On nonlocal de Sitter gravity and its cosmological solutions

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(joint work with I. Dimitrijević, B. Dragovich, and J. Stanković)

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Generic metric in these spaces is of the form (Friedmann-Robertson-Walker metric (FRW)):

$$ds^{2} = -dt^{2} + a^{2}(t) \left(\frac{dr^{2}}{1 - kr^{2}} + r^{2}d\theta^{2} + r^{2}\sin^{2}\theta d\phi^{2} \right), \ k \in \{-1, 0, 1\},$$
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where *a*(*t*) is a cosmic scale factor which describes the evolution (in time) of Universe and parameter *k* which describes the curvature of the space.

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$$S = \int \left(\frac{R-2\Lambda}{16 \pi G c^4} + \mathcal{L}_m\right) \sqrt{-g} d^4x$$

where *R* is scalar curvature, $g = det(g_{\mu\nu})$ is determinant of metric tensor, Λ is cosmological constant and \mathcal{L}_m is Lagrangian of matter.

The variation of the action S we obtain equations of motion:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad c = 1$$
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where $T_{\mu\nu}$ is the energy momentum tensor, $g_{\mu\nu}$ is metric tensor, $R_{\mu\nu}$ is Ricci tensor and R is scalar curvature.

The energy momentum tensor for ideal fluid (matter in cosmology) is

$$\Gamma = diag(-\rho \, g_{00}, g_{11}\rho, g_{22}\rho, g_{33}\rho), \tag{3}$$

where ρ is energy density and p is pressure.

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General theory of relativity

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- Einstein equation implies Friedmann equations

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 - the precession of Merkur perihelion.
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 - the gravitational lensing.
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Problems related to the Big Bang singularity.

There are two natural approaches:

Dark matter and energy

Modification of Einstein theory of gravity, i.e. modification of its Lagrangian L

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- If Einstein theory of gravity can be applied to the whole Universe then about 5% of ordinary matter, 27% of dark matter and 68% of dark energy.
- It means that 95% of total matter, or energy, represents dark side of the Universe, which nature is unknown.

Motivation for modification of Einstein theory of gravity

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 $S = \int \left(\frac{f(R)}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4x$

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$$S = \int \left(\frac{R-2\Lambda}{16\pi G} + \mathcal{L}_m\right) \sqrt{-g} d^4x$$

we have field equations

$$R_{\mu\nu} - rac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \qquad c = 1.$$

where $T_{\mu\nu}$ is stress-energy tensor, $g_{\mu\nu}$ is the metric tensor, $R_{\mu\nu}$ is Ricci tensor and *R* is scalar curvature.

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ight) \sqrt{-g} \, d^4 x, \qquad \mathcal{G} = R^2 - 4 \, R^{\mu
u} R_{\mu
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nonlocal modified gravity

$$S = \int \Big(rac{F(R,R_{\mu
u},R^{lpha}_{\mueta
u},\Box,...)}{16\pi\,G} + \mathcal{L}_{m}\Big)\sqrt{-g}\;d^{4}x$$

Under nonlocal modification of gravity we understand replacement of the scalar curvature *R* in the Einstein-Hilbert action by a suitable function $F(R, \Box)$, where $\Box = \nabla_{\mu} \nabla^{\mu}$ is d'Alembert operator and ∇_{μ} denotes the covariant derivative

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where $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$ is an analytic function of \Box , and Λ is cosmological constant.

In the case of FRW metric the scalar curvature and d'Alambert operator are given by

$$R = rac{6(a\ddot{a} + \dot{a}^2 + k)}{a^2}, \quad \Box R = -\ddot{R} - 3H\dot{R}, \quad H = rac{\dot{a}}{a}.$$

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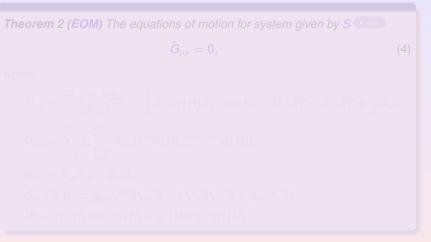
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Equations of motion



Let us note that $abla^{\mu} ilde{G}_{\mu
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EOM are invariant on the replacement of functions \mathcal{G} and \mathcal{H} in S.

