On nonlocal de Sitter gravity and its cosmological solutions

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(joint work with I. Dimitrijević, B. Dragovich, and J. Stanković)

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- GTR or ETG assumes that Universe is four dimensional homogeneous isotropic simple connected pseudo-Riemannian manifold M with metric $(g_{\mu\nu})$ of signature $(1,3)$.

- Generic metric in these spaces is of the form (Friedmann-Robertson-Walker metric (FRW)):

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right), \quad k \in \{-1, 0, 1\}, \quad (1)$$

where $a(t)$ is a cosmic scale factor which describes the evolution (in time) of Universe and parameter k which describes the curvature of the space.

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$$S = \int \left(\frac{R - 2\Lambda}{16\,\pi\,G\,c^4} + \mathcal{L}_m \right) \sqrt{-g}\; d^4x$$

where R is scalar curvature, $g = \det(g_{\mu\nu})$ is determinant of metric tensor, Λ is cosmological constant and $\mathcal{L}_m$ is Lagrangian of matter.

- The variation of the action S we obtain equations of motion:

$$R_{\mu\nu} - \frac{1}{2}\,R\,g_{\mu\nu} + \Lambda\,g_{\mu\nu} = 8\pi\,G\,T_{\mu\nu}, \quad c = 1 \tag{2}$$

where $T_{\mu\nu}$ is the energy momentum tensor, $g_{\mu\nu}$ is metric tensor, $R_{\mu\nu}$ is Ricci tensor and R is scalar curvature.

- The energy momentum tensor for ideal fluid (matter in cosmology) is

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  high orbital speeds of galaxies in clusters and stars in spiral galaxies,
  accelerated expansion of the Universe
  showed that they could not be explained by GTR without additional matter
- Problems related to the Big Bang singularity

There are two natural approaches:

- Dark matter and energy
- Modification of Einstein theory of gravity, i.e. modification of its Lagrangian $\mathcal{L}$

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Dark matter and energy

- Dark matter is responsible for orbital speeds in galaxies, and dark energy is responsible for accelerated expansion of the Universe.
- If Einstein theory of gravity can be applied to the whole Universe then the Universe contains about 5% of ordinary matter, 27% of dark matter and 68% of dark energy.
- It means that 95% of total matter, or energy, represents dark side of the Universe, which nature is unknown.

Motivation for modification of Einstein theory of gravity

- The validity of General Relativity on cosmological scale is not confirmed.
- Dark matter and dark energy are not yet detected in the laboratory experiments.

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Different approaches to modification of Einstein theory of gravity

From action

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$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \qquad c = 1.$$

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Different approaches to modification of Einstein theory of gravity

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- Under nonlocal modification of gravity we understand replacement of the scalar curvature R in the Einstein-Hilbert action by a suitable function $F(R, \Box)$, where $\Box = \nabla_\mu \nabla^\mu$ is d'Alembert operator and ∇_μ denotes the covariant derivative

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- Let M be a four-dimensional pseudo-Riemannian manifold with metric $(g_{\mu\nu})$ of signature (1,3). We consider a class of nonlocal gravity models without matter, given by the following action

$$S = \int_M \Big(\frac{R - 2\Lambda}{16\pi G} + \mathcal{H}(R)\,\mathcal{F}(\Box)\,\mathcal{G}(R) \Big) \sqrt{-g}\, d^4x,$$

where $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$ is an analytic function of $\Box$, and Λ is cosmological constant.

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Theorem 2 (EOM) *The equations of motion for system given by S are:*

$$\tilde{G}_{\mu\nu} = 0, \tag{4}$$

where

$$\tilde{G}_{\mu\nu} = \frac{G_{\mu\nu} + \Lambda g_{\mu\nu}}{16\pi G} - \frac{1}{2} g_{\mu\nu} \mathcal{H}(R)\mathcal{F}(\Box)\mathcal{G}(R) + R_{\mu\nu} W - K_{\mu\nu} W + \frac{1}{2}\Omega_{\mu\nu},$$

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for the following cases:

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for the following cases:

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- If $R = R_0 > 0$, then there exist non-singlar solutions for all three values of parameter $k = 0, \pm 1$, which are bounced in the cases $k = 0, 1$.
- If $R = R_0 = 0$ then exists Milne's solution $a(t) = |t + \frac{\sigma}{2}|$.
- If $R = R_0 < 0$, then there exists non-trivial singular cyclic solution $a(t) = \sqrt{\frac{-12}{R_0}} |\cos \frac{1}{2}(\sqrt{-\frac{R_0}{3}} t - \varphi)|$ za $k = -1$.
- Case $R_0 = 0$ is considered as an limit case when $R_0 \to 0$, and in both cases $R_0 < 0$ and $R_0 > 0$, we obtain Minkowski space.

