

Nonlinear density waves in galactic dynamics

Can the nonlinearity overcome missing mass?

Observations

Theoretical models

Nonlinearity working in the galaxy

Restoring the flat rotation curve

Future research

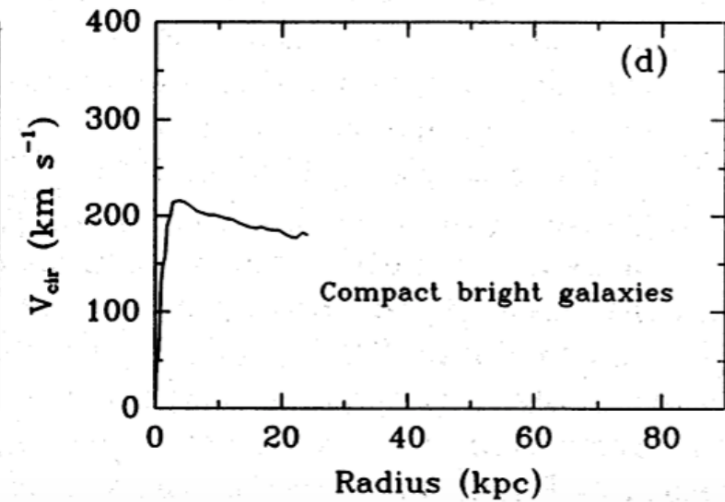
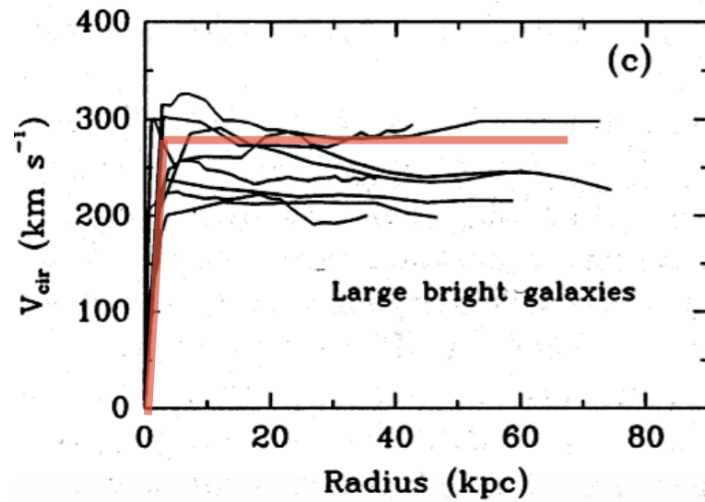
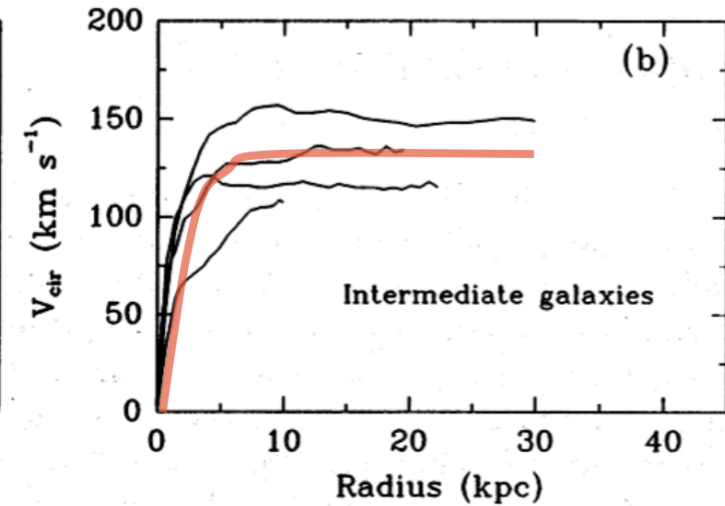
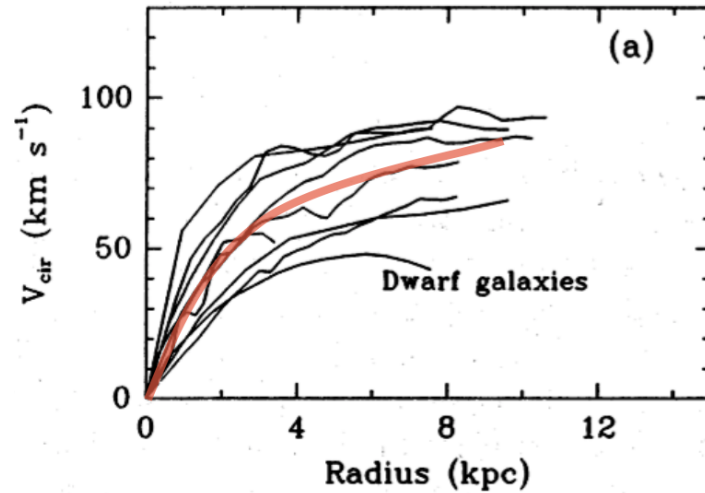
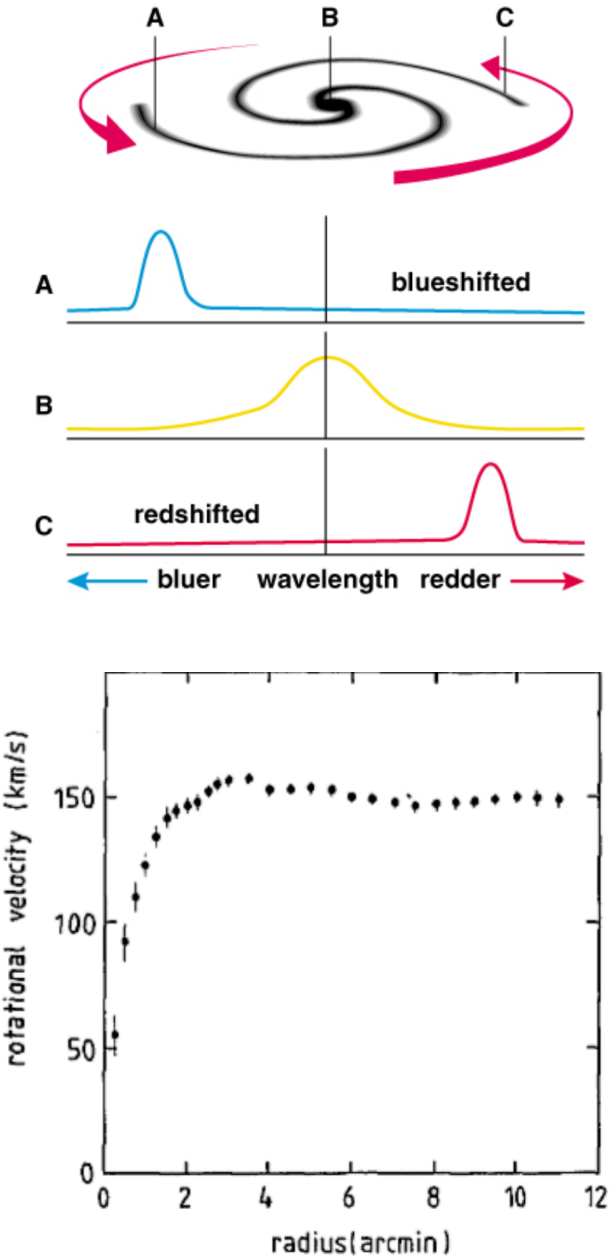
Miroslava Vukcevic

