Graduate School of
Advanced Science and Engineering
Hiroshima University

Nonlinear analysis of strong coupling gauge theory on Torus

Theoretical Particle and Hadron Physics Group, Hiroshima U.

Shoko Ono

Supervisor : Tomohiro Inagaki

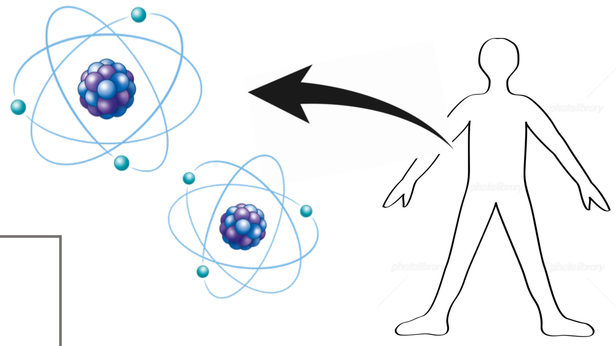
Outline of this talk

- **Introduction**
  - The purpose of studying the QCD phase diagram
- **Theories for QCD**
- **SDE with finite size and BC**
  - The features
- **Analytical calculation**
  - How to get the equations to analyze
- **Numerical results**
- **Conclusion and Summary**

Introduction

- QCD (Quantum Chromo Dynamics)

  - describes the dynamics of hadrons that make up our universe



  - spontaneous symmetry breaking
    → light quark mass is generated
    at about 300MeV
    - depends on the environment
      - temperature
      - density
      - size
      - curvature
      - electromagnetic field,…

chiral transformation

$$\begin{cases} \Psi_R = \dfrac{1+\gamma^5}{2}\Psi \\ \Psi_L = \dfrac{1-\gamma^5}{2}\Psi \end{cases} \qquad \Psi \to \mathrm{e}^{i\theta\gamma^5}\Psi$$

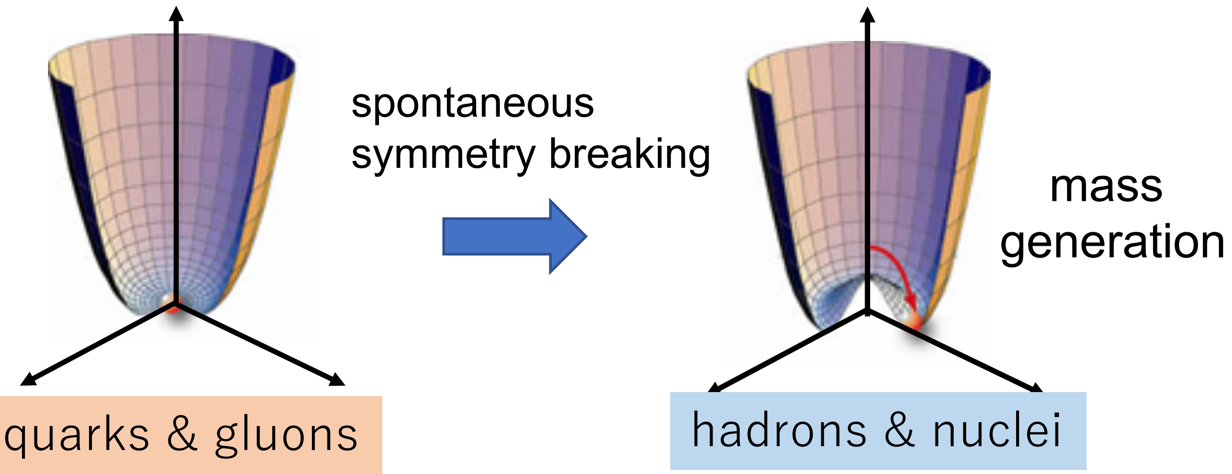
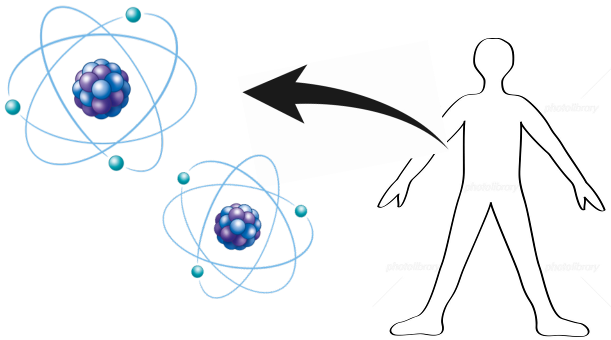


Fig1. spontaneous symmetry breaking

Introduction



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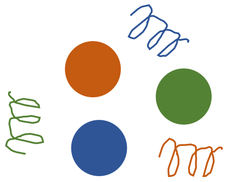
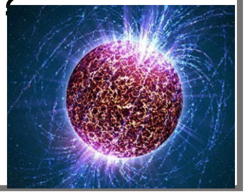
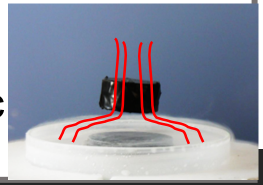
The early universe [1]
- the temperature was over 10^{14}℃
- symmetry restoration
- light quark mass is about 10MeV



Neutrons stars [2]
- have a high density core
- symmetry restoration?
- hyperon ?



Superconductivity [3]
- U(1) symmetry is breaking
- photons (appear to) have mass
- symmetry is restored by applying a magnetic field.



[1] A. Barducci, R. Casalbuoni, S. De Curtis, R. Gatto and G. Pettini, Phys. Lett. B 231, 463 (1989).
[2] Baym, G., Hatsuda, T., Kojo, T., et al. 2018, Reports on Progress in Physics, 81, 056902
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Image：Casey Reed / Penn State University

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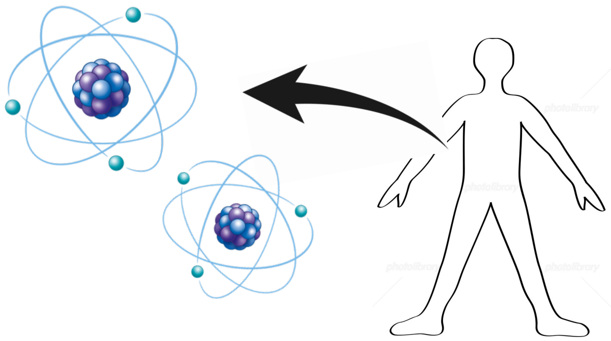

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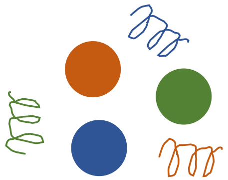

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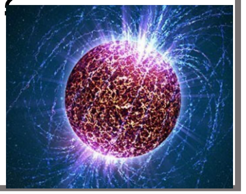

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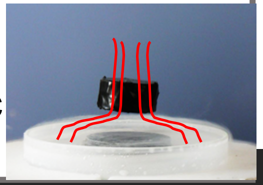

Studying phase transitions in QCD can be a good probe of various physical phenomena in our universe.

