Quantum aspects of gravitational collapse: non-singularity and non-locality

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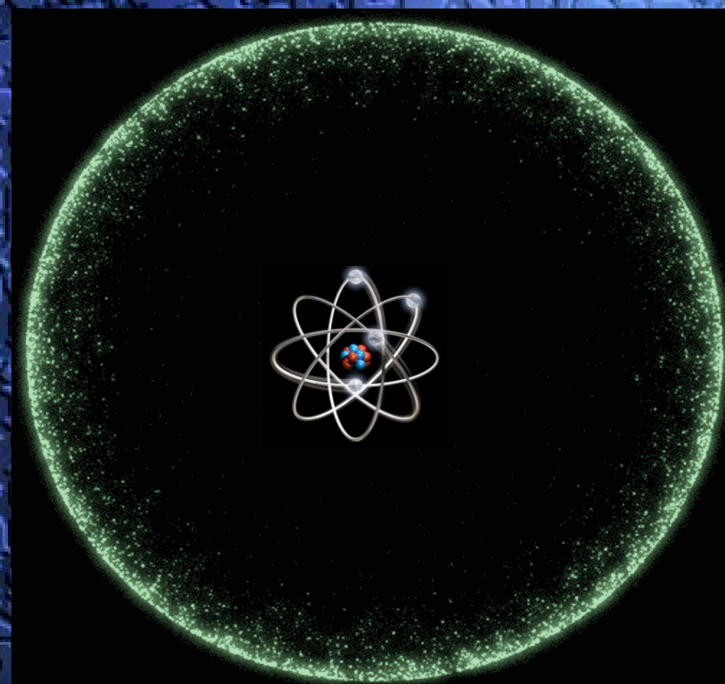

3rd CONFERENCE ON NONLINEARITY

"Nonlocal (but also nonsingular) physics at the last stages of gravitational collapse"

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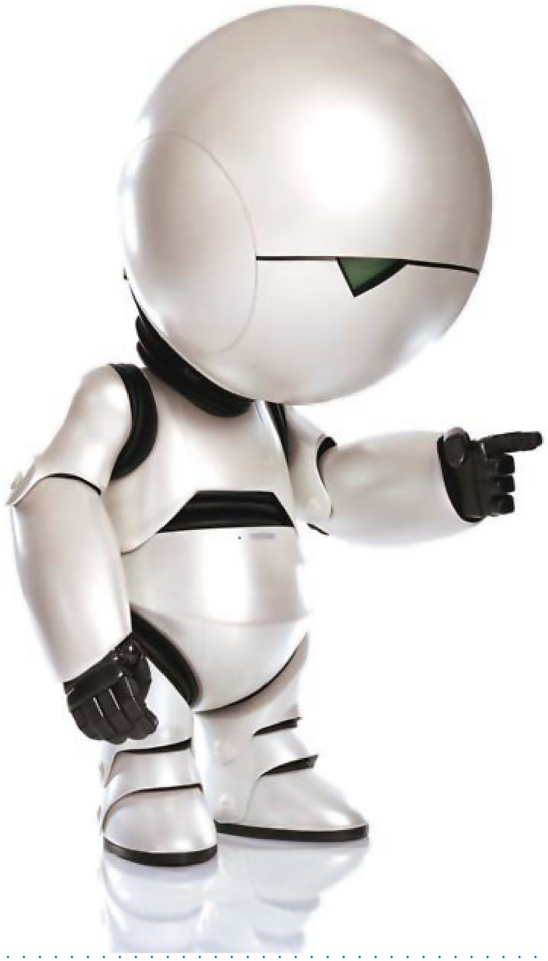
Phys.Rev. D89, 4, 044003