- Recently, we have considered the nonlocal gravity model with cosmological constant Λ and without matter, given by

$$\text{(MS)} \qquad S = \frac{1}{16\pi G} \int_M \left(R - 2\Lambda + \sqrt{R - 2\Lambda}\, \mathcal{F}(\Box) \sqrt{R - 2\Lambda} \right) \sqrt{-g}\, d^4x,$$

where $\mathcal{F}(\Box) = 1 + \sum_{n=1}^{+\infty} f_n \Box^n + \sum_{n=1}^{+\infty} f_{-n} \Box^{-n}$

- It is a [illegible] since the EOM (5), for $GG(R) = \sqrt{R - 2\Lambda}$, is simplified to

$$(G_{\mu\nu} + \Lambda g_{\mu\nu})(1 + \mathcal{F}(q)) + \frac{1}{2}\mathcal{F}'(q) S_{\mu\nu}(\sqrt{R - 2\Lambda}, \sqrt{R - 2\Lambda}) = 0, \quad (6)$$

where we take $q = \zeta\Lambda$.

- It is evident that EOM (6) are satisfied if $\mathcal{F}(q) = -1$ and $\mathcal{F}'(q) = 0$.
- One such nonlocal operator $\mathcal{F}(\Box)$ is

$$\mathcal{F}(\Box) = 1 + \sum_{n=1}^{+\infty} \tilde{f}_n \left[\left(\frac{\Box}{q}\right)^n + \left(\frac{q}{\Box}\right)^n \right] = 1 - \frac{1}{2e}\left(\frac{\Box}{q} e^{\frac{\Box}{q}} + \frac{q}{\Box} e^{\frac{q}{\Box}} \right), \quad q \neq 0.$$

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1. Cosmological solution in the flat Universe $(k = 0)$

1.1. Solutions of the form $a(t) = A\, t^{\alpha} e^{\beta t^2}$

- There are two solutions

1.2. New solutions of the form $a(t) = (\alpha e^{\lambda t} + \beta e^{-\lambda t})^{\gamma}$

- In this case for $\alpha\beta \neq 0$, $R \neq 2\Lambda$ and $q \neq 0$ we have solutions if

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$$a_3(t) = A \cosh^{\frac{2}{3}}\Big(\sqrt{\frac{3}{8}\Lambda}\, t\Big), \qquad \mathcal{F}\Big(\frac{3}{8}\Lambda\Big) = -1,\ \mathcal{F}'\Big(\frac{3}{8}\Lambda\Big) = 0,$$

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1. **Cosmological solution in the flat Universe** $(k = 0)$

- For $\alpha = 0$ or $\beta = 0$, we have also two cosmological solutions with $\gamma = \frac{2}{3}$:

$$a_7(t) = A \sin^{\frac{2}{3}} \Big(\sqrt{-\frac{3}{8}\Lambda}\, t\Big), \qquad \mathcal{F}\Big(\frac{3}{8}\Lambda\Big) = -1, \quad \mathcal{F}'\Big(\frac{3}{8}\Lambda\Big) = 0,$$

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   1.3. New solutions of the form $a(t) = (\alpha \ \sin \lambda t + \beta \ \cos \lambda t)^{\gamma}$

   - For $\alpha \neq 0$ and $\beta \neq 0$ there are only possibility for γ, $\gamma = \frac{2}{3}$. Taking $\beta = \pm\alpha$, and $A = \alpha^{\frac{2}{3}}$, we have the following two solutions:

$$a_5(t) = A\left(1 + \sin\left(2\sqrt{-\frac{3}{8}\Lambda\, t}\right)\right)^{\frac{1}{3}}, \qquad \mathcal{F}(\frac{3}{8}\Lambda) = -1, \ \ \mathcal{F}'(\frac{3}{8}\Lambda) = 0,$$

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2. Cosmological solution in the open and closed Universe $(k = \pm 1)$

2.2 New solutions of the form $a(t) = (\alpha e^{\lambda t} + \beta e^{-\lambda t})^{\gamma}$ $(k = \pm 1)$

- For $\alpha \neq 0$, $\beta \neq 0$, $R \neq 2\Lambda$, $\gamma \neq 0$ there are two following cosmological solutions:

$$a_{10}(t) = A \cosh^{\frac{1}{2}}\Big(\sqrt{\frac{2}{3}\Lambda}\, t\Big), \qquad k = \pm 1, \quad \mathcal{F}\big(\frac{1}{3}\Lambda\big) = -1, \ \mathcal{F}'\big(\frac{1}{3}\Lambda\big) = 0,$$

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2. **Cosmological solution in the open and closed Universe** $(k = \pm 1)$