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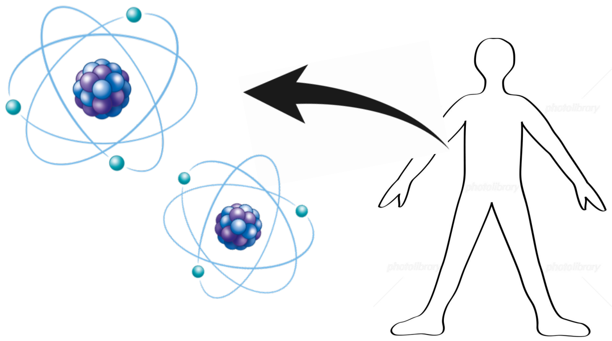

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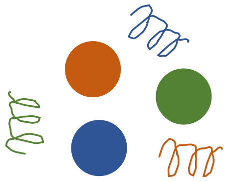

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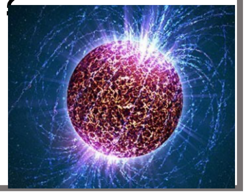

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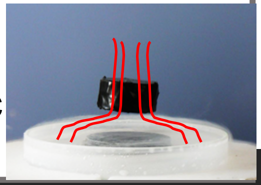

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Theories for QCD

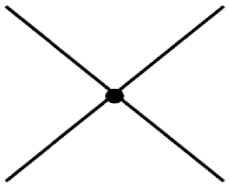
Because of the asymptotic freedom, perturbative expansion is not valid at low-energy.
Therefore, effective models are often used to evaluate low-energy QCD.

Four-Fermion Interaction Model [4][5]

$$\mathcal{L}_{NJL} = \overline{\psi} i \gamma_\mu \partial^\mu \psi + G[(\overline{\psi}\psi)^2 + (\overline{\psi} i \gamma^5 \tau^a \psi)^2]$$

- low energy effective model
- failure for phenomena in the high-energy region

interaction term



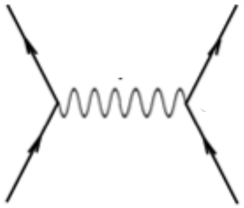
$$\frac{1}{2\sqrt{\pi}}\Gamma\left(1-\frac{D}{2}\right)\Gamma\left(\frac{D-1}{2}\right)\left[(-1)^a-\left(\frac{m^2}{m_a^2}\right)^{\frac{D}{2}-1}\right]$$
$$=-\int_0^\infty \frac{dK}{\sqrt{K^2+m^2}}\left(\frac{K}{m_a}\right)^{D-2}\frac{\exp\left(-L\sqrt{K^2+m^2}\right)-\cos(\pi\delta)}{\cosh\left(L\sqrt{K^2+m^2}\right)-\cos(\pi\delta)}$$

Nonlinear equation

Schwinger-Dyson Equation

$$\Sigma(p) = -ie^2\int \frac{d^Dq}{(2\pi)^D}\gamma^\mu i S(q)\gamma^\nu i D_{\mu\nu}(p-q)$$

- equation for the exact correlation function
- non-perturbative effects are included
- impossible to solve without appropriate approximations



$$B(p) = \frac{\alpha}{4\pi}\frac{1}{p}\int_0^\Lambda dq \frac{qB}{\sqrt{q^2+B^2}}\left(\frac{\sinh(L\sigma)}{(\cosh(L\sigma)-\cos(\delta\pi))}\right)$$
$$\times\left[2\log\frac{(p+q)^2}{(p-q)^2}+\log\frac{(p+q)^2+2Nm_{ph}^2}{(p-q)^2+2Nm_{ph}^2}\right]$$

Nonlinear equation

[4] Y. Nambu and G. Jona-Lasinio, "Dynamical model of elementary particles based on an analogy with superconductivity. I," Phys. Rev. 122, 345-358 (1961)
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GOAL :

- To analyze the effect of finite size and boundary condition for CSB using SDE
- To compare the results with those of the Four-Fermion Interaction Model (FFIM)

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SDE with finite size and BC

- SDE for fermion self-energy

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- propagator of fermions : $\mathcal{S}(p)$

$$\mathcal{S}(p) = \frac{1}{\not{p} - \Sigma(p) + i\epsilon} = \frac{1}{\not{p} + (A-1)\tilde{\not{p}} + (A_{D-1}-1)\not{p}^{D-1} - B + i\epsilon}$$

- propagator of photons : $D_{\mu\nu}(q)$

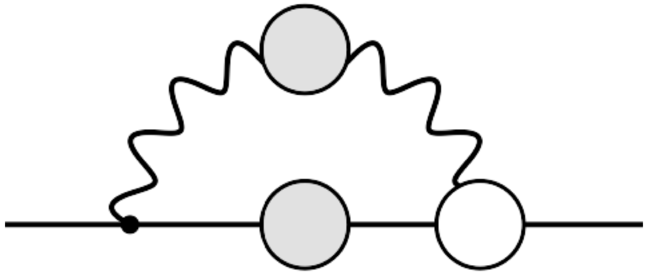


Fig2. Feynman diagram of $\Sigma(p)$

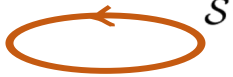
SDE with finite size and

$$p = (p^0, p^1, \cdots, p^{D-1}) = (\check{p}, p^{D-1})$$

$$p^{D-1} = \frac{2m\pi}{L} + \frac{\pi\delta}{L}$$

L : finite size

δ : boundary condition



$$\psi(\check{x}, x^{D-1} + L) = e^{i\pi\delta}\psi(\check{x}, x^{D-1})$$

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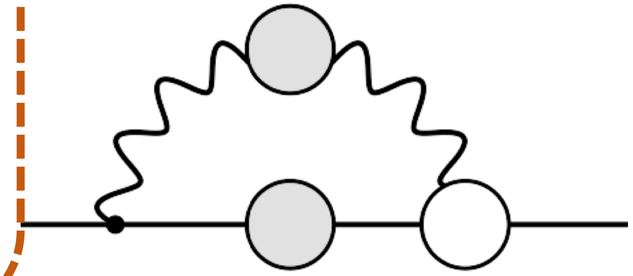

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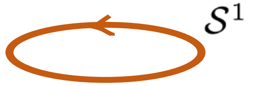