Motivation

Black Holes

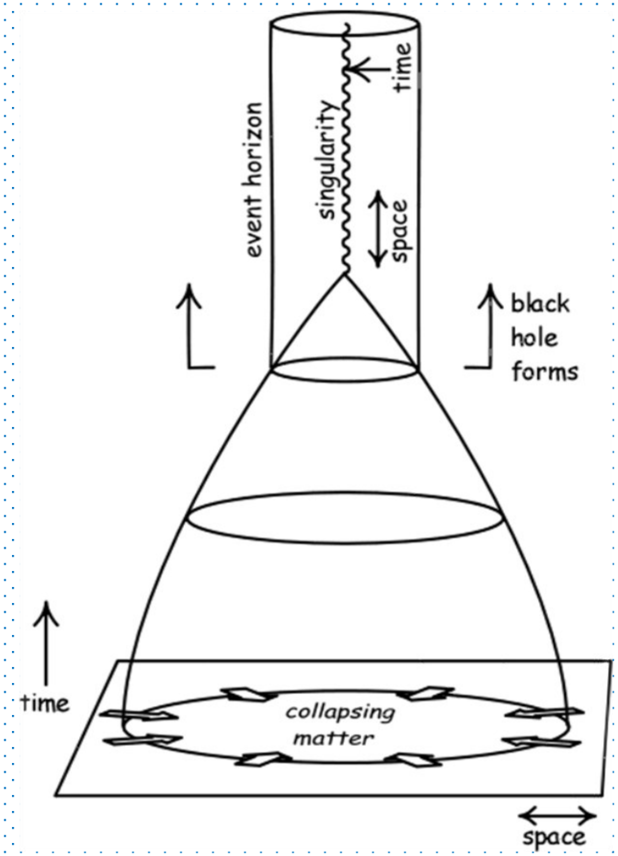
- Most interesting and intriguing objects in our Universe
- Singularity and horizon are roots of many problems
- Full QM treatment might resolve some of them
- A lot of words and nice pictures (complementarity, firewalls…)
- Very few concrete calculations

Outline



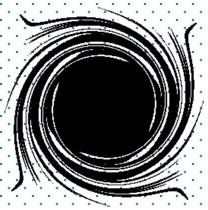
- Singularities – some basic facts
- Gravitational collapse – infalling observer - Schrodinger equation
- Non-local equation as $R \to 0$
- Non-singular wave function
- Quantization can remove singularities!

Gravitational collapse → singularity



Gravitational potential is singular as 1/r

Questions that need clear answer:



1. True singularity of the space-time?
2. We are extrapolating our theory beyond its region of validity?

Coulomb potential is also singular as 1/r

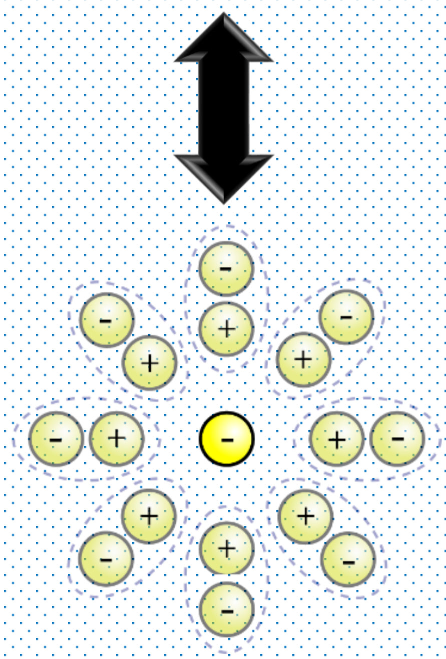
$$\frac{e^2}{r} \xrightarrow{r \to 0} \infty$$

Electron self-energy is divergent

$$\frac{e^2}{r} = m_e c^2$$

Something drastic should happen

$$r_c = \frac{e^2}{m_e c^2} = \underbrace{\frac{e^2}{\hbar c}}_{\alpha} \times \underbrace{\frac{\hbar}{m_e c}}_{r_0}$$



Singularity removed!

