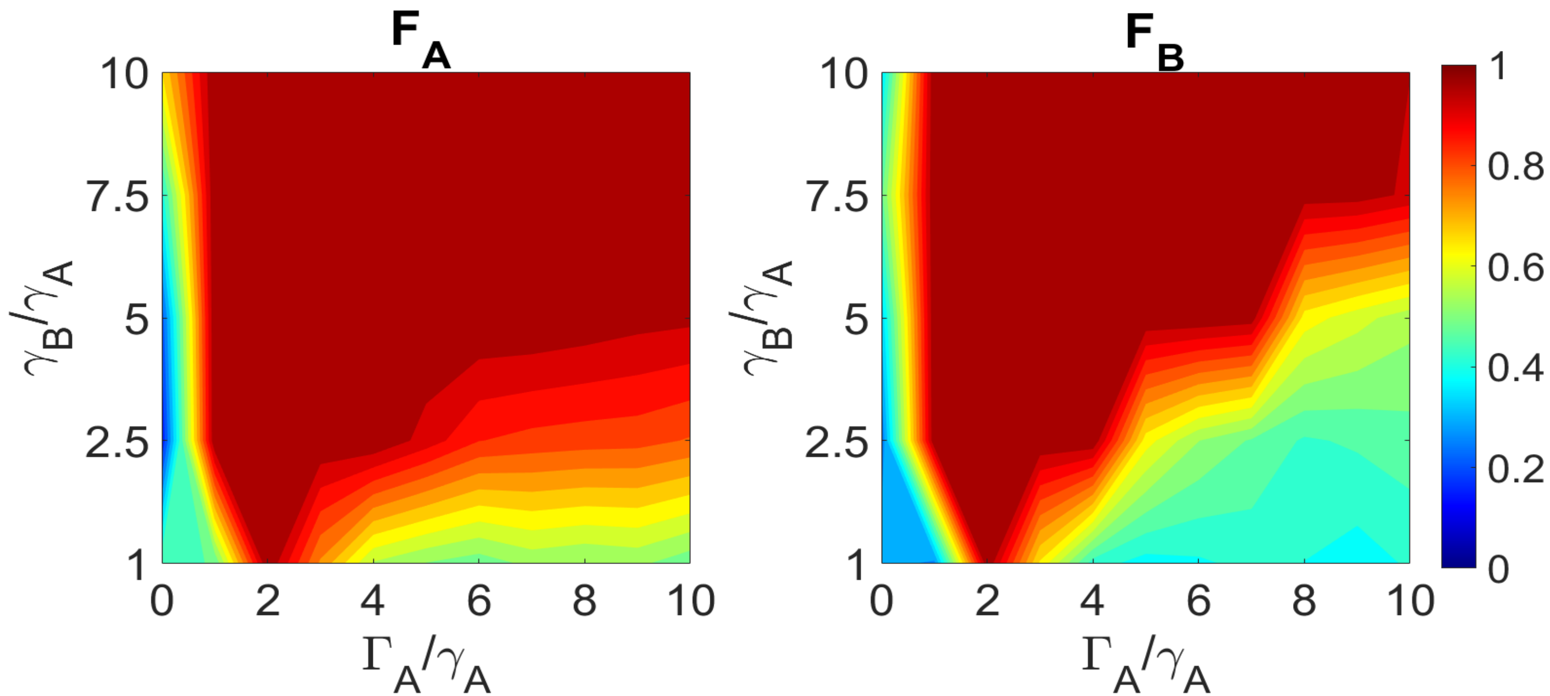


Fig. 6 Fidelity vs. gain in A and ratio of loss parameters, G=0 (similar for weak G).

Zero-mode lasing vs. local intensity-dependent nonlinear lattice response:

$|G| < 0.1$ –robust vortex-edge –component lasing → F is in the range [0.9, 1]
$0.1 < |G| < 0.9$ – robust vortex component lasing
$|G| > 0.9$ – delocalized radiation → destruction of the zero-mode lasing.