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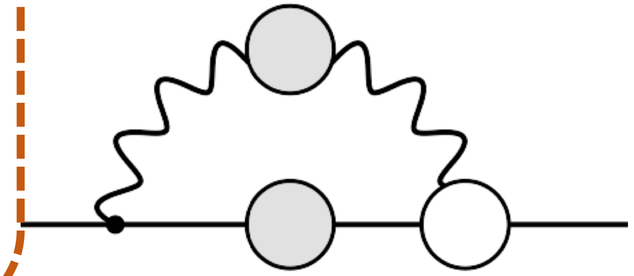

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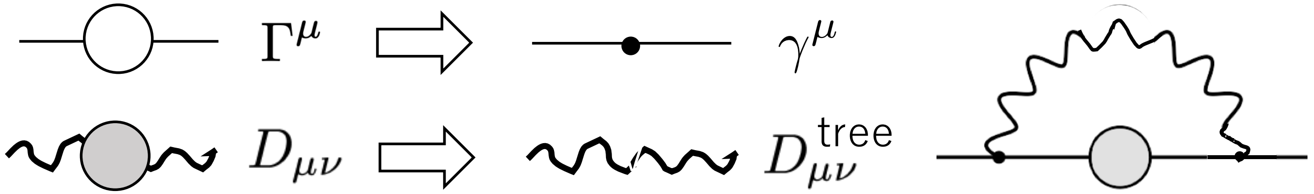

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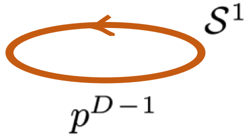

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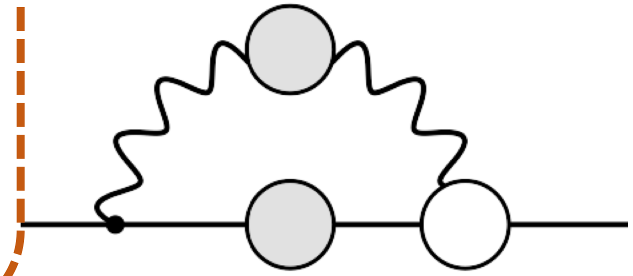

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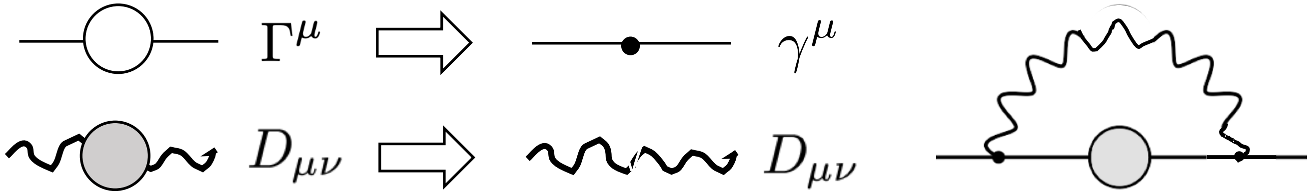

$$D_{\mu\nu}(q) = \frac{P^T_{\mu\nu}}{q^2 - \Pi^T(q) + i\epsilon} + \frac{P^L_{\mu\nu}}{q^2 - \Pi^L(q) + i\epsilon} - \frac{\alpha_{GF}}{q^2 + i\epsilon}\frac{q_\mu q_\nu}{q^2}$$

α_{GF} : gauge projection parameters

$P^T_{\mu\nu}$: Transverse like projection operator

$P^L_{\mu\nu}$: Longitudinal projection operator

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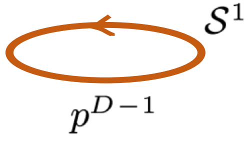

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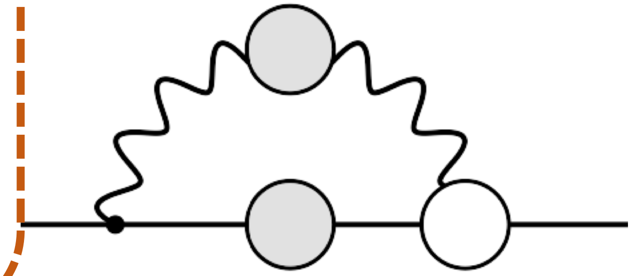

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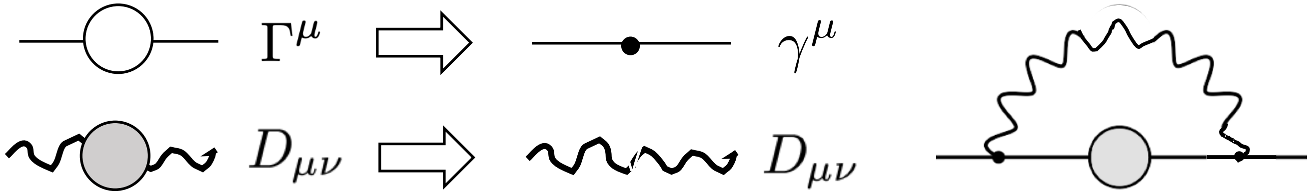

instantaneous exchange approximation

$$q^{D-1}\to 0 \qquad (\Pi^T\sim 0\ ,\Pi^L\sim\frac{Ne^2}{3L^2})$$

Landau gauge $\alpha_{\mathrm{GF}} = 0$

$$D_{\mu\nu}(q)=\frac{P^T_{\mu\nu}}{q^2-\Pi^T(q)+i\epsilon}+\frac{P^L_{\mu\nu}}{q^2-\Pi^L(q)+i\epsilon}-\frac{\alpha_{\mathrm{GF}}}{q^2+i\epsilon}\frac{q_\mu q_\nu}{q^2}$$

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Analytical calculation

Schwinger-Dyson Equation

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$$\int\frac{dq^{D-1}}{2\pi}\rightarrow\frac{1}{L}\sum_{m=-\infty}^{\infty}$$

$$p^{D-1}=\frac{2m\pi}{L}+\frac{\pi\delta}{L}$$

- ✓ imaginary-time formalism
- ✓ IE approximation
- ✓ angle Integral

$$B(p) \;=\; \frac{\alpha}{4\pi}\frac{1}{p}\int_0^\Lambda dq\,\frac{qB}{\sqrt{q^2+B^2}}\left(\frac{\sinh(L\sigma)}{(\cosh(L\sigma)-\cos(\delta\pi))}\right)$$

$$\times\left[2\log\frac{(p+q)^2}{(p-q)^2}+\log\frac{(p+q)^2+\frac{Ne^2}{3L^2}}{(p-q)^2+\frac{Ne^2}{3L^2}}\right]$$

$$\alpha \;=\; \frac{e^2}{4\pi}\quad,\sigma=\sqrt{q^2+B^2}$$

(D=4)

[6] T. Inagaki, Y. Matsuo, and H. Shimoji, Symmetry 11 (2019) no. 4, 451

Analytical calculation

Four-Fermion Interaction Model [6]

$$S = \int d^D x \left[\sum_{a=1}^{N} \bar{\psi}_a i\gamma^\mu \partial_\mu \psi_a + \frac{\lambda_0}{2N}\left(\left(\sum_{a=1}^{N} \bar{\psi}_a \psi_a \right)^2 + \left(\sum_{a=1}^{N} \bar{\psi}_a i\gamma^5 \psi_a \right)^2 \right) \right]$$