Gravitational potential is singular as 1/r

$$\frac{Gm_e^2}{r} \xrightarrow{r\to 0} \infty$$

Gravitational self-energy is divergent

$$\frac{Gm_e^2}{r} = m_e c^2$$

Something drastic should happen

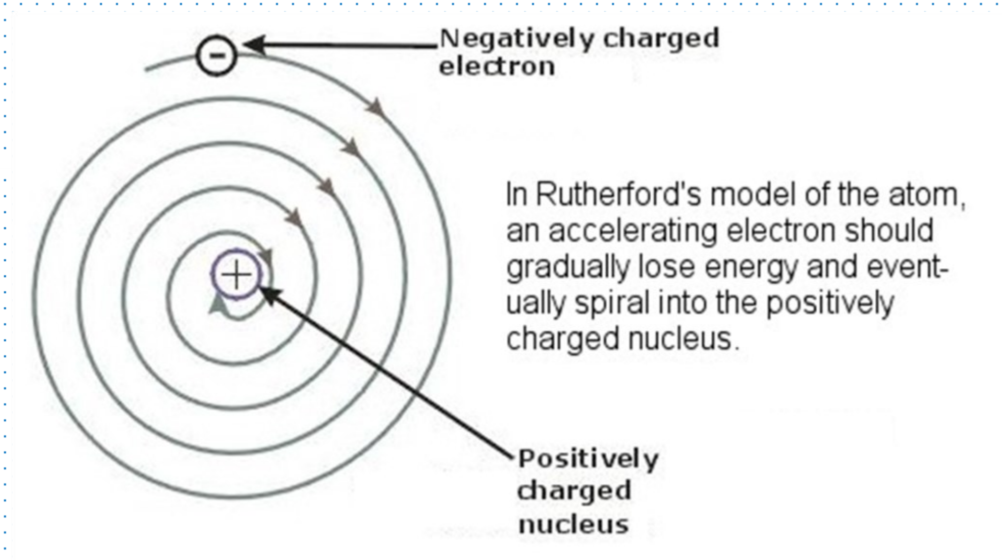
$$r_c = \frac{Gm_e}{c^2} = \frac{Gm_e^2}{\hbar c} \times \frac{\hbar}{m_e c}$$

$$\alpha_G = \left(\frac{m_e}{M_{Pl}}\right)^2 \qquad r_0$$

Singularity might be removed!

Coulomb potential is singular as 1/r

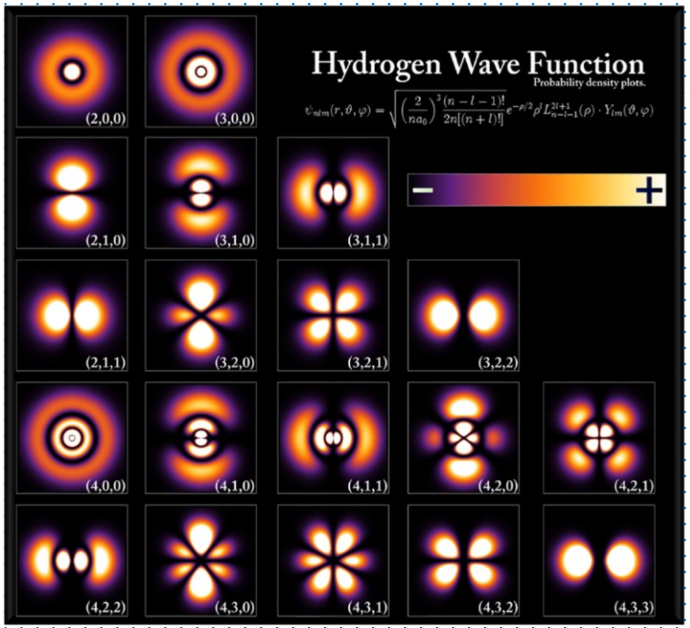
- Consequence: atom can't exist classically
- Electron would spiral down in a fraction of a second



- Quantization fixes the singularity and saves the atom

Hydrogen atom ground state wave function:

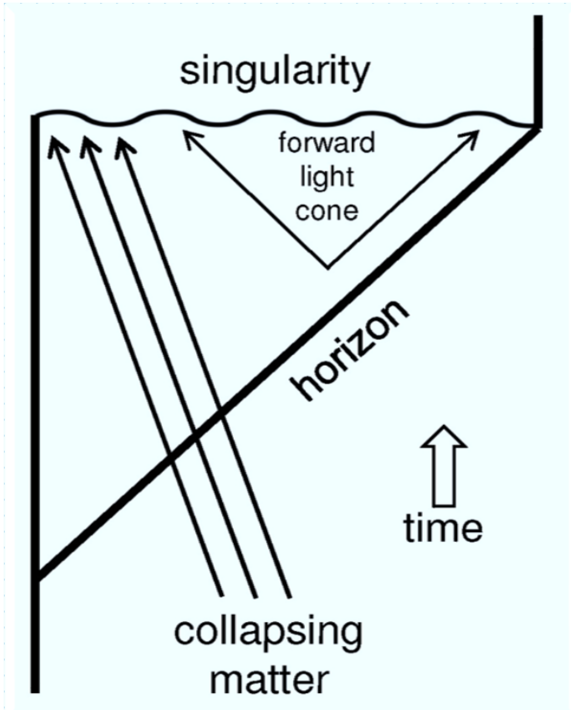
$$\psi(R) = const\ e^{-R/a_0} \xrightarrow{R \to 0} const$$