Theorem 2 (EOM) The equations of motion for system given by S erection $\tilde{G}_{\mu\nu}=0,$ (4)

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$$\tilde{G}_{\mu\nu} = \mathbf{0},\tag{4}$$

where

$$\begin{split} \tilde{G}_{\mu\nu} &= \frac{G_{\mu\nu} + \Lambda g_{\mu\nu}}{16\pi G} - \frac{1}{2} g_{\mu\nu} \mathcal{H}(R) \mathcal{F}(\Box) \mathcal{G}(R) + R_{\mu\nu} W - K_{\mu\nu} W + \frac{1}{2} \Omega_{\mu\nu}, \\ \Omega_{\mu\nu} &= \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} S_{\mu\nu} \big(\Box^l \mathcal{H}(R), \Box^{n-1-l} \mathcal{G}(R) \big), \\ K_{\mu\nu} &= \nabla_{\mu} \nabla_{\nu} - g_{\mu\nu} \Box, \\ S_{\mu\nu}(A, B) &= g_{\mu\nu} \nabla^{\alpha} A \nabla_{\alpha} B - 2 \nabla_{\mu} A \nabla_{\nu} B + g_{\mu\nu} A \Box B, \\ W &= \mathcal{H}'(R) \mathcal{F}(\Box) \mathcal{G}(R) + \mathcal{G}'(R) \mathcal{F}(\Box) \mathcal{H}(R). \end{split}$$

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$$G_{\mu\nu} + \Lambda g_{\mu\nu} - \frac{g_{\mu\nu}}{2} \mathcal{F}(q) \mathcal{G}^2 + 2\mathcal{F}(q) (R_{\mu\nu} - K_{\mu\nu}) \mathcal{G} \mathcal{G}' \qquad (5)$$
$$+ \frac{1}{2} \mathcal{F}'(q) S_{\mu\nu}(P, P) = 0.$$

If we suppose that the manifold *M* is endowed with FRW metric, then we have just suppose linearly independent equations: trace and 00-equation.

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If we take

- $\mathcal{G}(R) = \mathcal{H}(R)$ and
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Earlier, we considered models of nonlocal gravity without matter which are described by the action,

$$S = \int_{M} \left(\frac{R - 2\Lambda}{16\pi G} + \mathcal{H}(R) \mathcal{F}(\Box) \mathcal{G}(R) \right) \sqrt{-g} \, \mathrm{d}^{4}x,$$

for the following cases:

- 1. $\mathcal{H}(\mathbf{R}) = \mathbf{R}, \mathcal{G}(\mathbf{R}) = \mathbf{R},$
- 2. $\mathcal{H}(R) = R^{-1}, \mathcal{G}(R) = R,$
- 3. $\mathcal{H}(R) = R^{p}, \mathcal{G}(R) = R^{q},$
- 4. $\mathcal{H}(R) = (R + R_0)^m, \, \mathcal{G}(R) = (R + R_0)^m,$

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$$S = \int_{M} \left(\frac{R - 2\Lambda}{16\pi G} + \mathcal{H}(R) \,\mathcal{F}(\Box) \,\mathcal{G}(R) \right) \sqrt{-g} \, \mathrm{d}^{4}x,$$

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- Non-locality, $R^{-1}\mathcal{F}(\Box)R$, is invariant to transfor. $R \longrightarrow cR$, $c \in \mathbb{R}^*$.
- there are cosmological solutions of form $a(t) = a_0|t t_0|^{\alpha}$, in the case k = 0, for $\alpha \neq 0, 1/2$ and $3\alpha \in 1 + 2\mathbb{N}$, in cases $k \neq 0$, for $\alpha = 1$.
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3. model: $\mathcal{H}(R) = R^{p}, \ \mathcal{G}(R) = R^{q}, p \ge q.$

- We considered case with scale factor in the form $a(t) = a_0 \exp(-\frac{\gamma}{12}t^2)$
- For ρ = q = 1 there are infinite number of solutions, and constants γ and Λ satisfy γ = −12Λ.
- In other cases we proved existence of unique solution, for arbitrary
 γ ∈ ℝ. We explicitly found solutions for 1 ≤ q ≤ p ≤ 4.
- **4. model:** $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - We considered scale factor and ansatz of the form

 $a(t) = At^n \exp\left(-\frac{\gamma}{12}t^2\right)$ and $\Box (R+R_0)^m = r(R+R_0)^m$.

- Using this ansatz we obtined the followinf five solutions:
 - $r = m\gamma, \ n = 0, \ R_0 = \gamma, \ m = \frac{1}{2}$
 - $t = m\gamma, \ n = 0, \ B_0 = \frac{2}{3}, \ m = 1$
 - $t = m\gamma, \ n = \frac{1}{2}, \ R_0 = \frac{1}{2}\gamma, \ m = 1$
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 - $\mathbf{n} = \mathbf{n} \gamma, \ \mathbf{n} = \mathbf{0}, \ \mathbf{R}_0 = \gamma, \ \mathbf{m} = \frac{1}{2}$
 - $n = m \gamma_1, n = 0, R_0 = \frac{1}{2}, m = 1$
 - $t = m\gamma, \ n = \frac{1}{2}, \ R_0 = \frac{1}{2}\gamma, \ m = 1$
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 - $t = m\gamma, \ n = \frac{1}{2}, \ R_0 = \frac{1}{2}\gamma, \ m = 1$
 - $f = m\gamma, \eta = \lambda, R_0 = 3\gamma, \eta = 1$

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3. model: $\mathcal{H}(R) = R^{p}$, $\mathcal{G}(R) = R^{q}$, $p \geq q$.