- ✓ path integral
- ✓ 1/N expansion

$$\frac{\tilde{V}(\sigma)}{m_a^D} = \frac{\tilde{V}_0(\sigma)}{m_a^D} - \frac{\mathrm{tr}\, I}{(2\sqrt{\pi})^{D-1}\Gamma\left(\frac{D-1}{2}\right)} \frac{1}{L m_a} \times \int_0^\infty \frac{dK}{m_a} \left(\frac{K}{m_a}\right)^{D-2} \ln\left(2\frac{\cosh(L\sqrt{K^2+\sigma^2}) - \cos(\pi\delta)}{\exp(L\sqrt{K^2+\sigma^2})} \right)$$

$$\frac{\partial \tilde{V}(\sigma)}{\partial \sigma}\Big|_{\sigma=m} = 0$$

$$\frac{1}{2\sqrt{\pi}} \Gamma\left(1-\frac{D}{2}\right) \Gamma\left(\frac{D-1}{2}\right) \left[(-1)^a - \left(\frac{m^2}{m_a^2}\right)^{\frac{D}{2}-1} \right] = -\int_0^\infty \frac{dK}{\sqrt{K^2+m^2}} \left(\frac{K}{m_a}\right)^{D-2} \frac{\exp(-L\sqrt{K^2+m^2}) - \cos(\pi\delta)}{\cosh(L\sqrt{K^2+m^2}) - \cos(\pi\delta)}$$

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- ✓ IE approximation
- ✓ angle Integral

$$B(p) = \frac{\alpha}{4\pi} \frac{1}{p} \int_0^\Lambda dq \frac{qB}{\sqrt{q^2+B^2}} \left(\frac{\sinh(L\sigma)}{(\cosh(L\sigma) - \cos(\delta\pi))} \right) \times \left[2\log\frac{(p+q)^2}{(p-q)^2} + \log\frac{(p+q)^2 + \frac{Ne^2}{3L^2}}{(p-q)^2 + \frac{Ne^2}{3L^2}} \right]$$

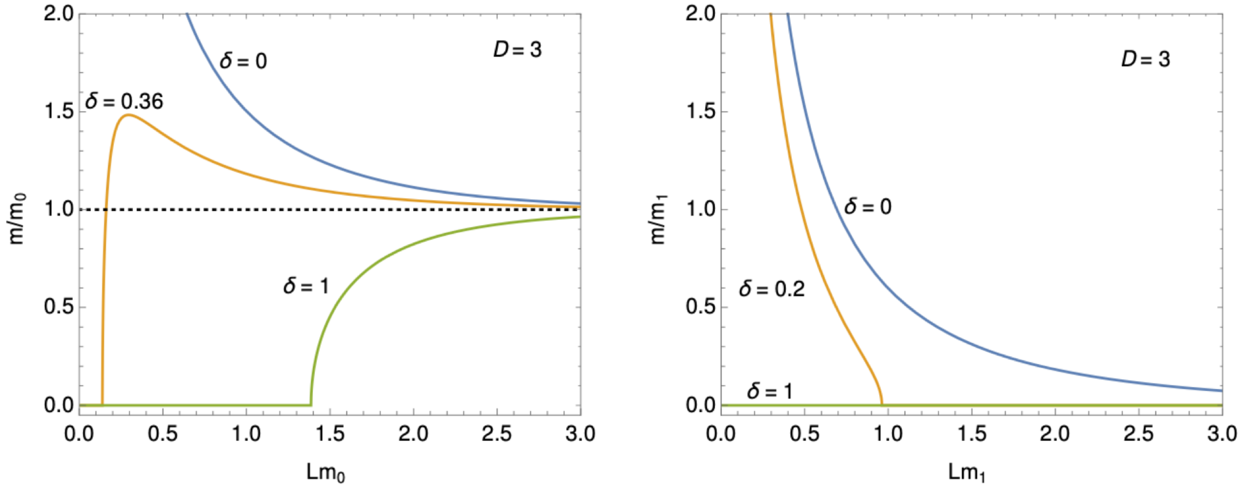
$$\alpha = \frac{e^2}{4\pi}, \quad \sigma = \sqrt{q^2+B^2}$$

(D=4)

[6] T. Inagaki, Y. Matsuo, and H. Shimoji, Symmetry 11 (2019) no. 4, 451

Numerical results 1 < finite size and BC effects >

Four-Fermion Interaction Model

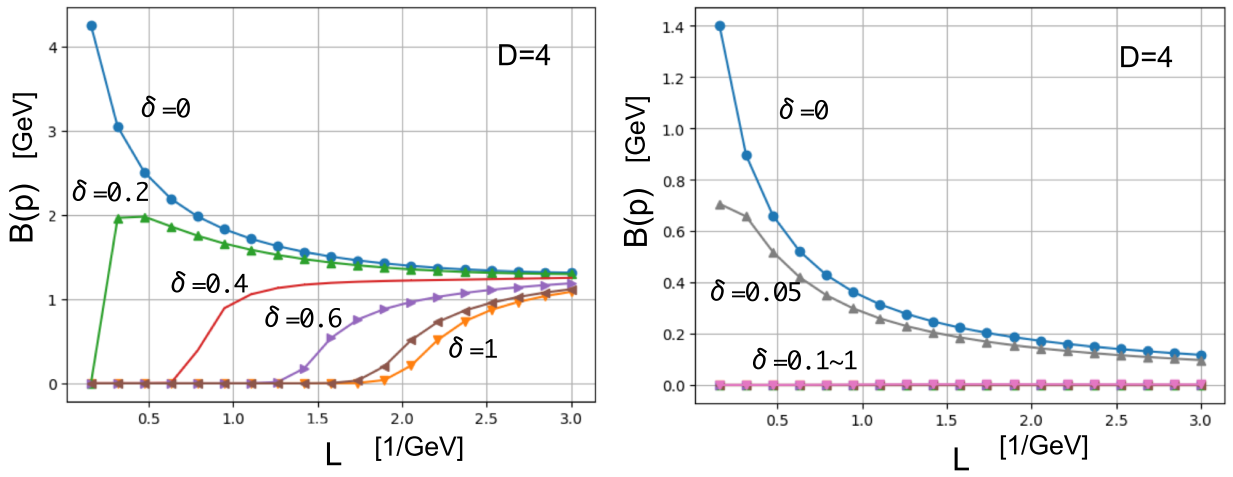


(a) $\lambda_r > \lambda_{cr}$ (b) $\lambda_r < \lambda_{cr}$

[6] Fig2. Dynamically generated fermion mass calculated by FFIM at D=3

- phase transition is second-order
- the finite-size effect restores the broken chiral symmetry at δ = 1 and strengthens the chiral symmetry breaking at δ = 0.