- Quantization can perhaps do the same in gravity

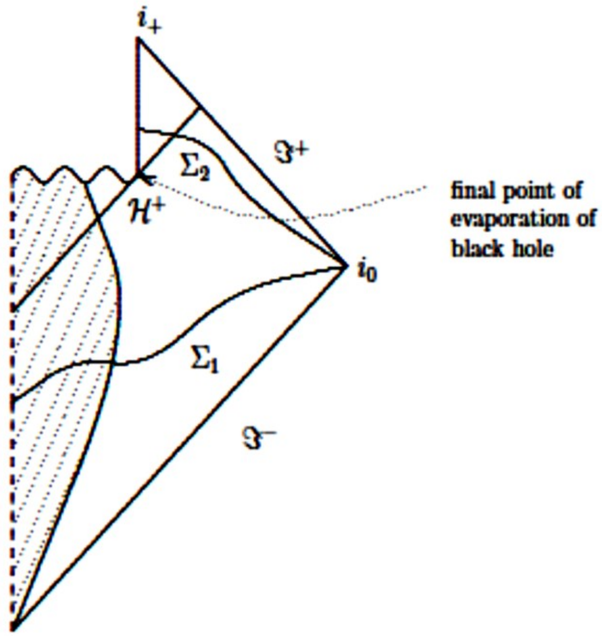
Benefits?

What do we gain by removing singularities from gravity?



- Black hole singularity makes the space-time incomplete
- Infalling observer reaches it in finite proper time

- Removing the singularity makes the space-time complete

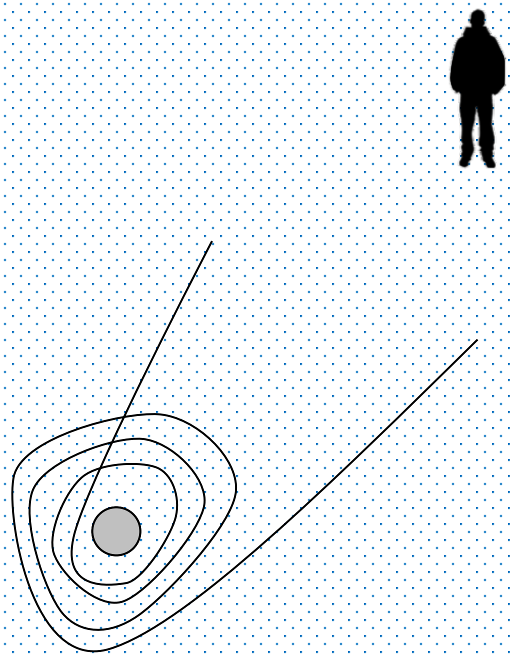


Standard Penrose diagram for evaporating black hole

- Σ_1 is the Cauchy surface but Σ_2 is not since its past domain of dependence does not include the black hole region (which disappears)
Evolution is not unitary!