- We considered case with scale factor in the form $a(t) = a_0 \exp(-\frac{\gamma}{12}t^2)$
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In other cases we proved existence of unique solution, for arbitrary
 γ ∈ ℝ. We explicitly found solutions for 1 ≤ q ≤ p ≤ 4.

- **4. model:** $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - We considered scale factor and ansatz of the form

 $a(t) = At^n \exp(-\frac{\gamma}{12}t^2)$ and $\Box (R+R_0)^m = r(R+R_0)^m$.

• Using this ansatz we obtined the followinf five solutions:

- $r = m\gamma, \ n = 0, \ R_0 = \gamma, \ m = \frac{1}{2}$
- $n = m \gamma_1, n = 0, R_0 = \frac{1}{2}, m = 1$
- $t = m\gamma, \ n = \frac{1}{2}, \ R_0 = \frac{1}{2}\gamma, \ m = 1$
- $\mathbf{v} = \mathbf{r} \mathbf{r} \mathbf{v}, \ \mathbf{r} = \frac{1}{2}, \ \mathbf{r} \mathbf{h} = \frac{1}{2}, \ \mathbf{r} \mathbf{h} = \frac{1}{2} \mathbf{v}, \ \mathbf{r} \mathbf{h} = \mathbf{v}$

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- **4. model:** $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
 - We considered scale factor and ansatz of the form
 - $a(t) = At^{n} \exp(-\frac{\gamma}{12}t^{2})$ and $\Box (R + R_{0})^{m} = r(R + R_{0})^{m}$.
 - Using this ansatz we obtined the followinf five solutions:
 - $\bullet \quad T = m\gamma, \ n = 0, \ R_0 = \gamma, \ m = \frac{1}{2}$
 - $a_{1} = m\gamma_{1}, n = 0, R_{0} = \frac{2}{2}, m = 1$
 - $\mathbf{n} = m\gamma, \ n = \frac{1}{2}, \ R_0 = \frac{1}{2}\gamma, \ m = 10$
 - $r = m\gamma, n = \frac{1}{2}, R_0 = 3\gamma, m = \gamma$

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 - $a(t) = At^n \exp\left(-\frac{\gamma}{12}t^2\right)$ and $\Box (R+R_0)^m = r(R+R_0)^m$.
 - Using this ansatz we obtined the followinf five solutions:
 - $\mathbf{e} = T = \mathbf{m} \gamma, \ \mathbf{n} = \mathbf{0}, \ \mathbf{R}_0 = \gamma, \ \mathbf{m} = \frac{1}{2}$
 - $a_{1} = m \gamma_{1}, n = 0, R_{0} = \frac{2}{2}, m = 1$
 - $\mathbf{n} = m\gamma, \ n = \frac{1}{2}, \ R_0 = \frac{1}{2}\gamma, \ m = 10$
 - $r = m\gamma, n = \frac{1}{2}, R_0 = 3\gamma, m = \gamma$

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 We considered scale factor and ansatz of the form
 a(t) = Atⁿ exp(-^γ/₁₂t²) and □(R + R₀)^m = r(R + R₀)^m

 Using this ansatz we obtined the following five solutions:

- $\mathbf{a} = \mathbf{I} = \mathbf{m} \gamma, \ \mathbf{n} = \mathbf{0}, \ \mathbf{R}_{\mathbf{0}} = \gamma, \ \mathbf{m} = \frac{1}{2}$
- $r = m \gamma_1, n = 0, R_0 = \frac{2}{3}, m = 1$
- $\mathbf{a} = t = m\gamma, \ \mathbf{n} = \frac{1}{2}, \ \mathbf{B}_0 = \frac{1}{2}\gamma, \ \mathbf{m} = 1$
- \mathbf{e} $r=m\gamma, n=\frac{1}{2}, P_0=3\gamma, m=1$

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 - We considered case with scale factor in the form $a(t) = a_0 \exp(-\frac{\gamma}{12}t^2)$
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- 4. model: $\mathcal{H}(R) = (R + R_0)^m$, $\mathcal{G}(R) = (R + R_0)^m$.
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• Using this ansatz we obtined the followinf five solutions:

•
$$r = m \gamma, \ n = 0, \ R_0 = \gamma, \ m = \frac{1}{2}$$

•
$$r = m\gamma, \ n = 0, \ R_0 = \frac{\gamma}{3}, \ m = 1$$

•
$$r = m \gamma, \ n = \frac{1}{2}, R_0 = \frac{4}{3} \gamma, \ m = 1$$

•
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•
$$r = m\gamma, \ n = \frac{2m+1}{3}, \ R_0 = \frac{7}{3}\gamma, \ m = \frac{1}{2}$$

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 - $r = m\gamma, \ n = \frac{1}{2}, \ R_0 = 3\gamma, \ m = -\frac{1}{4}$
 - $r = m\gamma, \ n = \frac{2m+1}{3}, \ R_0 = \frac{7}{3}\gamma, \ m = \frac{1}{2}.$

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 $a(t) = At^n \exp(-\frac{\gamma}{12}t^2)$ and $\Box (R+R_0)^m = r(R+R_0)^m$.