Schwinger-Dyson Equation



(a) $\alpha_r > \alpha_{cr}$ (b) $\alpha_r < \alpha_{cr}$

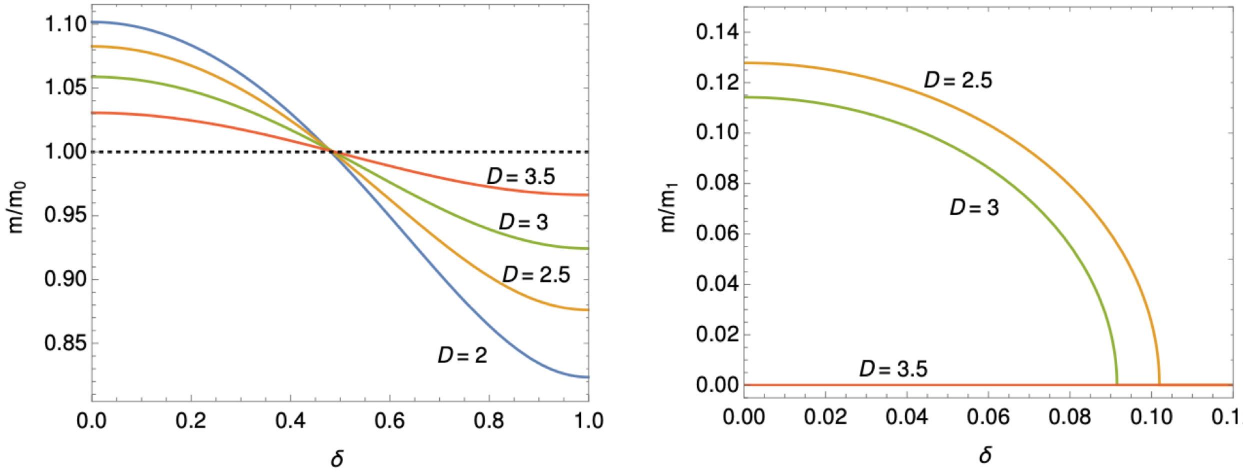
Fig3. Dynamically generated fermion mass calculated by SDE at D=4

- similar behavior at D=3 in FFIM and at D=4 in SDE
(under the effect of finite temperature, FFIM at D=2 and SDE at D=4 are known to behave similarly.[7])
→effect of IE approximation

[6] T. Inagaki, Y. Matsuo, and H. Shimoji, Symmetry 11 (2019) no. 4, 451
[7] D. Bailin, J. Cleymans and M. D. Scadron, Phys. Rev. D 31, 164 (1985). doi:10.1103/PhysRevD.31.164

Numerical results 2 < BC and dimension effects >

Four-Fermion Interaction Model

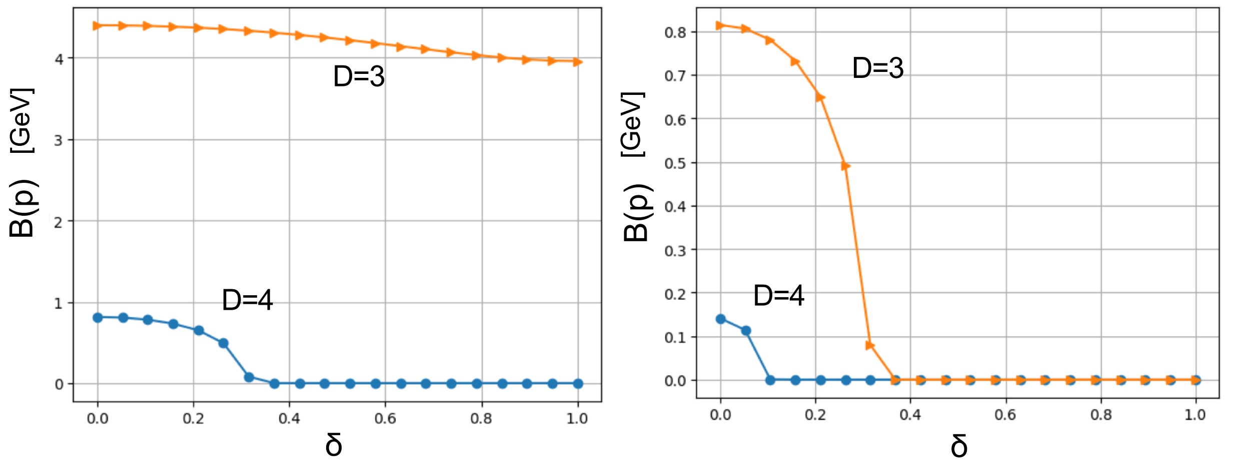


(a) $\lambda_r > \lambda_{cr},\ Lm_0 = 2.5$ (b) $\lambda_r < \lambda_{cr},\ Lm_1 = 2.5$

[6] Fig4. Dynamically generated fermion mass calculated by FFIM

- the generated mass monotonically decreases as δ → 1.
- in the weak coupling case (λr < λcr) the fermion mass disappears above a critical value of δ.

Schwinger-Dyson Equation



(a) $\alpha_r > \alpha_{cr}\quad L = 2.5$ [1/GeV] (b) $\alpha_r < \alpha_{cr}\quad L = 2.5$ [1/GeV]

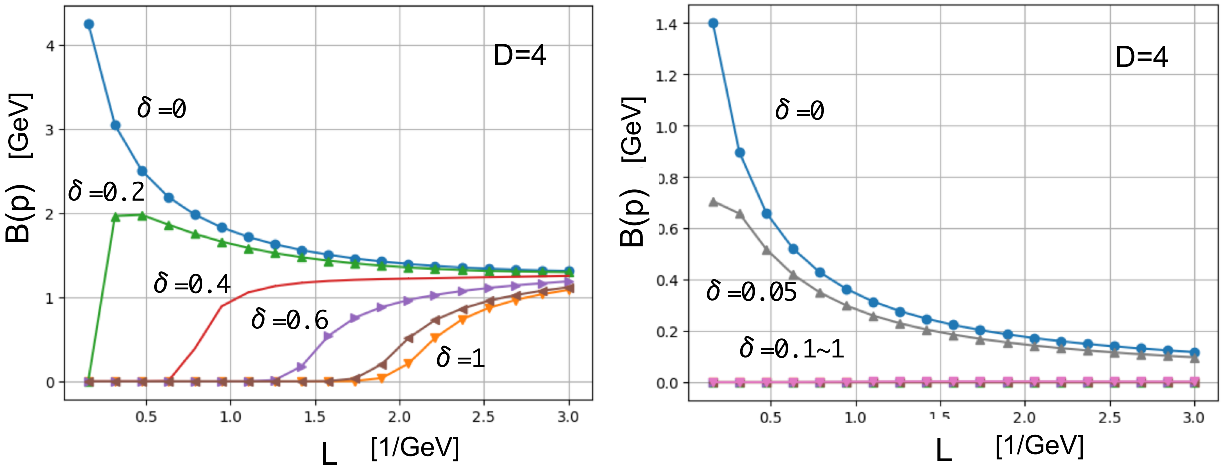
Fig5. Dynamically generated fermion mass calculated by SDE