Belgrade, 2023

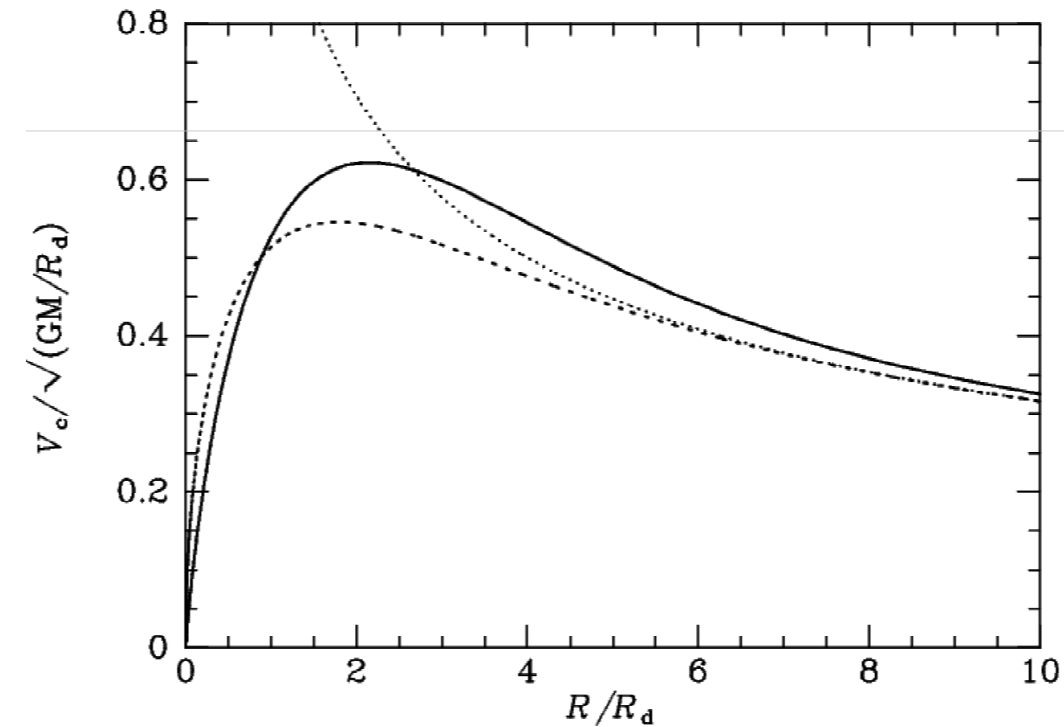


Observations

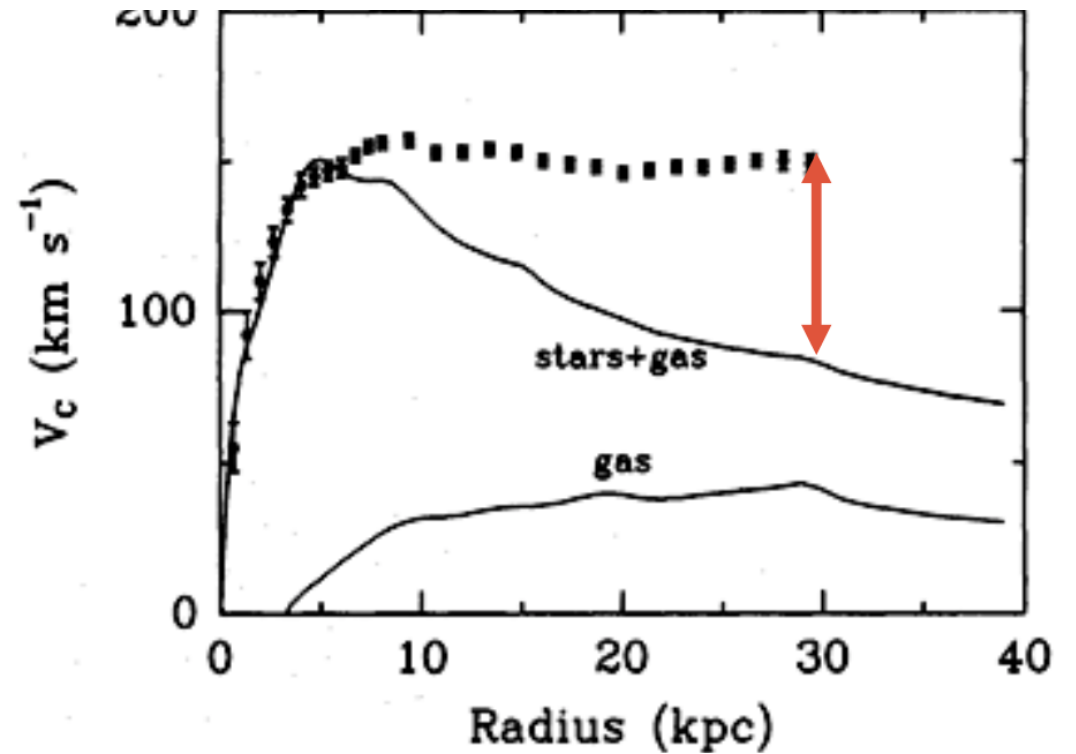
Rotational velocity curve of spiral galaxies



Theoretical models



Circular velocity for: an exponential disk (solid);
a spherical body with the same mass (dashed);
a point mass (dotted)



NGC 3198

Nonlinearity working in the galaxy



Infinitesimally thin disk approximation

Lin&Shu 1964

$$\frac{\partial \phi(r, z = 0)}{\partial r} = \pm 2\pi i G \sigma$$

Linearized \rightarrow dispersive waves \rightarrow winding dilemma



Fluid description based on Lin & Shu but with nonlinear terms

Reductive perturbation method (Jaffrey & Taniuti 1964)