- If we remove the singularity, the problem will perhaps be solved

- Quantum mechanical effects may turn the singularity into a regular region of strong gravity



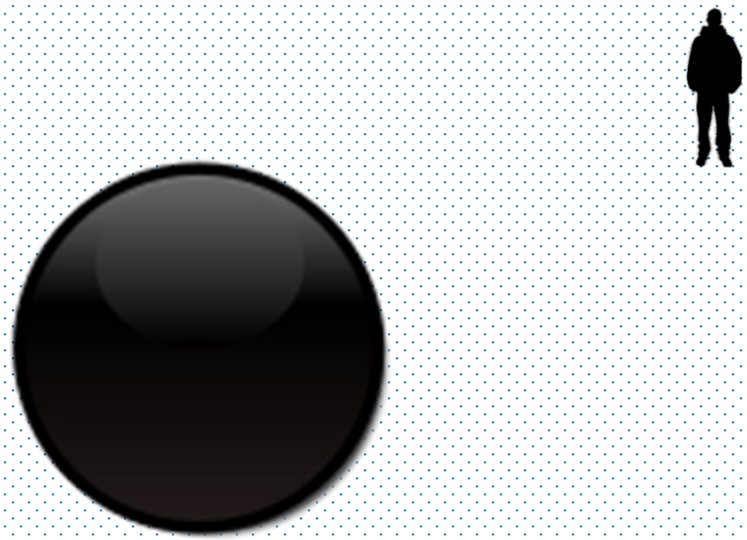
If singularity never forms, then the horizon can’t be a global event horizon!

It could trap light for a while, but when enough mass is lost to radiation, the light will be released out!

- The original black hole solution is Schwarzschild

$$ds^2 = -\left(1 - \frac{R_S}{r}\right) c^2 dt^2 + \frac{1}{1 - R_S / r} dr^2 + r^2 d\Omega^2$$

- It represents a point of view of a remote static observer



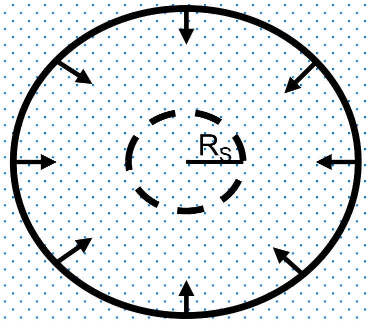
- However, we want to study the process of collapse from a point of view of an observer who can see past the horizon
- i.e. an infalling observer
- Classically, he can reach singularity in finite time
- and witness what is going on there

Question we explore:

Quantum aspects of gravitational collapse in the foliation of an infalling observer

The Setup

The collapsing spherical shell of matter is represented by an infinitely thin shell of mass M and radius R(t)



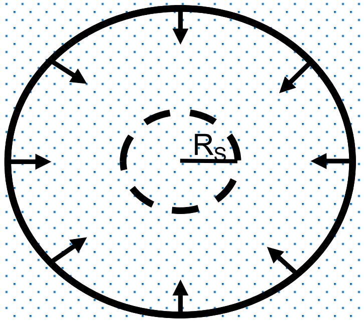
- **We work in Eddington-Finkelstein ingoing coordinates (t,r) → (v,r)**

$$v = t + r^* \qquad r^* = r + 2GM \ln\left|\frac{r}{2GM} - 1\right|$$

$$ds^2 = -\left(1 - \frac{R_S}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2$$

- **Metric is non-singular at the horizon**

Geometry of the problem



- **Metric outside the shell, i.e. for r > R(v), is Eddington-Finkelstein**

$$ds^2 = -\left(1-\frac{R_S}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2$$

- **Metric inside the shell, i.e. for r < R(v), is by Birkhoff's theorem flat**

$$ds^2 = -dT^2 + dr^2 + r^2 d\Omega^2$$

Use Gauss-Codazzi method to find conserved quantity:

$$M = \frac{\mu}{\sqrt{1 - R_T^2}} - \frac{\mu^2}{2R}$$

This quantity contains:

- rest mass of the shell $\mu = 4\pi\sigma R^2$, σ is the mass per unit area
- kinetic energy represented by $R_T \equiv \partial R / \partial T$
- gravitational self-energy $\mu^2/(2R)$

Actually, M is the total relativistic energy of the system

We identify M with the Hamiltonian of the system

$$H = \frac{\mu}{\sqrt{1 - R_T^2}} - \frac{\mu^2}{2R}$$

The corresponding action:

$$S = -\mu \int dT \left(\sqrt{1 - R_T^2} - \frac{\mu G}{2R} \right)$$

- we can now go from T → v (infalling time coordinate)
- and derive equations of motion