• Using this ansatz we obtined the followinf five solutions:

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$$r = m\gamma, \ n = 0, \ R_0 = \gamma, \ m = \frac{1}{2}$$

•
$$r = m\gamma, \ n = 0, \ R_0 = \frac{\gamma}{3}, \ m = 1$$

•
$$r = m\gamma, \ n = \frac{1}{2}, R_0 = \frac{4}{3}\gamma, \ m = 1$$

- $r = m\gamma, \ n = \frac{1}{2}, \ R_0 = 3\gamma, \ m = -\frac{1}{4}$
- $r = m\gamma, \ n = \frac{2m+1}{3}, \ R_0 = \frac{7}{3}\gamma, \ m = \frac{1}{2}.$

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■ Recently, we have considered the nonlocal gravity model with cosmological constant ∧ and without matter, given by

(MS)
$$S = \frac{1}{16\pi G} \int_M \left(R - 2\Lambda + \sqrt{R} - 2\Lambda \mathcal{F}(\Box) \sqrt{R} - 2\Lambda \right) \sqrt{-g} \, \mathrm{d}^4 x,$$

where $\mathcal{F}(\Box) = 1 + \sum_{n=1}^{+\infty} f_n \Box^n + \sum_{n=1}^{+\infty} f_{-n} \Box^{-n}$

It is a **GAUSSION** since the EOM (5), for $GG(R) = \sqrt{R - 2\Lambda}$, is simplified to

$$(G_{\mu\nu} + \Lambda g_{\mu\nu})(1 + \mathcal{F}(q)) + \frac{1}{2}\mathcal{F}'(q)S_{\mu\nu}(\sqrt{R - 2\Lambda}, \sqrt{R - 2\Lambda}) = 0, \quad (6)$$

where we take $q = \zeta \Lambda$.

- It is evident that EOM (6) are satisfied if $\mathcal{F}(q) = -1$ and $\mathcal{F}'(q) = 0$.
- One such nonlocal operator $\mathcal{F}(\Box)$ is

$\mathcal{F}(\Box) = 1 + \sum_{n=1}^{+\infty} \tilde{t}_n \Big[\Big(rac{\Box}{q} \Big)^n + \Big(rac{q}{\Box} \Big)^n \Big] = 1 - rac{1}{2e} \Big(rac{\Box}{q} e^{rac{\Box}{d}} + rac{q}{\Box} e^{rac{B}{d}} \Big), \quad q eq 0.$

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(MS)
$$S = \frac{1}{16\pi G} \int_M \left(R - 2\Lambda + \sqrt{R - 2\Lambda} \mathcal{F}(\Box) \sqrt{R - 2\Lambda} \right) \sqrt{-g} \, \mathrm{d}^4 x,$$

where $\mathcal{F}(\Box) = 1 + \sum_{n=1}^{+\infty} f_n \Box^n + \sum_{n=1}^{+\infty} f_{-n} \Box^{-n}$

It is a very special case since the EOM (5), for $GG(R) = \sqrt{R - 2\Lambda}$, is simplified to

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$$S = \frac{1}{16\pi G} \int_M \left(R - 2\Lambda + \sqrt{R - 2\Lambda} \mathcal{F}(\Box) \sqrt{R - 2\Lambda} \right) \sqrt{-g} \, \mathrm{d}^4 x,$$

where $\mathcal{F}(\Box) = 1 + \sum_{n=1}^{+\infty} f_n \Box^n + \sum_{n=1}^{+\infty} f_{-n} \Box^{-n}$

It is a very special case since the EOM (5), for $GG(R) = \sqrt{R - 2\Lambda}$, is simplified to

$$(G_{\mu\nu} + \Lambda g_{\mu\nu})(1 + \mathcal{F}(q)) + \frac{1}{2}\mathcal{F}'(q)S_{\mu\nu}(\sqrt{R - 2\Lambda}, \sqrt{R - 2\Lambda}) = 0,$$
 (6)

where we take $q = \zeta \Lambda$.

It is evident that EOM (6) are satisfied if $\mathcal{F}(q) = -1$ and $\mathcal{F}'(q) = 0$.