Equilibrium + **stretched coordinates** + variable expansion

Linear dispersion relation → **marginal stability of the disk**

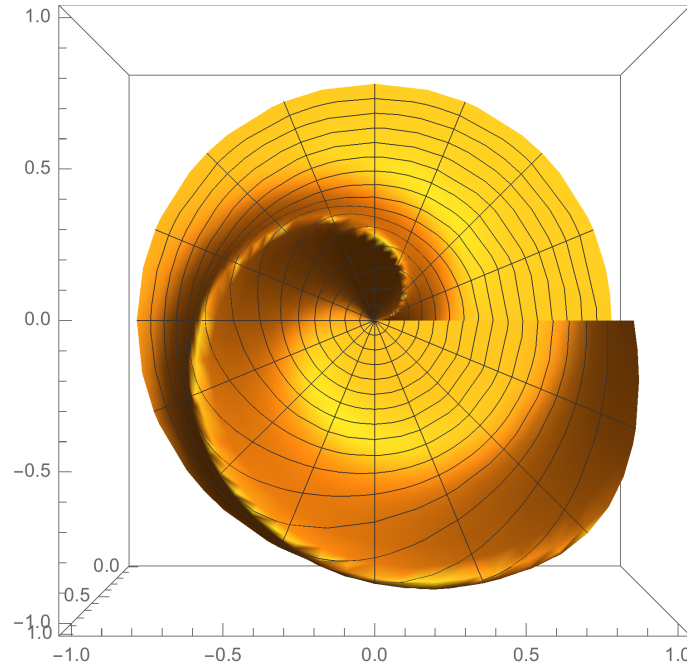
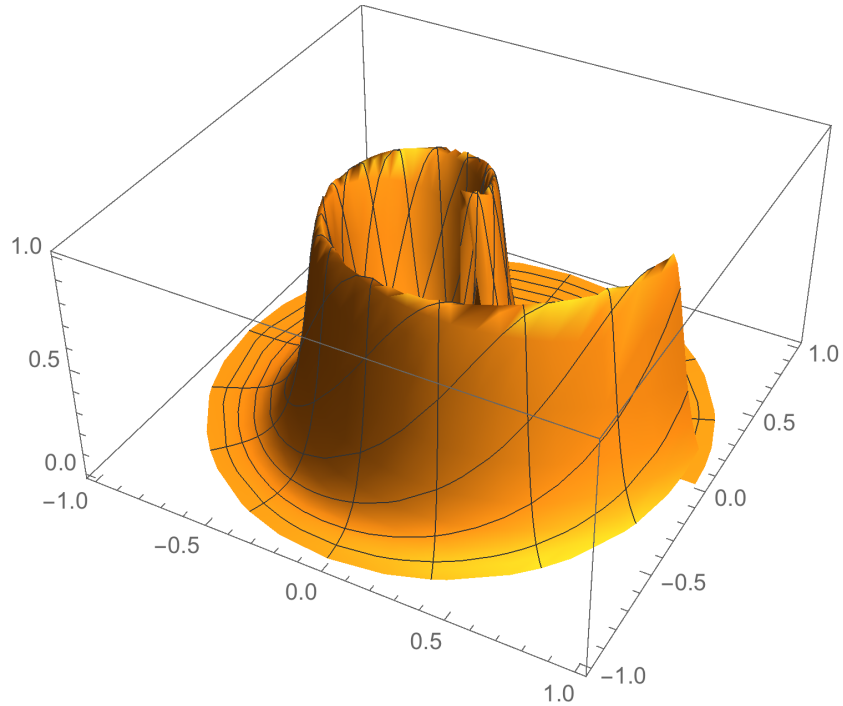
Frequency goes to zero → group velocity tends to infinity (Watanabe 1969)

$$i \frac{\partial}{\partial \eta} \rho^{1,1} + P \frac{\partial^2}{\partial \xi^2} \rho^{1,1} + Q |\rho^{1,1}|^2 \rho^{1,1} = 0.$$

$$\rho^{1,1}(\xi, \eta) = \rho_a \frac{e^{i\psi}}{\text{ch}\left(\sqrt{\frac{Q}{P}} \rho_a (\xi - P\eta)\right)}.$$

Soliton \rightarrow constant amplitude and group velocity due to balance in dispersion and nonlinearity

One-dimensional curved wave \rightarrow moving along the spiral



Vukcevic 2014, MNRAS

Back to the Rotation Curve of the galaxy (P. Jovanovic and E. Bon)

$$V^2(r) = r \frac{\partial \phi}{\partial r}, \quad \frac{\partial \phi}{\partial r} = r\Omega^2 + \sum_{n=1}^{\infty} \sum_{m=-\infty}^{\infty} 2\pi G \epsilon^n \Re(\rho^{(n,m)}(\xi, \eta) e^{i(kr - \omega\tau)}),$$

$$\rho^{1,1}(\xi, \eta) = \rho_a \frac{e^{i\psi}}{\text{ch}(\sqrt{\frac{Q}{P}} \rho_a (\xi - P\eta))}$$