Exact expressions cumbersome

We are interested in behavior near the singularity, i.e. $R \to 0$

$$R_v \approx -\frac{1}{2} \left(\frac{\mu G}{R} \right)^2$$

Hamiltonian in the near singularity limit : $R \to 0$

$$H = -\mu R_v \left[\frac{1}{\sqrt{2|R_v|}} - \frac{\mu G}{2RR_v} \right]$$

$$\Pi = \mu \left[\frac{1}{\sqrt{2|R_v|}} + \frac{\mu G}{2R} \right]$$

$\Pi \to$ **momentum corresponding to the coordinate R**

Hamiltonian in terms of momentum:

Non-local Hamiltonian!

$$H = \frac{-R}{G}\left[1 - \frac{2\Pi R}{\mu^2 G}\right]^{-1} + \frac{\mu^2 G}{2R}$$

Radical Conservatism

Use the things that we understand well

And push them as far as we can…

We might get a glimpse at how the ultimate theory should look like

We are tempted to write down Schrodinger equation:

$$H\psi = i\hbar \frac{\partial \psi}{\partial v}$$

But Wheeler-de Witt says:

$$H\psi = 0$$

No global time in gravity!

Wheeler-de Witt: $$H_{tot}\psi_{tot} = 0$$

Decompose the Hamiltonin: $$H_{tot} = H_{sys} + H_{obs}$$

Introduce observer's time: $$H_{obs}\psi_{obs} = i\hbar\frac{\partial\psi_{obs}}{\partial t}$$

Assume the wave function separates: $$\psi_{tot} = \psi_{sys} \times \psi_{obs}$$

Schrodinger equation: $$H_{sys}\psi_{sys} = i\hbar\frac{\partial\psi_{sys}}{\partial t}$$

Observer's time

$$H = \frac{-R}{G}\left[1 - \frac{2\Pi R}{\mu^2 G}\right]^{-1} + \frac{\mu^2 G}{2R}$$

This Hamiltonian governs the evolution of the system as $R \to 0$

- **Quantization:**

$$\Pi = -i\hbar \frac{\partial}{\partial R}$$

Write down Schrodinger equation:

$$H\psi = i\hbar \frac{\partial \psi}{\partial v}$$

Wave function for the collapsing shell

$$\psi = \psi(R, v)$$

Dealing with the non-local Hamiltonian

• Isolate the non-local operator:

$$\hat{A} = \left[1 - \frac{2\Pi R}{\mu^2 G}\right]^{-1}$$

• Inverse is ok!

$$\hat{A}^{-1} = 1 - \frac{2\Pi R}{\mu^2 G}$$

Normal ordering:

$$\hat{A}^{-1} = 1 - \frac{1}{\mu^2 G}\left(\Pi R + R\Pi\right)$$

• Define action of A

$$\hat{A}\psi = \varphi \qquad \text{or} \qquad \hat{A}^{-1}\varphi = \psi$$

$\hat{A}^{-1}\varphi = \psi$ **becomes**

$$\frac{\partial \varphi}{\partial R} + \frac{1}{2R}\left(1 - i\mu^2 G\right)\varphi + \frac{i\mu^2 G}{2R}\psi = 0$$

$$\frac{\partial\varphi}{\partial R}+\frac{1}{2R}\left(1-i\mu^2G\right)\varphi+\frac{i\mu^2G}{2R}\psi=0$$

Solution:

$$\varphi=-\frac{i\mu^2G}{2}\frac{\int R^{-\frac{1+i\mu^2G}{2}}\psi\,dR}{R^{-\frac{1+i\mu^2G}{2}}}$$

$$\varphi=\hat{A}\psi$$

Placeholder for a function on which A is acting

Action of A:

$$\hat{A}=-\frac{i\mu^2G}{2}\frac{\int R^{-\frac{1+i\mu^2G}{2}}(\ldots)\,dR}{R^{-\frac{1+i\mu^2G}{2}}}$$

Total Hamiltonian:

$$H=-\frac{R\hat{A}}{G}+\frac{\mu^2G}{2R}$$

Operator ordering:

$$H=-\frac{1}{2G}\left(R\hat{A}+\hat{A}R\right)+\frac{\mu^2G}{2R}$$

Hamiltonian: $$H = -\frac{1}{2G}\left(R\hat{A} + \hat{A}R\right) + \frac{\mu^2 G}{2R}$$