- in SDE, there is the symmetry breaking phase at D=4
- similar behavior at D=3 in FFIM and at D=3 in SDE for changes in BC

[6] T. Inagaki, Y. Matsuo, and H. Shimoji, Symmetry 11 (2019) no. 4, 451

Numerical results 3 < Debye mass effects >

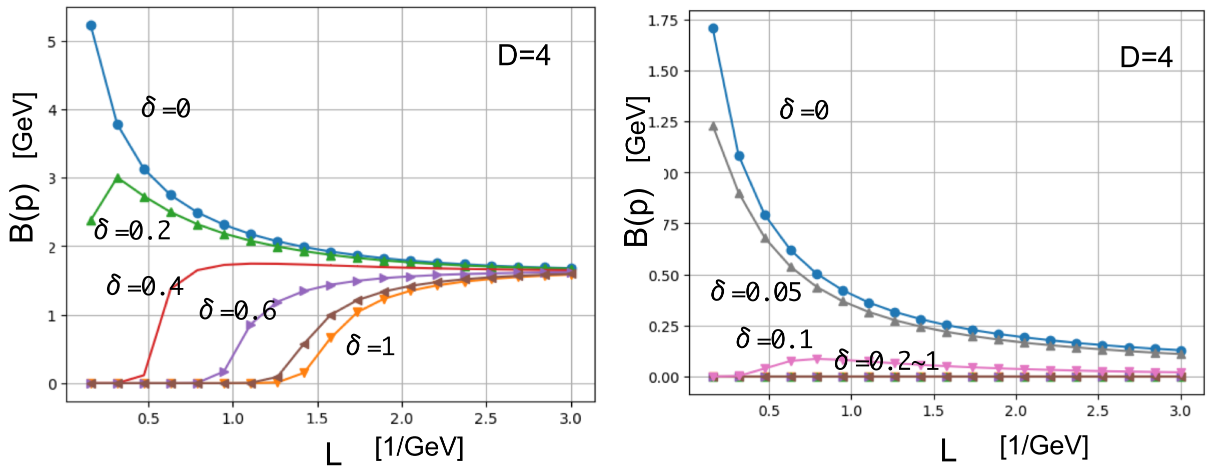
With Debye mass



(a) $\alpha_r > \alpha_{cr}$ (b) $\alpha_r < \alpha_{cr}$

Fig5. Dynamically generated fermion mass calculated by SDE

Without Debye mass



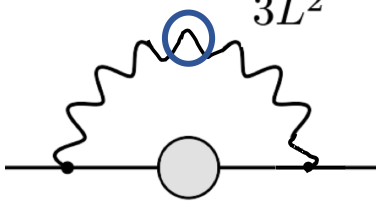
(a) $\alpha_r > \alpha_{cr}$ (b) $\alpha_r < \alpha_{cr}$

Fig6. Dynamically generated fermion mass calculated by SDE

$$D_{\mu\nu}(q) = \frac{P^T_{\mu\nu}}{q^2 - \Pi^T(q) + i\epsilon} + \frac{P^L_{\mu\nu}}{q^2 - \Pi^L(q) + i\epsilon} - \frac{\alpha_{\rm GF}}{q^2 + i\epsilon}\frac{q_\mu q_\nu}{q^2}$$