One such nonlocal operator $\mathcal{F}(\Box)$ is

$$\mathcal{F}(\Box) = 1 + \sum_{n=1}^{+\infty} ilde{f}_n \Big[\Big(rac{\Box}{q} \Big)^n + \Big(rac{q}{\Box} \Big)^n \Big] = 1 - rac{1}{2e} \Big(rac{\Box}{q} e^{rac{\Box}{q}} + rac{q}{\Box} e^{rac{q}{\Box}} \Big), \quad q
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1. Cosmological solution in the flat Universe (k = 0)

1.1. Solutions of the form $a(t) = A t^n e^{\gamma t}$

There are two solutions:

 $a(t) = A t^{2} o^{\frac{1}{2}t^{2}}, \qquad \mathcal{F}(-\frac{2}{7}h) = -1, \quad \mathcal{F}(-\frac{2}{7}h) = 0,$ $a(t) = A o^{\frac{1}{2}t^{2}}, \qquad \mathcal{F}(-h) = -1, \quad \mathcal{F}'(-h) = 0.$

Now solutions of the form $a(t) = (\alpha e^{\lambda t} + \beta e^{-\lambda t})^{\gamma}$ for 1/2. Now solutions of the form $a(t) = (\alpha e^{\lambda t} + \beta e^{-\lambda t})^{\gamma}$.

 $\gamma = \frac{2}{2}, \quad \eta = \frac{3}{6}\Lambda, \quad \lambda = \pm \sqrt{\frac{3}{6}\Lambda}.$

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$$a_{1}(t) = A t^{\frac{3}{2}} e^{\frac{A}{4}t^{2}}, \qquad \qquad \mathcal{F}(-\frac{3}{7}\Lambda) = -1, \quad \mathcal{F}'(-\frac{3}{7}\Lambda) = 0,$$

$$a_{2}(t) = A e^{\frac{A}{4}t^{2}}, \qquad \qquad \mathcal{F}(-\Lambda) = -1, \quad \mathcal{F}'(-\Lambda) = 0.$$

1.2. New solutions of the form $a(t) = (\alpha e^{\lambda t} + \beta e^{-\lambda t})^{\gamma}$ In this case for $\alpha\beta \neq 0$, $\beta \neq 2\Lambda$ and $\sigma \neq 0$ we have solution

$$\gamma=rac{2}{3}, \qquad q=rac{3}{8}\Lambda, \qquad \lambda=\pm\sqrt{rac{3}{8}\Lambda}.$$

When $\alpha\beta \neq 0$, we have the following two special solutions:

$$a_{3}(t) = A \cosh^{\frac{2}{3}} \left(\sqrt{\frac{3}{8}} \Lambda t \right), \qquad \qquad \mathcal{F}\left(\frac{3}{8} \Lambda\right) = -1, \ \mathcal{F}'\left(\frac{3}{8} \Lambda\right) = 0,$$
$$a_{4}(t) = A \sinh^{\frac{2}{3}} \left(\sqrt{\frac{3}{8}} \Lambda t \right), \qquad \qquad \mathcal{F}\left(\frac{3}{8} \Lambda\right) = -1, \ \mathcal{F}'\left(\frac{3}{8} \Lambda\right) = 0.$$

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1.2. New solutions of the form a(t) = (α e^{λt} + β e^{-λt})^γ
In this case for αβ ≠ 0, R ≠ 2Λ and q ≠ 0 we have solutions if

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- 1. Cosmological solution in the flat Universe (k = 0)
 - 1.3. New solutions of the form $a(t) = (\alpha \sin \lambda t + \beta \cos \lambda t)^{\gamma}$
 - If For $\alpha \neq 0$ and $\beta \neq 0$ there are only possibility for $\gamma, \gamma = \frac{1}{2}$. Taking $\beta = \pm \alpha$, and $A = \alpha^{\frac{1}{2}}$, we have the following two solutions:



 $h_0(0) = h \cosh\left(\sqrt{-\frac{2}{3}}h(0), \cdots, \sqrt{-\frac{2}{3}}h(0)\right) = -1, \quad \forall e \in [\frac{2}{3}h(0) = 0, \forall e \in [\frac{2}{3}h(0)] = -1, \quad \forall e \in [\frac{2}{3}h(0)] = -1, \quad$

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$$\begin{aligned} \mathbf{a}_{5}(t) &= A\left(1 + \sin\left(2\sqrt{-\frac{3}{8}}\Lambda t\right)\right)^{\frac{1}{3}}, \qquad \qquad \mathcal{F}(\frac{3}{8}\Lambda) = -1, \ \mathcal{F}'(\frac{3}{8}\Lambda) = 0, \\ \mathbf{a}_{6}(t) &= A\left(1 - \sin\left(2\sqrt{-\frac{3}{8}}\Lambda t\right)\right)^{\frac{1}{3}}, \qquad \qquad \mathcal{F}(\frac{3}{8}\Lambda) = -1, \ \mathcal{F}'(\frac{3}{8}\Lambda) = 0. \end{aligned}$$