$$V(r) = \sqrt{\Omega^2 r^2 + \frac{ar}{\cosh b(T - cr)}}$$

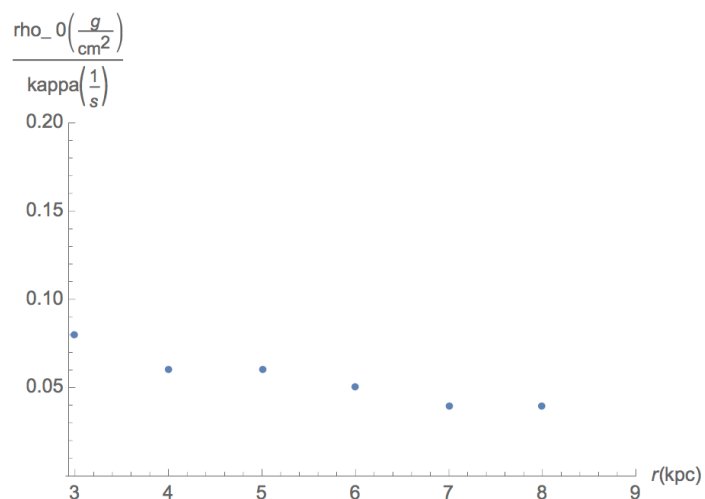
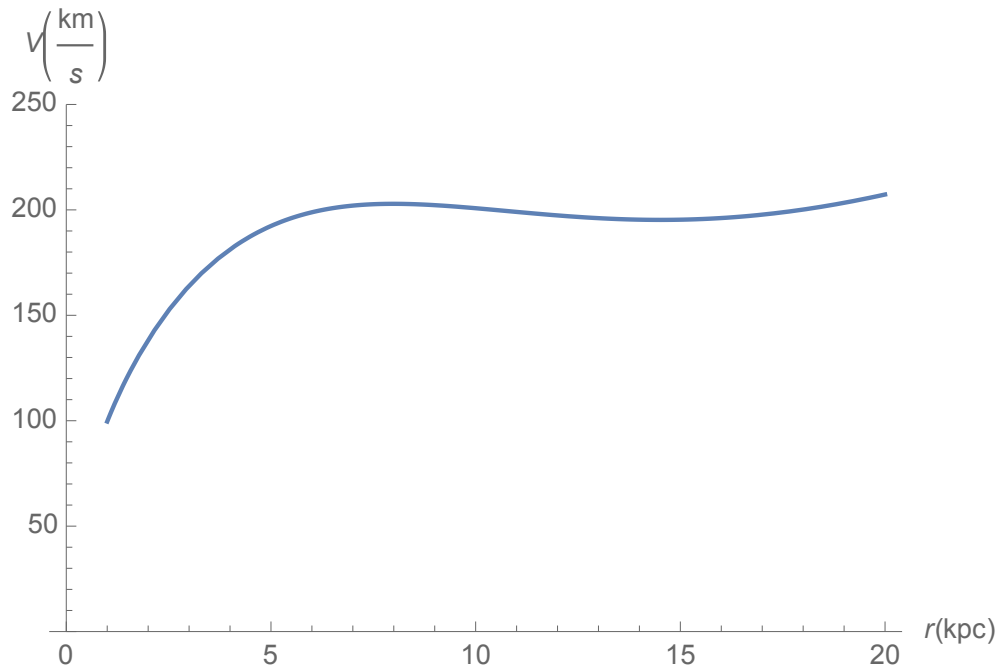
$$a = 2\pi G \rho_0 \rho_a [km/s^2]$$

$$b = \kappa \rho_a [1/s]$$

$$c = 1/V_g [s/km]$$

$$V_g = \pi G \rho_0 / \kappa \sim 200 km/s : \quad (\kappa \sim 10^{-15} 1/s, \rho_0 \sim (4 - 6) \times 10^{-2} g/cm^2 = (200 - 300) M_{\odot}/pc^2).$$

Restoring the flat rotation curve



Surface mass density $\rho_0=0.045g/cm^2=200M_{\odot}/pc^2$

Angular velocity $\Omega \sim 30 \times 10^{-16} 1/s$ (Luna et al. 2006)

$V_g=200km/s$

$\Omega^2=90 [km/(s kpc)]^2$ $a=6 \times 10^3$ $b=4.5 \times 10^{-16}$ $c=4 \times 10^{14}$

$\Omega=\Omega(r)$

$\rho_0=\rho_0(r)$

$\kappa=\kappa(r)=V2\Omega$

$a, b, V_g \sim \rho_0/\kappa=const!$

Corrections:

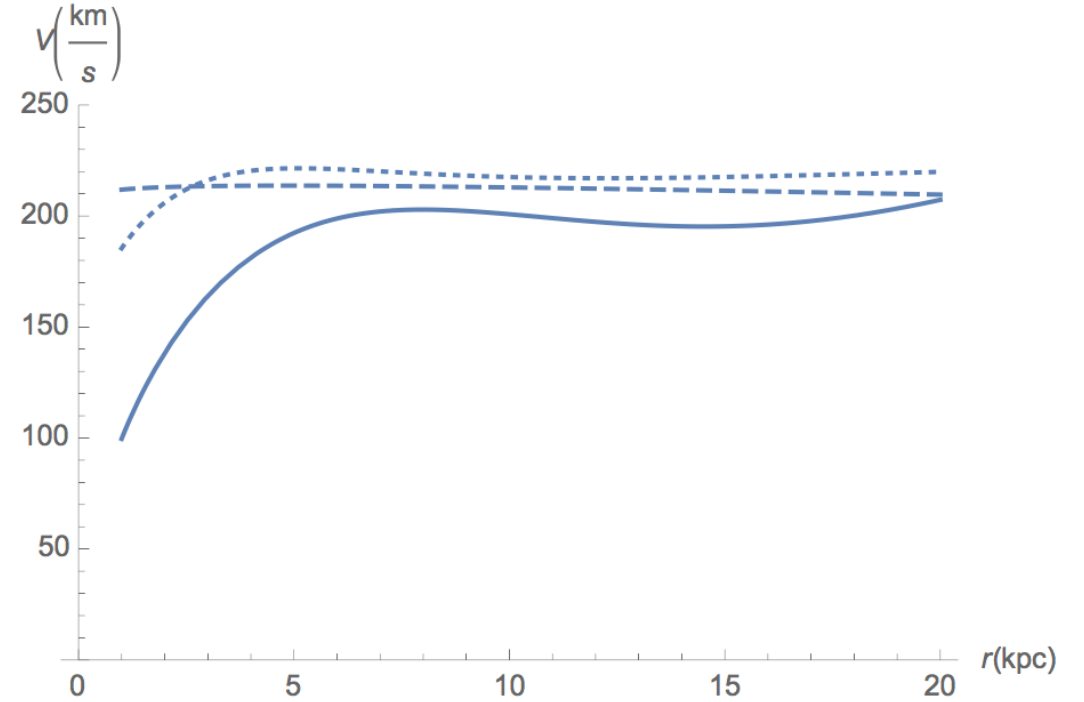
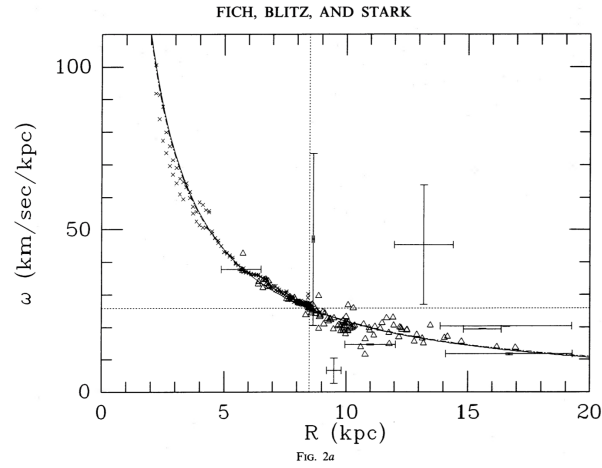
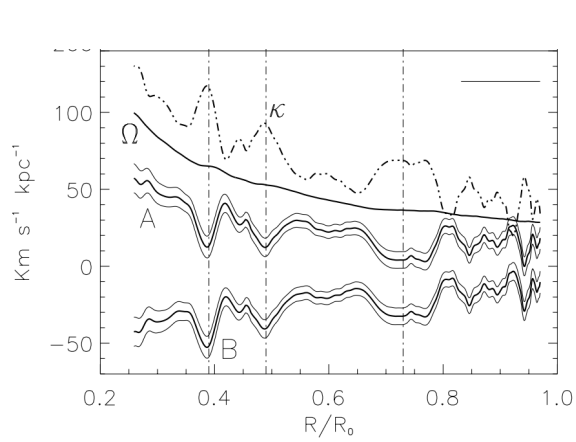