2.1. Solutions of the form $a(t) = A\, e^{\pm\sqrt{\frac{\Lambda}{6}}t}$, $(k = \pm 1)$

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- 1. Cosmological solution for $a_1(t) = A\, t^{\frac{2}{3}}\, e^{\frac{\Lambda}{14}t^2}$, $k = 0$

- The corresponding Hubble parameter, acceleration and the scalar
2 curvature are:

$$H_1(t) = \frac{\dot{a}_1}{a_1} = \frac{2}{3}\frac{1}{t} + \frac{1}{7}\Lambda t,$$

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- Friedman equations gives

$$\bar{\rho}(t) = \frac{2t^{-2} + \frac{9}{98}\Lambda^2 t^2 - \frac{9}{14}\Lambda}{12\pi G}, \quad \bar{p}(t) = -\frac{\Lambda}{56\pi G}\Big(\frac{3}{7}\Lambda t^2 - 1\Big), \qquad (7)$$

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- The expressions (7) implies that $\bar{w}(t) \to -1$ when $t \to \infty$, what corresponds to an analog of Λ dark energy dominance in the standard cosmological model.
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- Let us recall, the second Friedman equation

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}, \qquad (8)$$

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- Then we can rewrite the previous equation as,

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- Observational data obtained by Planck-2018 for the ΛCDM model:

$t_0 = (13.801 \pm 0.024) \times 10^9$yr – age of the universe,

$H(t_0) = (67.40 \pm 0.50)$ km/s/Mpc – Hubble parameter,

$\Omega_m = 0.315 \pm 0.007$– matter density parameter,

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$$\frac{H^2}{H_0^2} = \frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \frac{\Omega_k}{a^2} + \Omega_\Lambda$$

- Observational data obtained by Planck-2018 for the ΛCDM model:

$t_0 = (13.801 \pm 0.024) \times 10^9$yr – age of the universe,

$H(t_0) = (67.40 \pm 0.50)$ km/s/Mpc – Hubble parameter,

$\Omega_m = 0.315 \pm 0.007$– matter density parameter,

$\Omega_\Lambda = 0.685 - \Lambda$ density parameter,

$w_0 = -1.03 \pm 0.03$– ratio of pressure to energy density.

- Then we can rewrite the previous equation as,

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho_r + \frac{8\pi G}{3}\rho_m - \frac{k}{a^2} + \frac{\Lambda}{3}$$
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- From,

$$H_1(t) = \frac{\dot{a}_1}{a_1} = \frac{2}{3}\frac{1}{t} + \frac{1}{7}\Lambda t,$$

  taking $H_1(t_0) = H(t_0)$ we calculate $\Lambda_1 = 1.05 \times 10^{-35}\mathrm{s}^{-2}$ that differs from $\Lambda = 3H^2(t_0)\,\Omega_\Lambda = 0.98 \times 10^{-35}\mathrm{s}^{-2}$ (by ΛCDM model).

- We also computed

$$\ddot{a}_1(t_0)/a_1(t_0) = 2.7 \times 10^{-36} s^{-2}$$

$$R(t_0) = 4.5 \times 10^{-35} s^{-2} \quad \text{and consequently}$$

$$R(t_0) - 2\Lambda = 2.4 \times 10^{-35} s^{-2}.$$

- Replacing solution $a_1(t)$ with $k = 0$, Friedman equations give

$$\bar{\rho}_1(t) = \frac{3}{8\pi G}\Big(H_1^2(t) - \frac{\Lambda_1}{3}\Big) = \frac{3}{8\pi G}\Big(\frac{4}{9}t^{-2} - \frac{1}{7}\Lambda_1 + \frac{1}{49}\Lambda_1^2 t^2\Big),$$

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- For $t = t_0$, from previous formula, and from ΛCDM model we have

$$\bar{\rho}_1(t_0) = 2.26 \times 10^{-30} \ \frac{g}{cm^3},$$

$$\rho(t_0) = \frac{3}{8\pi G}\left(H_0^2 - \frac{\Lambda}{3}\right) = 2.68 \times 10^{-30} \ \frac{g}{cm^3}.$$

- Then, for vacuum energy density of background solution $a_1(t)$ and ΛCDM model, we have

$$\rho(t_0) - \bar{\rho}_1(t_0) = \frac{\Lambda_1 - \Lambda}{8\pi G} = \rho_{\Lambda_1} - \rho_\Lambda = 0.42 \times 10^{-30} \ \frac{g}{cm^3},$$