For $\alpha = 0$ or $\beta = 0$, we have also two cosmological solutions with $\gamma = \frac{2}{3}$:

$$a_{7}(t) = A \sin^{\frac{3}{4}} \left(\sqrt{-\frac{3}{8}} \Lambda t \right), \qquad \qquad \mathcal{F}\left(\frac{3}{8} \Lambda\right) = -1, \quad \mathcal{F}'\left(\frac{3}{8} \Lambda\right) = 0,$$
$$a_{8}(t) = A \cos^{\frac{2}{3}} \left(\sqrt{-\frac{3}{8}} \Lambda t \right), \qquad \qquad \mathcal{F}\left(\frac{3}{8} \Lambda\right) = -1, \quad \mathcal{F}'\left(\frac{3}{8} \Lambda\right) = 0.$$

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$$a_{8}(t) = A \cos^{\frac{2}{3}} \left(\sqrt{-\frac{3}{8}} \Lambda t \right), \qquad \qquad \mathcal{F}\left(\frac{3}{8} \Lambda\right) = -1, \quad \mathcal{F}'\left(\frac{3}{8} \Lambda\right) = 0.$$

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- 1. Cosmological solution in the flat Universe (k = 0)
 - 1.3. New solutions of the form $a(t) = (\alpha \sin \lambda t + \beta \cos \lambda t)^{\gamma}$

For $\alpha \neq 0$ and $\beta \neq 0$ there are only possibility for γ , $\gamma = \frac{2}{3}$. Taking $\beta = \pm \alpha$, and $A = \alpha^{\frac{2}{3}}$, we have the following two solutions:

$$a_{5}(t) = A \left(1 + \sin\left(2\sqrt{-\frac{3}{8}\Lambda t}\right)\right)^{\frac{1}{3}}, \qquad \mathcal{F}\left(\frac{3}{8}\Lambda\right) = -1, \ \mathcal{F}'\left(\frac{3}{8}\Lambda\right) = 0,$$

$$a_{6}(t) = A \left(1 - \sin\left(2\sqrt{-\frac{3}{8}\Lambda t}\right)\right)^{\frac{1}{3}}, \qquad \mathcal{F}\left(\frac{3}{8}\Lambda\right) = -1, \ \mathcal{F}'\left(\frac{3}{8}\Lambda\right) = 0.$$

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- 2. Cosmological solution in the open and closed Universe $(k=\pm 1)$
 - 2.1. Solutions of the form $a(t) = A e^{\pm \sqrt{b}t}$, $(k = \pm 1)$
 - In Eq. (a) $\neq 0, \beta = 0$ or $\alpha = 0, \beta \neq 0$ we have the following solution: $a_{i}(t) = A e^{t \sqrt{2}t}, \qquad k = \pm 1, \qquad \pi (\frac{1}{2}A) = -1, \quad \pi' (\frac{1}{2}A) = 0, \quad A > 0$
 - New solutions of the form $a(t) := (a \cdot a^{\lambda} + b \cdot a^{-\lambda})$ ($b := (b := \pm 1)$
 - For q > 0, y > 0 there are two following cosmological solutions:
 - $$\begin{split} & a_{11}(0) = A \ \text{subs} \left\{ \sqrt{\frac{2}{3}} A \left(0 \right), \quad A = A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) = -A \left(-\frac{2}{3} A \left(-\frac{2}{3} A \right) \right) =$$

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- 2. Cosmological solution in the open and closed Universe $(k = \pm 1)$
 - 2.1. Solutions of the form $a(t) = A e^{\pm \sqrt{6}t}$, $(k = \pm 1)$
 - For $\alpha \neq 0, \beta = 0$ or $\alpha = 0, \beta \neq 0$ we have the following solution: $a_0(t) = A e^{\pm \sqrt{\frac{1}{6}}t}, \qquad k = \pm 1, \quad \mathcal{F}(\frac{1}{2}\Lambda) = -1, \quad \mathcal{F}'(\frac{1}{2}\Lambda) = 0, \quad \Lambda > 0$
 - 2.2. New solutions of the form $a(t) = (\alpha \ e^{\lambda t} + \beta \ e^{-\lambda t})^{\gamma}, \ (k = \pm 1)$
 - For $\alpha \neq 0$, $\beta \neq 0$, $R \neq 2\Lambda$, $q \neq 0$ there are two following cosmological solutions:

$$\begin{aligned} \mathbf{a}_{10}(t) &= A \, \cosh^{\frac{1}{2}} \left(\sqrt{\frac{2}{3}} \Lambda \, t \right), \qquad k = \pm 1, \quad \mathcal{F}\left(\frac{1}{3}\Lambda\right) = -1, \ \mathcal{F}'\left(\frac{1}{3}\Lambda\right) = 0, \\ \mathbf{a}_{11}(t) &= A \, \sinh^{\frac{1}{2}} \left(\sqrt{\frac{2}{3}} \Lambda \, t \right), \qquad k = \pm 1, \quad \mathcal{F}\left(\frac{1}{3}\Lambda\right) = -1, \ \mathcal{F}'\left(\frac{1}{3}\Lambda\right) = 0. \end{aligned}$$