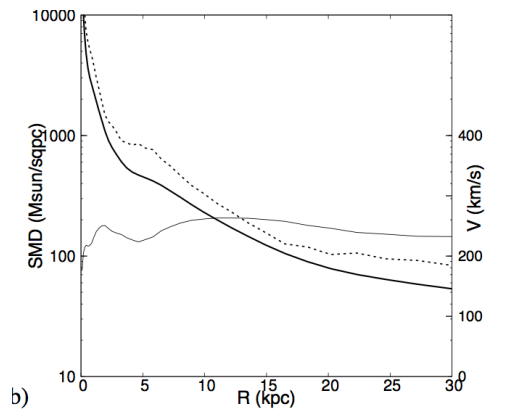
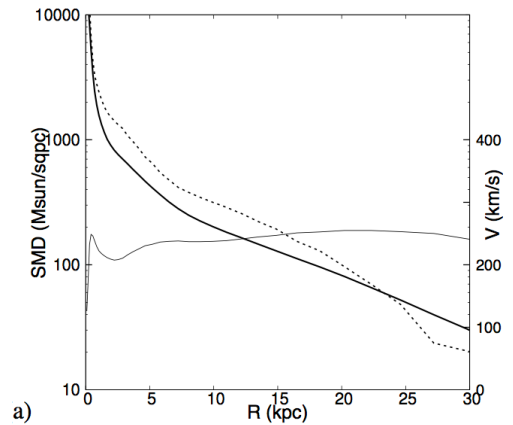
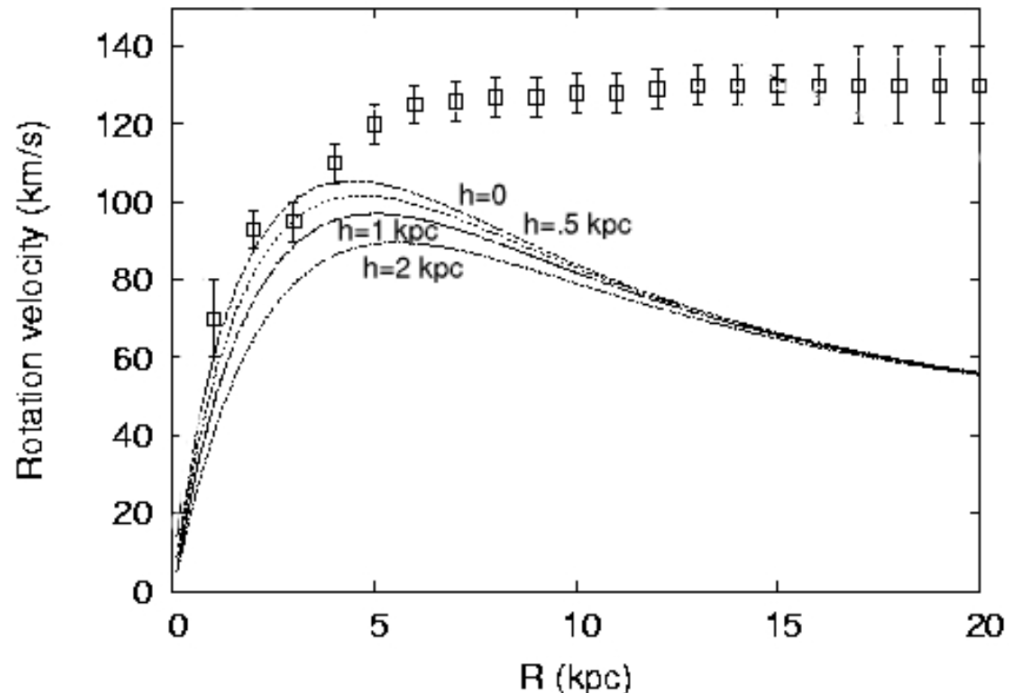