- Critical energy density: $\rho_c = \dfrac{3\,H_0^2}{8\,\pi\,G} = 8.51 \times 10^{-30} \ \dfrac{g}{cm^3}$
- and consequently,

$$\Omega_{\Lambda_1} = \frac{\rho_{\Lambda_1}}{\rho_c} = 0.734, \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} = 0.685, \quad \Delta\Omega_\Lambda = \Omega_{\Lambda_1} - \Omega_\Lambda = 0.049, \tag{9}$$

$$\Omega_{m_1} = \frac{\bar{\rho}_1(t_0)}{\rho_c} = 0.266,\ \Omega_m = \frac{\rho(t_0)}{\rho_c} = 0.315,\ \Delta\Omega_m = \Omega_m - \Omega_{m_1} = 0.049. \tag{10}$$

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- According to (9) and (10), we obtain that $\Omega_{m_1} = 26,6\%$ corresponds to dark matter and $\Delta\Omega_m = \Delta\Omega_\Lambda = 4.9\%$ is related to visible matter, what is in a very good agreement with the standard model of cosmology.
- Efective presure. At the beginning, $\bar{p}_1(0) = \frac{\Lambda_1}{56\pi G} > 0$, then decreases and equals zero at $t = \sqrt{\frac{7}{3\Lambda_1}} = 4.71 \times 10^{17}\, s = 14,917 \times 10^9 yr$.
- According to (7), we have parameter $\bar{w}_1(t) = \frac{\bar{p}_1(t)}{\bar{\rho}_1(t)}$ which has future behavior in agreement with standard model of cosmology, i.e. $\bar{w}_1(t \to \infty) \to -1$.
- Note that the Hubble parameter has minimum at $t_{min} = 21.1 \times 10^9 yr$ and it is $H_1(t_{min}) = 61.72 km/s/Mpc$. It also, follows that the Universe experiences decelerated expansion during matter dominance, that turns to acceleration at time $t_{acc} = 7.84 \times 10^9 yr$ when, $\ddot{a} = 0$.

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THANK YOU FOR

YOUR ATTENTION !!!

Non-trivial Christoffel symbols of Friedman – Roberton – Walker metric

$$\Gamma^1_{01} = \frac{\dot a}{a} \qquad \Gamma^2_{02} = \frac{\dot a}{a} \qquad \Gamma^3_{03} = \frac{\dot a}{a}$$

$$\Gamma^0_{11} = \frac{a\,\dot a}{1 - k\,r^2} \qquad \Gamma^1_{11} = \frac{k\,r}{1 - k\,r^2} \qquad \Gamma^2_{12} = \frac{1}{r}$$

$$\Gamma^3_{13} = \frac{1}{r}$$

$$\Gamma^0_{22} = r^2\,a\,\dot a \qquad \Gamma^1_{22} = r\,(k\,r^2 - 1) \qquad \Gamma^3_{23} = \cot\theta$$

$$\Gamma^0_{33} = r^2\,a\,\dot a\,\sin^2\theta \qquad \Gamma^1_{33} = r\,(k\,r^2 - 1)\sin^2\theta \qquad \Gamma^2_{33} = -\sin\theta\,\cos\theta$$

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Non-trivial components of curvature tensor

$$R_{0110} = \frac{a\,\ddot{a}}{1 - k\,r^2} \qquad R_{1221} = -\frac{r^2\,a^2\,(\dot{a}^2 + k)}{1 - k\,r^2}$$

$$R_{0220} = r^2\,a\,\ddot{a} \qquad R_{1331} = -\frac{r^2\,a^2\,\sin^2\theta\,(\dot{a}^2 + k)}{1 - k\,r^2}$$

$$R_{0330} = r^2\,a\,\ddot{a}\,\sin^2\theta \qquad R_{2332} = -r^4\,a^2\,\sin^2\theta\,(\dot{a}^2 + k)$$

Ricci tensor

$$R_{\mu\nu} = \begin{pmatrix} -\frac{3\ddot{a}}{a} & 0 & 0 & 0 \\ 0 & u\,g_{11} & 0 & 0 \\ 0 & 0 & u\,g_{22} & 0 \\ 0 & 0 & 0 & u\,g_{33} \end{pmatrix}, \qquad u = \frac{a\,\ddot{a} + 2\,(\dot{a}^2 + k)}{a^2}$$

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Scalar curvature

$$R = \frac{6\,(a\,\ddot{a} + \dot{a}^2 + k)}{a^2}$$

Einstein tensor

$$G_{\mu\nu} = \begin{pmatrix} \frac{3\,(\dot{a}^2 + k)}{a^2} & 0 & 0 & 0 \\ 0 & -v\,g_{11} & 0 & 0 \\ 0 & 0 & -v\,g_{22} & 0 \\ 0 & 0 & 0 & -v\,g_{33} \end{pmatrix}, \qquad v = \frac{2\,a\,\ddot{a} + \dot{a}^2 + k}{a^2}$$

FRW metric EOM EOM-2

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