- 2. Cosmological solution in the open and closed Universe ($k = \pm 1$)
 - 2.1. Solutions of the form $a(t) = A \ e^{\pm \sqrt{\frac{\alpha}{6}}t}, \ (k=\pm 1)$

For $\alpha \neq 0, \beta = 0$ or $\alpha = 0, \beta \neq 0$ we have the following solution:

$$a_9(t) = A e^{\pm \sqrt{\frac{\Lambda}{6}}t}, \qquad k = \pm 1, \quad \mathcal{F}(\frac{1}{3}\Lambda) = -1, \quad \mathcal{F}'(\frac{1}{3}\Lambda) = 0, \quad \Lambda > 0.$$

- 2.2. New solutions of the form $a(t) = (\alpha \ e^{\lambda t} + \beta \ e^{-\lambda t})^{\gamma}$, $(k = \pm 1)$
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$$\begin{aligned} \mathbf{a}_{10}(t) &= \mathbf{A} \, \cosh^{\frac{1}{2}}\left(\sqrt{\frac{2}{3}}\Lambda \, t\right), \qquad k = \pm 1, \quad \mathcal{F}(\frac{1}{3}\Lambda) = -1, \ \mathcal{F}'(\frac{1}{3}\Lambda) = 0, \\ \mathbf{a}_{11}(t) &= \mathbf{A} \, \sinh^{\frac{1}{2}}\left(\sqrt{\frac{2}{3}}\Lambda \, t\right), \qquad k = \pm 1, \quad \mathcal{F}(\frac{1}{3}\Lambda) = -1, \ \mathcal{F}'(\frac{1}{3}\Lambda) = 0. \end{aligned}$$

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 - For $\alpha \neq 0$, $\beta \neq 0$, $R \neq 2\Lambda$, $q \neq 0$ there are two following cosmological solutions:

 $a_{10}(t) = A \cosh^{\frac{1}{2}} \left(\sqrt{\frac{2}{3}} \Lambda t \right), \qquad k = \pm 1, \quad \mathcal{F}\left(\frac{1}{3}\Lambda\right) = -1, \ \mathcal{F}'\left(\frac{1}{3}\Lambda\right) = 0,$ $a_{11}(t) = A \sinh^{\frac{1}{2}} \left(\sqrt{\frac{2}{3}} \Lambda t \right), \qquad k = \pm 1, \quad \mathcal{F}\left(\frac{1}{3}\Lambda\right) = -1, \ \mathcal{F}'\left(\frac{1}{3}\Lambda\right) = 0.$

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- 1. Cosmological solution for $a_1(t) = A t^{\frac{5}{3}} e^{\frac{h}{14}t^2}$, k = 0
- The corresponding **Constant and the scalar** 2 curvature are:

$$H_{1}(t) = \frac{\dot{a}_{1}}{a_{1}} = \frac{2}{3}\frac{1}{t} + \frac{1}{7}\Lambda t,$$

$$\ddot{a}_{1}(t) = \left(-\frac{2}{9}\frac{1}{t^{2}} + \frac{1}{3}\Lambda + \frac{1}{49}\Lambda^{2}t^{2}\right)a_{1}(t)$$

$$R_{1}(t) = \frac{4}{3}\frac{1}{t^{2}} + \frac{22}{7}\Lambda + \frac{12}{49}\Lambda^{2}t^{2},$$

Friedman equations gives

$$\bar{\rho}(t) = \frac{2t^{-2} + \frac{9}{98}\Lambda^2 t^2 - \frac{9}{14}\Lambda}{12\pi G}, \quad \bar{\rho}(t) = -\frac{\Lambda}{56\pi G} (\frac{3}{7}\Lambda t^2 - 1), \qquad (7)$$

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Observational data obtained by Planck-2018 for the ACDM model:

 $t_0 = (13.801 \pm 0.024) \times 10^9$ yr – age of the universe,

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We also computed

 $\ddot{a}_1(t_0)/a_1(t_0) = 2.7 \times 10^{-36} s^{-2}$ $R(t_0) = 4.5 \times 10^{-35} s^{-2}$ and consequently $R(t_0) - 2\Lambda = 2.4 \times 10^{-35} s^{-2}$.

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• For $t = t_0$, from previous formula, and from ACDM model we have $\bar{\rho}_1(t_0) = 2.26 \times 10^{-30} \frac{g}{cm^3}$, $\rho(t_0) = \frac{3}{8\pi G} \left(H_0^2 - \frac{\Lambda}{3}\right) = 2.68 \times 10^{-30} \frac{g}{cm^3}$.