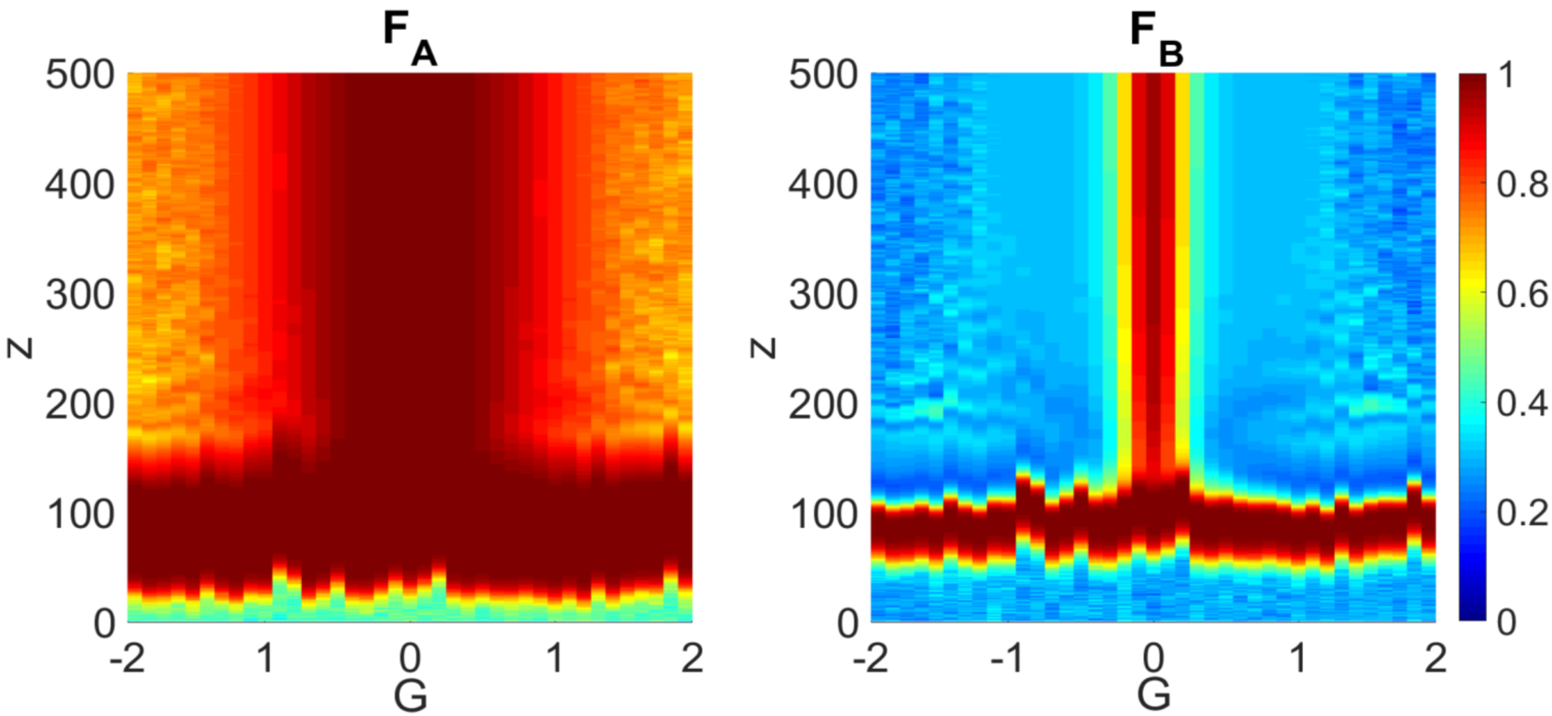


Fig. 7 F(z) vs. G. : $\Gamma_A/\gamma_A = 10$, $\Gamma_B = 0$, $\gamma_A = 0.01$, $\gamma_B/\gamma_A = 10$.

IV SUMMARY

Key points:

- Excitation of the zero-mode from a noisy background by tuning sublattice dependent saturable gain and linear loss

- Creation of stable lasing in an unfolded lattice resonator

- Deterioration of lasing by intensity-dependent nonlinearity in the limit of huge powers of light – strong nonlinearity.

Our study opens the door to an unhindered propagation and amplification of a coherent zero-mode and proposes the design of new topological lasers in active photonic media [6] : WGAs, MCFs, lattices of ring resonators.

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