Figure 1. Rotational velocity curve for $\Omega = const.$ presented by solid line. Dotted line is rotational velocity curve for power-law fit given by Eq. (22) in Fich et al. (Fich et al. 1989), namely $\Omega(r) \sim r^{-0.9}$, while dashed line is result obtained from Eq. (6) taking all variables r dependent ($\rho(r) \sim r^{(-0.7)}$); power-law for SMD is approximated according to result obtained by Sofue for Milky Way and M31 (Sofue 2018), and $\Omega(r) \sim r^{-0.9}$.



Vukcevic 2021, AJ

Thickness effects:



V_g overestimated due to infinitesimally thin disk approximation

but it can be compensated by neglected gas contribution

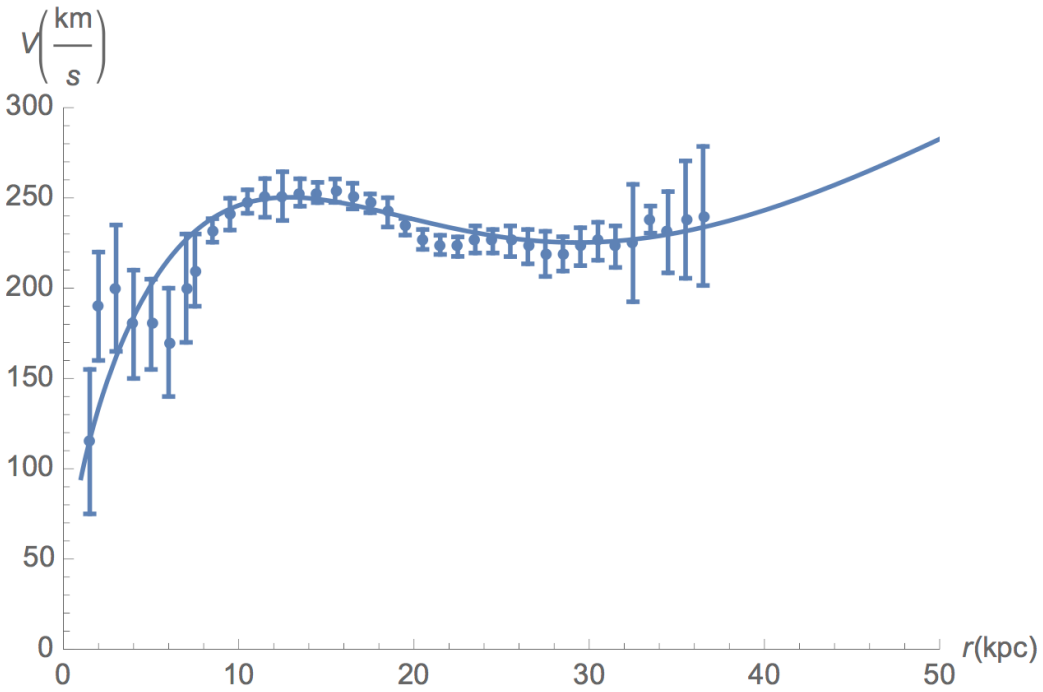


Figure 6. The rotational velocity curve for the galaxy M31. Dots are observed data [45], and the solid curve is the result of Equation (9) for the following parameters: $a = 8 \times 10^3$, $b = 0.03$, $c = 3.5$.

Vukcevic 2022, Universe

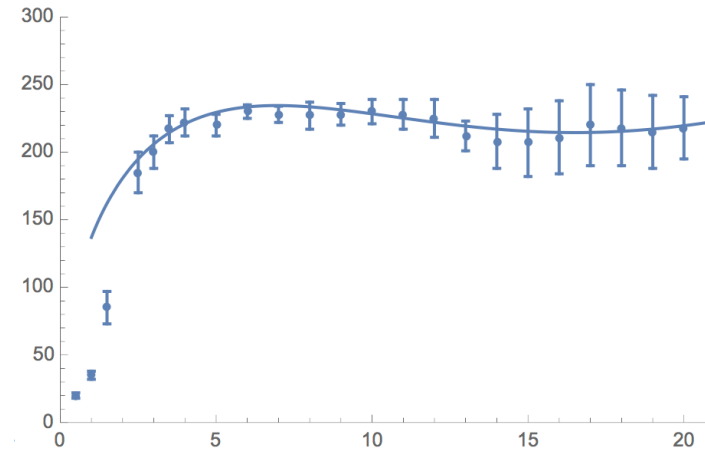
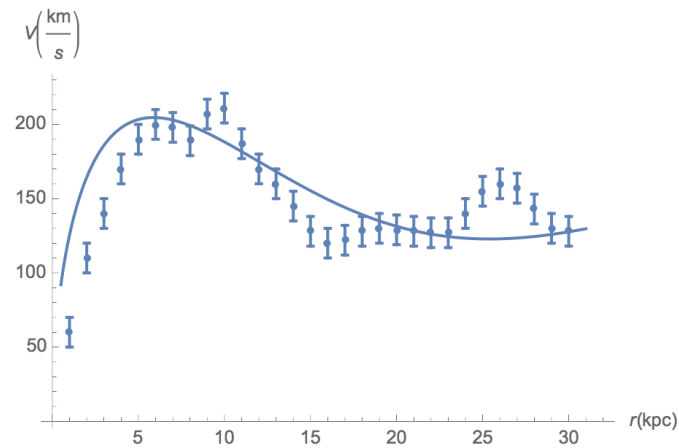


Fig. 7: RC NGC 3521. dots observations (de Blok et al. 2008); solid line: $\Omega^2=80$, $a=6.5 \times 10^3$, $b=4.2 \times 10^{-16}$, $c=3.8 \times 10^{14}$, for observed values $\Omega=28 \times 10^{-16}$, $\rho_0=4.3 \times 10^{-2} \text{g/cm}^2$, $V_g=220 \text{km/s}$



Peculiar NGC 157
 $\Omega=7 \times 10^{-16} 1/s$, $\rho_0=2.4 \times 10^{-2} \text{g/cm}^2$, $V_g=180 \text{km/s}$
 $\Omega^2=15$, $a=6 \times 10^3$, $b=3.8 \times 10^{-16}$, $c=3.6 \times 10^{14}$

Further research

Numerical simulations: N-body simulations V. Zekovic & M. Radeta (Vukcevic et al. 2021, AN)

Indication that spirals evolve into ring → implication on the galaxy evolution

Gas contribution: implication on the accretion disk dynamics and dwarf gas rich galaxies

Analysis of elliptical galaxies dynamics

Baryonic Tully-Fisher relation by fitting as many as possible spiral galaxies: Dj. Savic

Larger scale nonlinearity contribution – galaxy clusters

Before involving some exotic undetectable matter it is always useful to reinvestigate assumptions done in theoretical models (if observations are trustful)

Thank you for your attention!