 Then, for vacuum energy density of background solution a₁(t) and ACDM model, we have

$$\rho(t_0) - \bar{\rho}_1(t_0) = \frac{\Lambda_1 - \Lambda}{8\pi G} = \rho_{\Lambda_1} - \rho_{\Lambda} = 0.42 \times 10^{-30} \frac{g}{cm^3},$$

• Critical energy density: $\rho_c = \frac{3 H_0^2}{8 \pi G} = 8.51 \times 10^{-30} \frac{g}{cm^3}$

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$$\Omega_{\Lambda_{1}} = \frac{\rho_{\Lambda_{1}}}{\rho_{c}} = 0.734, \quad \Omega_{\Lambda} = \frac{\rho_{\Lambda}}{\rho_{c}} = 0.685, \quad \Delta\Omega_{\Lambda} = \Omega_{\Lambda_{1}} - \Omega_{\Lambda} = 0.049, \quad (9)$$
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• Then, for vacuum energy density of background solution $a_1(t)$ and Λ CDM model, we have

$$\rho(t_0) - \bar{\rho}_1(t_0) = \frac{\Lambda_1 - \Lambda}{8\pi G} = \rho_{\Lambda_1} - \rho_{\Lambda} = 0.42 \times 10^{-30} \frac{g}{cm^3},$$

• Critical energy density:
$$\rho_c = \frac{3 H_0^2}{8 \pi G} = 8.51 \times 10^{-30} \frac{g}{cm^3}$$

and consequently,

$$\Omega_{\Lambda_{1}} = \frac{\rho_{\Lambda_{1}}}{\rho_{c}} = 0.734, \quad \Omega_{\Lambda} = \frac{\rho_{\Lambda}}{\rho_{c}} = 0.685, \quad \Delta\Omega_{\Lambda} = \Omega_{\Lambda_{1}} - \Omega_{\Lambda} = 0.049, \quad (9)$$

$$\Omega_{m_{1}} = \frac{\bar{\rho}_{1}(t_{0})}{\rho_{c}} = 0.266, \quad \Omega_{m} = \frac{\rho(t_{0})}{\rho_{c}} = 0.315, \quad \Delta\Omega_{m} = \Omega_{m} - \Omega_{m_{1}} = 0.049. \quad (10)$$

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- Effective presure. At the beginning, $\bar{p}_1(0) = \frac{N_1}{56\pi d} > 0$, then decreases and equals zero at $t = \sqrt{\frac{7}{3N_1}} = 4.71 \times 10^{17} s = 14,917 \times 10^9 yr$.

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Note that the Hubble parameter has minimum at $t_{min} = 21.1 \times 10^9 yr$ and it is $H_1(t_{min}) = 61.72 km/s/Mpc$. It also, follows that the Universe experiences decelerated expansion during matter dominance, that turns to acceleration at time $t_{acc} = 7.84 \times 10^9 yr$ when, $\ddot{a} = 0$.

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THANK YOU FOR

YOUR ATTENTION !!!

Zoran Rakić On nonlocal de Sitter gravity and its cosmological solutions

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Non-trivial Christoffel symbols of Friedman – Roberton – Walker metric

$$\Gamma_{01}^{1} = \frac{\dot{a}}{a} \qquad \Gamma_{02}^{2} = \frac{\dot{a}}{a} \qquad \Gamma_{03}^{3} = \frac{\dot{a}}{a}$$

$$\Gamma_{11}^{0} = \frac{a\dot{a}}{1 - kr^{2}} \qquad \Gamma_{11}^{1} = \frac{kr}{1 - kr^{2}} \qquad \Gamma_{12}^{2} = \frac{1}{r}$$

$$\Gamma_{13}^{3} = \frac{1}{r}$$

$$\Gamma_{22}^{0} = r^{2}a\dot{a} \qquad \Gamma_{22}^{1} = r(kr^{2} - 1) \qquad \Gamma_{23}^{3} = \cot\theta$$

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Ricci tensor

$$R_{\mu\nu} = \begin{pmatrix} -\frac{3a}{a} & 0 & 0 & 0\\ 0 & u g_{11} & 0 & 0\\ 0 & 0 & u g_{22} & 0\\ 0 & 0 & 0 & u g_{33} \end{pmatrix}, \qquad u = \frac{a\ddot{a} + 2(\dot{a}^2 + k)}{a^2}$$

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$$R_{0110} = \frac{\ddot{a}\ddot{a}}{1 - k r^2} \qquad R_{1221} = -\frac{r^2 a^2 (\ddot{a}^2 + k)}{1 - k r^2}$$
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Einstein tensor

$$G_{\mu\nu} = \begin{pmatrix} \frac{3(\dot{a}^2 + k)}{a^2} & 0 & 0 & 0\\ 0 & -v g_{11} & 0 & 0\\ 0 & 0 & -v g_{22} & 0\\ 0 & 0 & 0 & -v g_{33} \end{pmatrix}, \qquad v = \frac{2 \, a \ddot{a} + \dot{a}^2 + k}{a^2}$$

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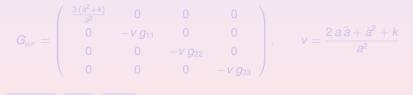
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