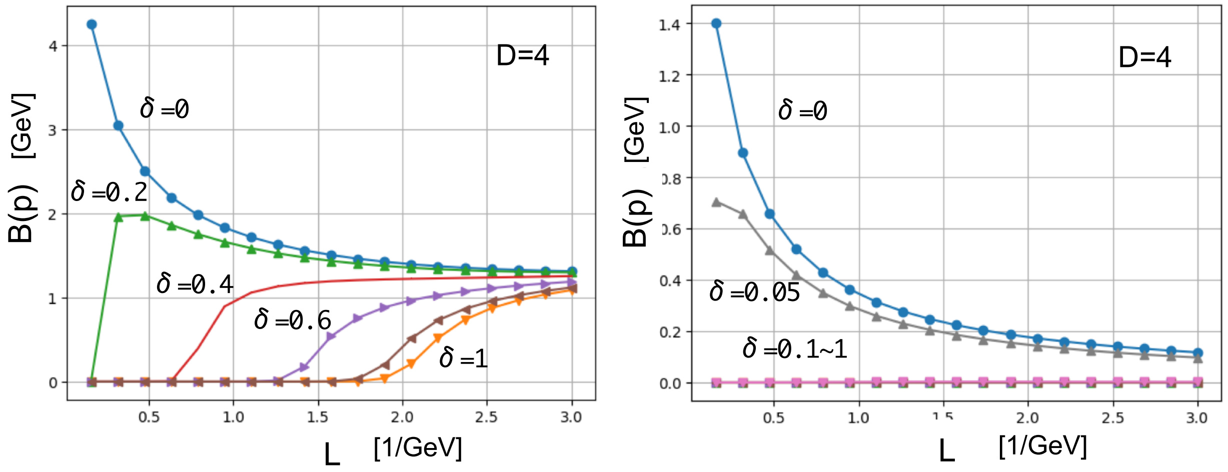
$$\frac{N}{3L^2}e^2$$

$$\frac{N}{3L^2}e^2$$



Numerical results 3 < Debye mass effects >

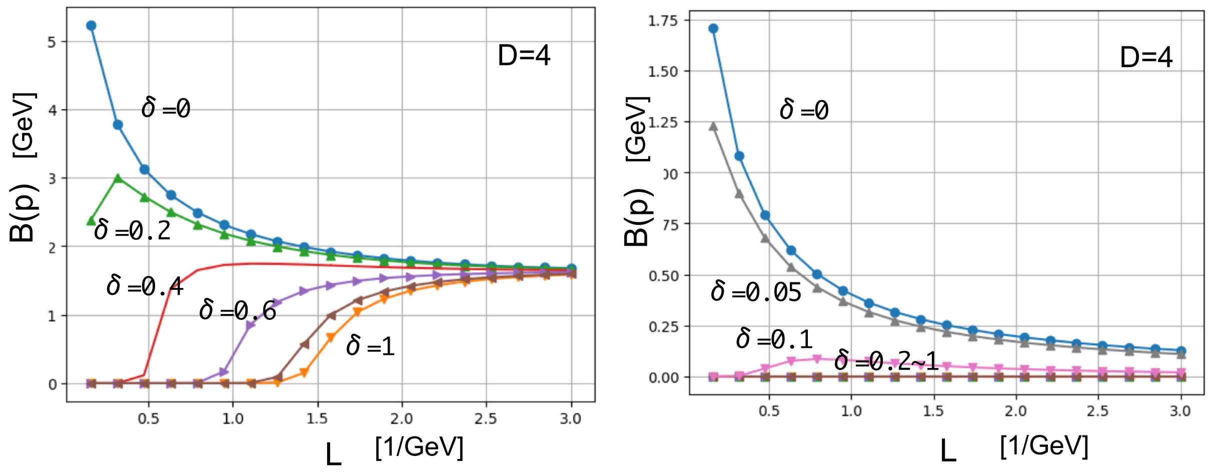
With Debye mass



(a) $\alpha_r > \alpha_{cr}$ (b) $\alpha_r < \alpha_{cr}$

Fig5. Dynamically generated fermion mass calculated by SDE

Without Debye mass



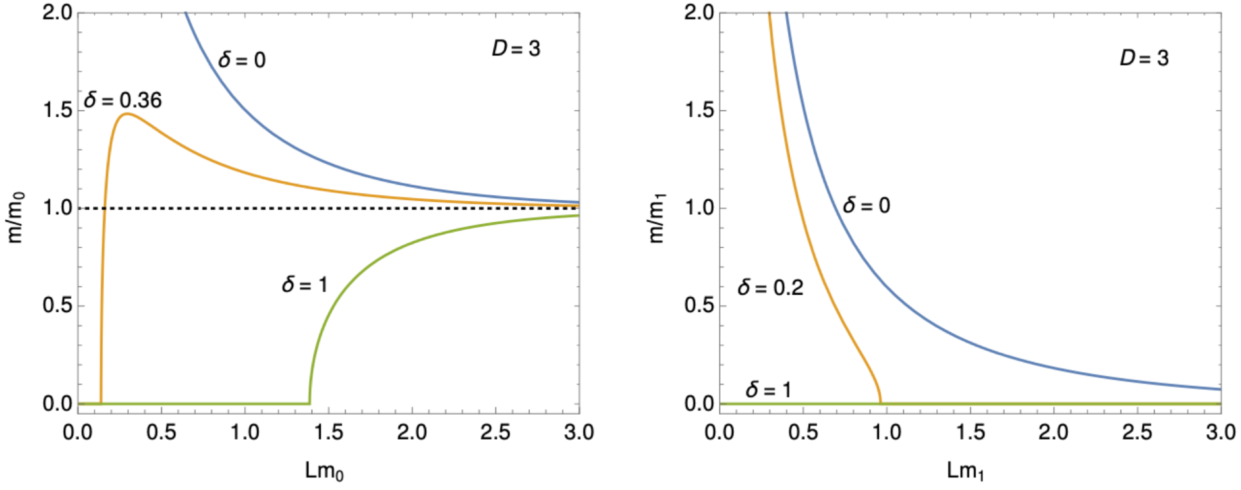
(a) $\alpha_r > \alpha_{cr}$ (b) $\alpha_r < \alpha_{cr}$

Fig6. Dynamically generated fermion mass calculated by SDE

・without Debye mass, the critical value of L is smaller (L=about 1.5).

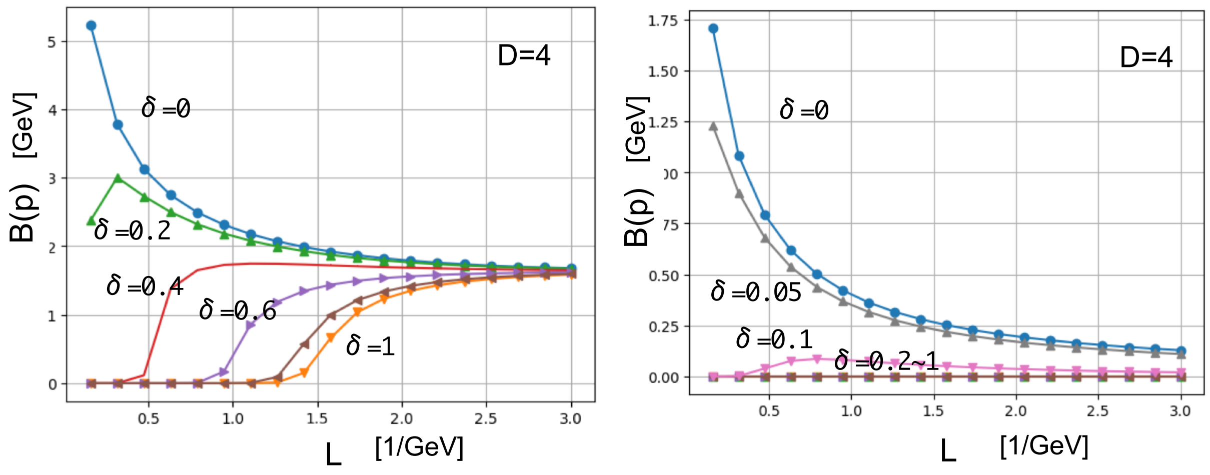
Numerical results 3 < Debye mass effects >

Four-Fermion Interaction Model ⟷ Schwinger-Dyson Equation | Without Debye mass



(a) $\lambda_r > \lambda_{cr}$ (b) $\lambda_r < \lambda_{cr}$

[6] Fig4. Dynamically generated fermion mass calculated by FFIM



(a) $\alpha_r > \alpha_{cr}$ (b) $\alpha_r < \alpha_{cr}$

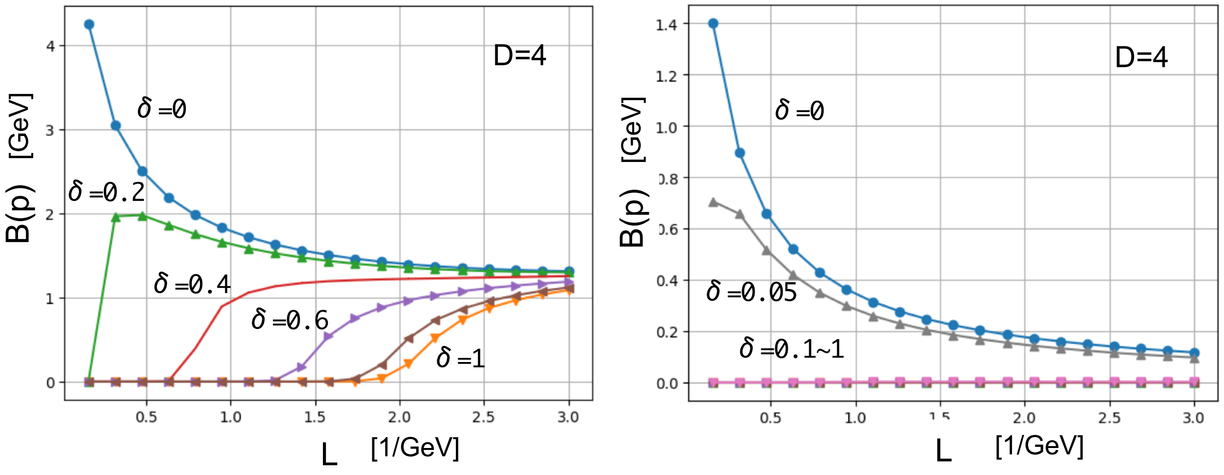
Fig6. Dynamically generated fermion mass calculated by SDE