Ansatz: $$\psi(R,v) = \psi(R)e^{iEv/\hbar}$$ i.e. stationary solutions

Schrodinger equation: $$H\psi = i\hbar\frac{\partial\psi}{\partial v}$$

Contains integral $$\hat{A} = -\frac{i\mu^2 G}{2}\frac{\int R^{-\frac{1+i\mu^2 G}{2}}(\ldots)\,dR}{R^{-\frac{1+i\mu^2 G}{2}}}$$

Explicitly: $$\frac{-1}{2G}\left(R\hat{A}\psi - \frac{i\mu^2 GR}{i\mu^2 G - 3}\psi\right) + \frac{\mu^2 G}{2R}\psi = E\psi$$

Take a derivative: $$\frac{d\psi}{\psi} = \int f(R)\,dR$$

Keeping only the leading terms in R → 0 limit:

$$\ln\psi \approx \int \frac{3+i\mu^2 G}{2R}\,dR$$

Solution: $\psi(R) = \lambda R^{\frac{3+i\mu^2 G}{2}}$ λ is a constant

Probability density: $|\psi(R)|^2 = \lambda^2 R^3$

Obviously non-singular in R → 0 limit!

Comparison with the hydrogen atom

Hydrogen ground state wave function:

$$|\psi(R)|^2 = const\; e^{-2R/a_0} \xrightarrow{R\to 0} const$$

Collapsing shell case:

$$|\psi(R)|^2 = const\; R^3 \xrightarrow{R\to 0} 0$$

The shell has zero probability to be found at R =0

Quantum effects might help resolving singularities in gravity

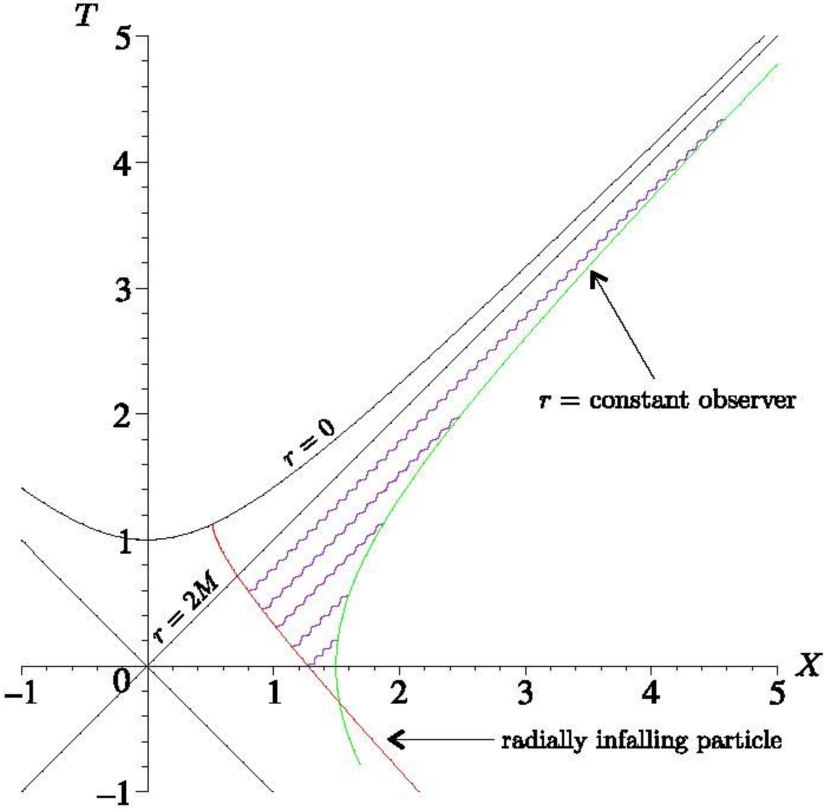
Conclusions

Solving Schrodinger equation for gravitational collapse we learned:

1. Quantum effects might be able to remove the singularity

2. Physics becomes non-local in strong gravity regime

THANK YOU



T
5
4
3
2
1
0
−1
−1
1
2
3
4
5
X
r = 0
r = 2M
r = constant observer
radially infalling particle