・without Debye mass, the critical value of L is smaller (L=about 1.5).
→this value is close to the one of L in FFIM.
→ similar behavior at D=3 in FFIM and at D=4 in SDE without Debye mass

[6] T. Inagaki, Y. Matsuo, and H. Shimoji, Symmetry 11 (2019) no. 4, 451

Numerical results 3 < Debye mass effects >

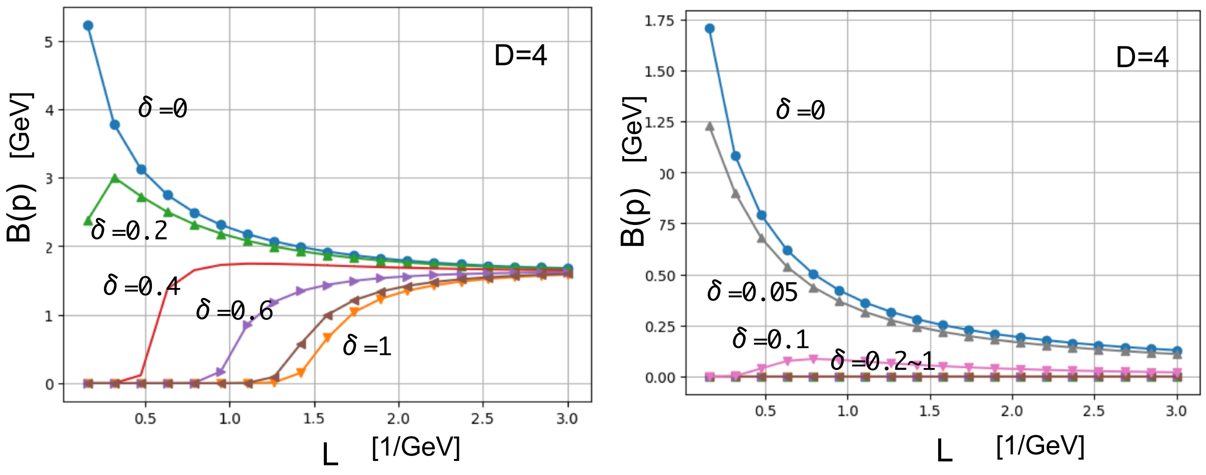
With Debye mass



(a) $\alpha_r > \alpha_{cr}$ (b) $\alpha_r < \alpha_{cr}$

Fig5. Dynamically generated fermion mass calculated by SDE

Without Debye mass

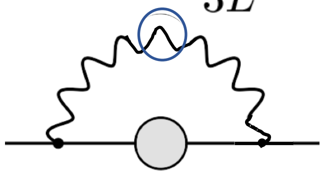


(a) $\alpha_r > \alpha_{cr}$ (b) $\alpha_r < \alpha_{cr}$

Fig6. Dynamically generated fermion mass calculated by SDE

- without Debye mass, the critical value of L is smaller (L=about 1.5).
  →this value is close to the one of L in FFIM.
  → similar behavior at D=3 in FFIM and at D=4 in SDE without Debye mass
- the finite size effect is expected to increase as the number of charged particles is increased.

$$\frac{N}{3L^2}e^2$$



Conclusion and Summary

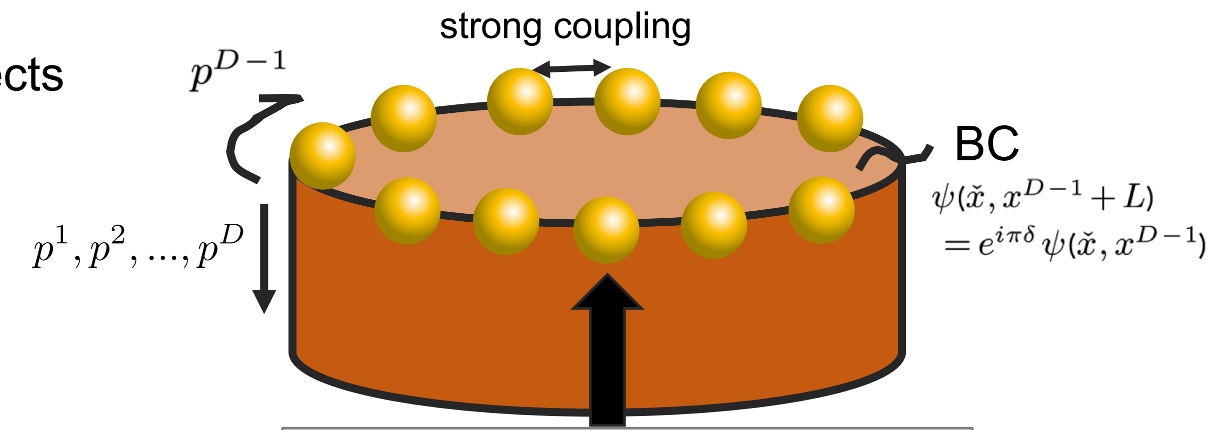
- We have studied spontaneous chiral symmetry breaking in Schwinger-Dyson Equation on torus with the U(1)-valued boundary condition and found the difference from the Four-Fermion Interaction Model.

  - chiral symmetry breaking under finite size and BC effects
    - varies significantly with boundary conditions

      → more easily restored in δ = 1

      → relationship with Aharonov–Bohm phase ？[8]
    - restored in higher dimension (D=4)

      →even higher dimension？



L→ 0 , δ → 1
Dynamically generated mass → 0

  - similar behavior at D=3 in FFIM and at D=4 in SDE for changes in finite size, and similar behavior at D=3 in FFIM and at D=3 in SDE for changes in BC
  - the significant difference between SDE and FFIM lies in the fermion interaction part.

    one of the advantages of SDE is its ability to account for the effects of changing the Debye mass.

[8] R. Yoshii, M. Nitta (2019),” Nambu-Jona Lasinio and Nonlinear Sigma Models in Condensed Matter Systems “